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The Transient Force History on a Body Started From Rest

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1 Introduction

The question of how rapidly transients associated with the abrupt motions of a floating body decay is one of fundamental theoretical interest as well as practical importance. The rate at which transient oscillations vanish and measurements taken is of some concern in model tests especially for unsteady and local effects. The question of the behavior of transients comes up also in almost all numerical simulations in the time domain and directly affects our ability to extract steady-state predictions for resistance problems and to obtain meaningful results for general seakeeping problems.

Despite the obvious importance, the problem appears to have been addressed only for the idealized case of a single translating source of *known* strength. Havelock (1949) considered the two-dimensional problem of the wave resistance of a submerged circular cylinder impulsively started from rest. By approximating the body as a point dipole of constant strength, he derived a closed-form solution for the wave resistance. The significant finding is that for a given forward speed U, the resistance oscillates about the steady value with the frequency $\omega_c = g/4U$, where g is the gravitational acceleration, and the oscillation decays only like $t^{-\frac{1}{2}}e^{i\omega_c t}$ as $t \to \infty$. This result was extended to three dimensions by Wehausen (1964) who considered a constant source started abruptly and obtained that the unsteady resistance vanishes like $t^{-1}e^{i\omega_c t}$ as $t \to \infty$.

The above results can be understood by considering the associated classical seakeeping problem in the frequency domain, wherein it is known that (for a single source) the solution is in fact singular at the frequency $U\omega_c/g = \frac{1}{4}$ (Haskind 1954). Recently, we (Liu & Yue 1993) showed that for an actual body the solution at the critical frequency is finite for a general class of bodies (*admissible* bodies) subject to a single geometric condition. An immediate consequence of this finding is that the decay rate of transients must necessarily be an order faster than the single-source predictions of Havelock (1949) and Wehausen (1964) for this class of geometries.

In this work, we consider the starting from rest to steady speed of a general body. To solve the problem, we use a transient source distribution on the wetted surface of the body, where in general the singularity strengths are unsteady and part of the solution of the problem. Our analyses show that for *admissible* bodies, the transient decay rate is indeed more rapid than for constant strength isolated singularities, with the unsteady wave resistance in two and three dimensions decaying respectively like $O(t^{-1}, t^{-\frac{3}{2}}e^{i\omega_c t})$ and $O(t^{-2}, t^{-2}e^{i\omega_c t})$ as $t \to \infty$. These results are substantially confirmed by carefully controlled numerical simulations in the time domain.

2 The Transient Solution of a Body Started From Rest

As with the demonstration for the frequency-domain problem (Liu & Yue 1993), the transient analysis is general for submerged and floating bodies with the extra complication in the latter of a waterline integral. For simplicity, we focus in this presentation on the case of a submerged body only. We construct the velocity potential, $\Phi(\vec{x},t)$, in terms of a source distribution:

$$\Phi(\vec{x},t) = \int_{S_B} \Psi(\vec{x},\vec{x}',\sigma(\vec{x}',t))\,\mathrm{d}s' , \tag{1}$$

where $\vec{x}$ represents the body-fixed Cartesian coordinates, $\sigma(\vec{x}',t)$ the source distribution on the body, and Ψ the single-source potential. Unlike Havelock (1949) and Wehausen (1964), we allow the source strength σ to be time-dependent and as yet unknown. For convenience, we decompose $\sigma(\vec{x}',t)$ into steady and unsteady components:

$$\sigma(\vec{x}',t) = \bar{\sigma}(\vec{x}') + \check{\sigma}(\vec{x}',t) + \hat{\sigma}(\vec{x}',t) \tag{2}$$

where $\bar{\sigma}$ is independent of time, and $\check{\sigma}$ is assumed to be a continuous function of time for $t \in (0,t_0)$ and vanishes for $t > t_0$ while $\hat{\sigma}$ is identically zero for $t \in (0,t_0)$ and decays smoothly as $t \to \infty$. With (2), the potential in (1) can then be rewritten as:

$$\Phi(\vec{x},t) = \int_{S_B} \Psi(\vec{x},\vec{x}',\bar{\sigma}(\vec{x}'))\,\mathrm{d}s' + \int_{S_B} \Psi(\vec{x},\vec{x}',\check{\sigma}(\vec{x}',t))\,\mathrm{d}s' + \int_{S_B} \Psi(\vec{x},\vec{x}',\hat{\sigma}(\vec{x}',t))\,\mathrm{d}s' . \tag{3}$$

