

A Contribution to the Theory of Slender Ships in Large Amplitude Motion with Forward Speed — A Proposed Method of Nonlinear Computation —

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Introduction

Existing methods of the prediction of ship motion in rough seas depend on the linearized theory in general. The theory is based on the assumption of small amplitude of the incident wave and the ship's response. When finite forward speed is introduced, another condition, that the hull geometry is slender in the longitudinal direction, is necessary in order to keep the small disturbance, by which the linearization of the fluid motion is justified. The linearization of the boundary value problem is derived by the perturbation expansion with two small parameters, one of which represents the smallness of the amplitude of incident waves as well as ship's oscillations and the other represents the slenderness of the hull. These parameters are assumed to be mutually independent. The application of the slender body theory provides a typical method of this kind. The two parameter expansion by the slender body theory derives the first order solution in the near field, which is determined from the linear boundary value problem in two dimensions in the plane perpendicular to the longitudinal axis of the hull. Several versions of the slender body solution for ships moving through ambient regular waves have been published so far.

The implication of the two parameters expansion is the requirement that the amplitude of the vertical movement of the hull and the free surface elevation are both small compared with the dimension of the cross section of the hull i.e. breadth and draft. However the motion amplitude is not necessarily small in comparison with the draft of the ship, even though the motion amplitude and the wave amplitude are small in comparison with the ship's length, which is taken as the reference length in the perturbation scheme. This contradiction is the consequence of the assumption that the slenderness ratio and the amplitude ratio are taken as mutually independent parameters. As a matter of fact, these parameters are of the same order, and the separation of two variables may not be a suitable way to describe the actual situation. When the amplitude is comparable with the draft of the ship, nonlinearity is likely to appear in the fluid motion considerably. The nonlinear effect is remarkable in several problems of seakeeping of ships, such as slamming and deck wetness, and the linearized theory is not available in these problems.

In order to find a method suitable to such a large amplitude motion, it seems to be more rational, that both the slenderness ratio and the amplitude ratio are expressed by a common parameter by which the perturbation expansion is carried out. The perturbation expansion with respect to a single parameter will lead to another system of boundary value problems, which is different from that of the existing slender ship theory based on the two parameter expansion. A great advantage of this scheme is that the formulation allows the large amplitude motion in the local scale at the cross section, though a small amplitude is assumed in the global scale based on the ship's length. Since the amplitude of motion is small in ship's length scale in general, this condition is quite acceptable. Different from the existing theory of slender ships, the boundary value problem in the near field is nonlinear in the new approach. This may bring some complication in the solution technique. However the remarkable progress of the computational method in hydrodynamics in recent years has enabled the solution of fully nonlinear boundary value problem of the free surface flow to be tractable at least in two dimensions.

There is another difficulty in this approach to be overcome. The inner solution obtained in the near field contains an indefinite term which should be determined by matching with the outer solution in the far field. Since the inner solution obtained by a numerical method is not expressed in the analytical form, the matching with the outer solution is not straightforward. A method of resolving this issue is discussed in this paper.

Formulation of the problem

Take the coordinate system x, y, z , fixed in space with x and y -axes on the undisturbed free surface and z -axis in the vertically upward direction. The fluid is assumed as inviscid and incompressible. Consider the irrotational motion and the fluid motion is specified by the velocity potential Φ which satisfies the Laplace equation in the space occupied by the fluid.

$$[L] \quad \Phi_{xx} + \Phi_{yy} + \Phi_{zz} = 0 \quad (1)$$

Consider a ship moving through longitudinal waves either regular or irregular, in the direction of positive x -axis with average forward speed U . Take another coordinate system X, Y, Z , fixed to the ship, with X -axis along the longitudinal axis of the hull, Y -axis athwartships and Z -axis in upward direction. The ship makes oscillations with three degrees of freedom, i.e. surge, heave and pitch. The motion of the ship is expressed

by coordinates of the center of gravity $(x_g, 0, z_g)$ in terms of the x, y, z coordinates, and the pitching angle θ , positive in the bow down rotation. The following transformation is valid between the two coordinate systems.

$$X = (x - x_g) \cos \theta - (z - z_g) \sin \theta, \quad Y = y, \quad Z = (x - x_g) \sin \theta + (z - z_g) \cos \theta \quad (2)$$