For a given source strength, the source potential Ψ is known and its asymptotic time dependence for large time can be explicitly extracted. Upon using large-time expansions of $\Psi(\vec{x},\vec{x}',\bar{\sigma}(\vec{x}'))$ and $\Psi(\vec{x},\vec{x}',\check{\sigma}(\vec{x}',t))$ in (3), we obtain

$$\Phi(\vec{x},t) = \int_{S_B} \bar{\sigma}(\vec{x}')\bar{G}(\vec{x},\vec{x}')\,\mathrm{d}s' + \int_{S_B} \Psi(\vec{x},\vec{x}',\hat{\sigma}(\vec{x}',t))\,\mathrm{d}s' + \frac{\alpha \mathrm{e}^{-\mathrm{i}\omega_c t}}{t^\gamma} \mathrm{e}^{\kappa(\mathrm{i}x+z)} + \text{c.c.} + (\text{h.o.t}) \tag{4}$$

as $t \to \infty$. In the above, the wavenumber $\kappa = g/(4U^2)$, $\gamma = \frac{1}{2}$ and 1 for two- and three-dimensional bodies respectively, and $\bar{G}$ is the steady source potential. Here, c.c. represents the complex conjugate of the preceding term, and (h.o.t.) higher-order terms. The constant α is the Kochin function and depends on the source strengths $\bar{\sigma}$ and $\check{\sigma}$. Note that if $\hat{\sigma} = 0$, the results of Havelock (1949) and Wehausen (1964) directly follow from (4). In order to find the exact decay rate of the unsteady potential in (4), it is necessary to determine the time dependence of the unsteady source $\hat{\sigma}$ first. To do that, we impose the body boundary condition on Φ in (4) and obtain an integral equation for the unsteady source $\hat{\sigma}$:

$$\pi\hat{\sigma}(\vec{x},t) + \int_{S_B} \Psi_n(\vec{x},\vec{x}',\hat{\sigma}(\vec{x}',t))\,\mathrm{d}s' + \frac{\mathrm{e}^{-\mathrm{i}\omega_c t}}{t^\gamma}\alpha\kappa(\mathrm{i}n_x + n_z)\mathrm{e}^{\kappa(\mathrm{i}x+z)} + \text{c.c.} = (\text{h.o.t}) \tag{5}$$

as $t \to \infty$ for $\vec{x} \in S_B$.

Based on the large-time asymptotic expansion of the single-source potential $\Psi(\vec{x},\vec{x}',t^{-\gamma}\mathrm{e}^{-\mathrm{i}\omega_c t})$, we can show that the solution of (5) depends on the geometric parameter (Liu & Yue 1993):

$$\Gamma = \int_{S_B} n_z \mathrm{e}^{2\kappa z}\,\mathrm{d}s . \tag{6}$$

If $\Gamma \neq 0$, we find that the Kochin function $\alpha = 0$, and the unsteady source $\hat{\sigma}(\vec{x}',t) = O(t^{-1}, t^{-\frac{3}{2}}\mathrm{e}^{-\mathrm{i}\omega_c t})$ and $O(t^{-2}, t^{-2}\mathrm{e}^{-\mathrm{i}\omega_c t})$ for two- and three-dimensional bodies respectively. Thus, the unsteady potential in (4) decays like $O(t^{-1}, t^{-\frac{3}{2}}\mathrm{e}^{\mathrm{i}\omega_c t})$ in two dimensions and $O(t^{-2}, t^{-2}\mathrm{e}^{\mathrm{i}\omega_c t})$

in three dimensions. If $\Gamma = 0$, on the other hand, the unsteady source $\hat{\sigma}$ is at least $O(t^{-\gamma}e^{-i\omega_c t})$ so that the unsteady potential in (4) may not decay at all.

As mentioned earlier, the above analysis and results can be generalized to surface-piercing bodies by including the contribution of the waterline source distribution in (1). Furthermore, by using Fourier transformation, the present results have also been extracted from the frequency domain and in particular from the behaviors of the solution near $\omega = 0$ and ω_c.

Note that the necessary and sufficient condition for $\Gamma \neq 0$ for a submerged body is that it has non-zero volume. For surface-intersecting bodies, the condition $\Gamma \neq 0$ has a simple geometric interpretation similar to that of John (1950). Details can be found in Liu & Yue (1993).

3 Numerical Confirmation

We confirm the above results through direct long-time numerical simulations in the time domain. The first problem we consider is that of a two-dimensional submerged circular cylinder started impulsively from rest to constant forward speed. The numerical code is a forward-speed extension of our spectral method (Liu, Dommermuth & Yue 1992) which provides efficient high-resolution transient results. Figure 1 shows the comparison between the numerical result and the fitted asymptotic solution based on the above analysis for the unsteady wave resistance on the body. The behavior of the decaying transient solution is well corroborated. Figure 2 shows the time-dependent behavior of the source strength on the body. For simplicity, only the first (circumferential) Fourier mode is plotted. The comparison between theoretical prediction and numerical result is excellent.

As a second problem we consider the unsteady resistance of a surface-piercing three-dimensional body. Specifically, we choose a Wigley hull at a Froude number of 0.15. The numerical simulation is performed using a time-domain transient Green function method of Lin & Yue (1990). The comparison between the theoretical asymptotic solution and numerical calculation for the time-dependent resistance is shown in figure 3. The agreement is again excellent and confirms the $O(t^{-2})$ approach to steady-state resistance.

4 References

HASKIND, M.D. 1954 On wave motion of a heavy fluid. *Prikl. Mat. Mekh.* **18**, 15-26.

HAVELOCK, T.H. 1949 The wave resistance of a cylinder started from rest. *Quarterly Journal of Mechanics and Applied Mathematics* **2**, 325-334.

JOHN, F. 1950 On the motion of floating bodies. Part II. *Comm. Pure Appl. Math.* **3**, 45-101.

LIN, W.M. & YUE, D.K.P. 1990 Numerical solutions for large-amplitude ship motions in the time-domain. *Proc. 18th Symp. Naval Hydro.* U. Michigan, Ann Arbor, MI, USA.

LIU, Y. & YUE, D.K.P. 1993 On the solution near the critical frequency for an oscillating and translating body in or near a free surface. *J. Fluid Mech.* **254**, 251-266.

LIU, Y., DOMMERMUTH, D.G. & YUE, D.K.P. 1992 A high-order spectral method for nonlinear wave-body interactions. *J. Fluid Mech.* **245**, 115-136.

WEHAUSEN, J.V. 1964 Effect of the initial acceleration upon the wave resistance of ship models. *J. Ship Res.* **7**, 38-50.

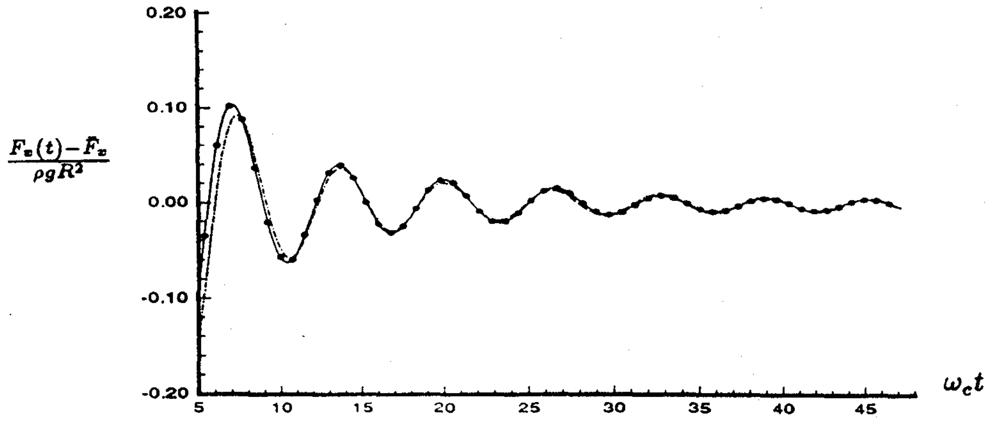

Figure 1: Comparison between numerical simulation result (•) and fitted asymptotic solution (— · —), $U/(gR)^{\frac{1}{2}} = 1.$, $H/R = 2.$, R: radius, H: submergence.

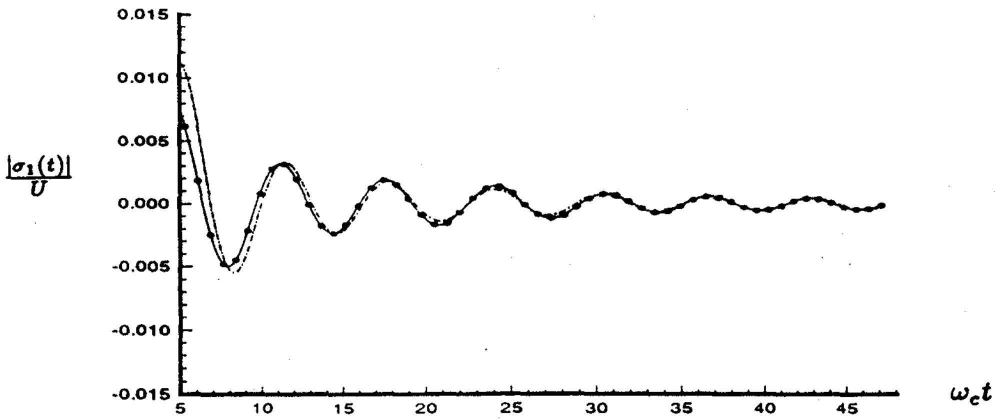

Figure 2: Comparison between numerical result (•) and fitted asymptotic solution (— · —) for the source distribution on the cylinder, $U/(gR)^{\frac{1}{2}} = 1.$, $H/R = 2.$.

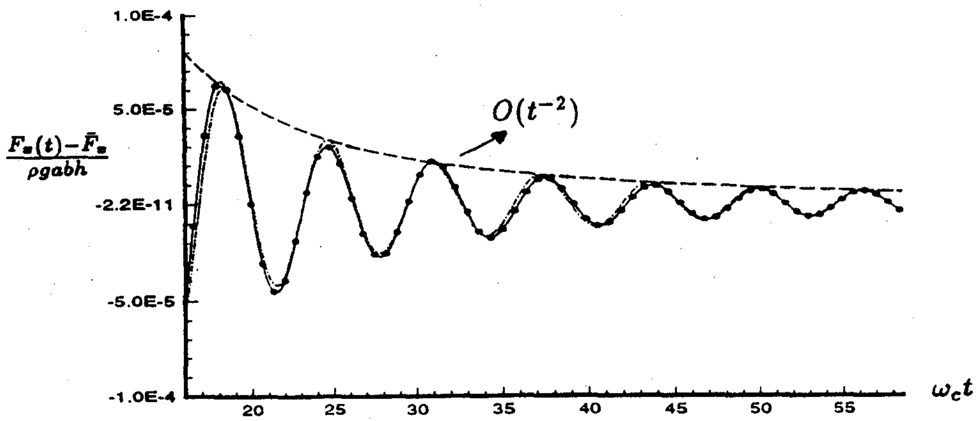

Figure 3: Comparison between numerical result (•) and fitted asymptotic solution (— · —), $U/(ga)^{\frac{1}{2}} = 0.15$, $b/a = 0.1$, $h/a = 0.0625$, a: length, b: width, h: draft.

DISCUSSION

Tulin M.: How does the amplitude of the transient waves in the wave group traveling with the ship (responsible for the encounter frequency ω_c) vary with time? Is the relationship between these transient waves and the interaction force linear? If the amplitude of the transient waves does not vary as $t^{-1/2}$ (2D) or t^{-1} (3D) (as it would for the waves from an initial impulse), then why not? In any event, your results are quite remarkable and the analysis is formidable.

Liu Y. & Yue D.K.P.: Our analysis is completely linear and the interaction force is linearly proportional to the wave amplitude. The amplitude of the transient wave (at frequency ω_c) traveling with the body decays as $t^{-3/2}$, t^{-2} for $t \gg 1$ for 2D and 3D bodies respectively. The amplitude does not vary as $t^{-1/2}$, t^{-1} which would be expected for isolated singularities (or arbitrary distributions of such singularities).

The key distinction between the present case and the classical analyses of Havelock (1949) and Wehausen (1964) is that their results are for a single source (or dipole) while the present analysis applies to an actual body on which a body boundary condition (BBC) must be satisfied. We show that for bodies satisfying $\Gamma \neq 0$, the net strength of the $t^{-1/2}$, t^{-1} disturbance, represented by the associated Kochin function α, vanishes as a result of satisfying the BBC. The remaining contribution is then higher order in time for $t \gg 1$. When $\Gamma = 0$ (which incidentally includes the case of a point singularity), the part of the kernel which multiplies α in the governing integral equation itself vanishes and the $t^{-3/2}$, t^{-2} decay rates for 2D, 3D do not obtain. This result is consistent with the frequency-domain analysis which gives bounded solution at $\tau = 1/4$ for $\Gamma \neq 0$ bodies (Liu & Yue 1993).

That this is the case can be argued physically. It is somewhat simpler from the frequency domain. The induced velocity at any point s due to a source distribution $\sigma(s')$ on the body contains a free surface part $V^\sigma(s)$ which depends on the non-Rankine portion of the Green function. For $\delta^2 \equiv |4\tau - 1| \ll 1$, one can show from the asymptotics that $V^\sigma(s) \sim \alpha_\sigma f(s)$, where α_σ is associated with a Kochin function and $f(s)$ is a property of the geometry (and frequency) but is independent of σ. As $\delta \to 0$, $f(s)$ becomes unbounded (everywhere) like δ^{-1}. In order for a body boundary condition (for finite forcing) to be satisfied as $\delta \to 0$, it follows that $\alpha_\sigma \sim O(\delta)$. For bodies satisfying $\Gamma = 0$, however, we find that $f(s) \equiv 0$ at ω_c and the preceding result does not obtain. Note that Γ depends only on the body geometry and frequency but not the forward speed U. Physically, $\Gamma = 0$ for a body implies that it possesses an irregular frequency ω_I, at which a non-trivial σ exists for a homogeneous $f(s)$. The solution is then not bounded at $\omega_I = \omega_c = 4g/U$. This condition is independent of U and not surprisingly has the same interpretation as the condition of John (1950) for the uniqueness of the radiation and diffraction problems without forward speed.

Yeung R.W.: In reference to the 3D results, I notice the major difference in the conclusion on the decay rate of the oscillating wave resistance from that of Wehausen (1964). Is it appropriate to state that the main cause of this difference is simply related to assuming the source strength to be oscillating in ω_c and decay like t^{-2}?

Liu Y. & Yue D.K.P.: No. The main reason is the fact that an actual body rather than a point singularity is considered (see discussion to **Tulin** above). In this case, it is of course unreasonable to assume constant (or an a *priori* prescribed) temporal dependence of the source strength which should be determined from the body boundary condition.

Roberts A.J.: There exists a simple resolution of Tulin versus Yue; that is, a source generates fluid whereas the movement of a body conserves fluid. This qualitative difference could easily account for the qualitative difference of t^{-1} decay (source) with t^{-2} decay (body). However, what if the flow around the 2D body involves non-zero circulation (e.g. hydrofoil)? Is the transient decay t^{-1} or t^{-2}?

Liu Y. & Yue D.K.P.: The asymptotic decay rate of the transient waves due to a dipole (or vortex), which conserves fluid, is in fact identical to that for a single source. The actual mechanism for a more rapid decay associated with a body is somewhat more involved (see discussion to **Tulin** above).