The geometry of the hull surface is expressed by the equation

$$Y = F(X, Z) = f(x, z, t) \quad (3)$$

The body boundary condition at the hull surface S_H is expressed in the form

$$[H] \quad f_t + \bar{\Phi}_x f_x + \bar{\Phi}_z f_z - \bar{\Phi}_y = 0 \quad \text{on } S_H \quad (4)$$

where subscripts mean the partial derivatives. The free surface condition consists of the kinematic condition [K] and the dynamic condition [D] on the disturbed free surface $z = \zeta$, such as

$$[K] \quad \zeta_t + \bar{\Phi}_x \zeta_x + \bar{\Phi}_y \zeta_y - \bar{\Phi}_z = 0 \quad \text{at } z = \zeta \quad (5)$$

$$[D] \quad \bar{\Phi}_t + \frac{1}{2} (\bar{\Phi}_x^2 + \bar{\Phi}_y^2 + \bar{\Phi}_z^2) + g\zeta = 0 \quad \text{at } z = \zeta \quad (6)$$

The velocity potential is the sum of the incident wave potential ϕ_w and the disturbance potential ϕ , such as $\bar{\Phi} = \phi_w + \phi$. We have the condition at infinity as $\bar{\Phi} = \phi_w$ at $\sqrt{x^2 + y^2} \rightarrow \infty$. If the depth of water is infinite, we have $\phi_z = 0$ at $z \rightarrow -\infty$. In order to solve the boundary value problem, the flow field is divided to the near field and the far field.

Inner problem

Consider a slender ship of length L , breadth B and draft d . Define the slenderness ratio $\epsilon = B/L \ll 1$, and assume $d/B = O(1)$. The singular perturbation technique is applied to the disturbance potential ϕ in the near field. Since the partial differentiation with respect to y or z reduces the order of magnitude by ϵ^{-1} , the Laplace equation is reduced to the two-dimensional form by omitting higher order terms, such as

$$[L] \quad \phi_{yy} + \phi_{zz} = 0 \quad (7)$$

Assuming $\zeta/L = O(\epsilon)$, $z_g/L = O(\epsilon)$, $\theta = O(\epsilon)$, the hull surface condition becomes

$$[H] \quad \frac{\partial \phi}{\partial n} = -(UF_X + VF_Z)(1 + F_Z^2)^{-1/2} \quad (8)$$

on the hull surface contour Γ_H , where $V = U\theta + \dot{z}_g - (x - x_g)\dot{\theta} - \dot{\zeta}_w$, n is the outward normal to the hull contour Γ_H and $z = \zeta_w$ is the incident wave elevation. If we consider the case of $U^2/gL = O(1)$ and $V^2/gd = O(1)$, $\phi = O(\epsilon^{3/2})$. Writing $\zeta_1 = \zeta - \zeta_w$ for the free surface elevation due to the disturbance, the free surface condition [K] and [D] are reduced to

$$[K] \quad \zeta_{1t} + \phi_y \zeta_{1y} - \phi_z = 0 \quad (9)$$

$$[D] \quad \phi_t + \frac{1}{2} (\phi_y^2 + \phi_z^2) + \phi_z \dot{\zeta}_w + g\zeta_1 = 0 \quad (10)$$

at $z = \zeta = \zeta_1 + \zeta_w$. (9)(10) represent the evolution equation for ζ_1 and ϕ on the free surface respectively. If the free surface elevation and the velocity on the free surface are given at a certain instant $t = t_1$, ζ_1 and ϕ at the free surface at subsequent time are determined by integration of the above equations with time. The fluid velocities ϕ_y and ϕ_z are determined by the solution of the two-dimensional Laplace equation (7) which satisfies both the hull surface condition (8) and the free surface conditions (9)(10). It should be noted that the condition of infinity, $\phi = 0$, at $y \rightarrow \pm\infty$, is not applied, because the solution is valid only in the near field. The solution of (7) is generally expressed in the form $\phi = \phi_1 + g(x)$, where ϕ_1 satisfies the boundary conditions and vanishes at infinity and $g(x)$ is an arbitrary function of x only, which brings some indeterminateness to the solution. In order to make the solution definite, one has to choose the function $g(x)$ to match with the solution in the far field. The method of determination of ϕ_1 is in the following way. Take a transverse plane at a certain position $x = x_1$, as the plane of solution. The domain in this plane occupied by the fluid is bounded by the hull cross section Γ_H and the intersection of the free surface Γ_F . Take a large contour Γ_∞ encompassing the hull section at a great distance in such a way that $\Gamma = \Gamma_H + \Gamma_F + \Gamma_\infty$ makes a closed contour around the hull section. Assume that the value of ϕ on Γ_F is given at a certain instant $t = t_1$. The normal velocity ϕ_n on Γ_H is given by the hull surface condition (8), if the motion of the hull is known. If Γ_∞ is taken at infinite distance, ϕ_1 and normal velocity vanish on it. The application of Green's theorem provides an integral equation for ϕ_n on Γ_F and ϕ on Γ_H as unknowns to be determined. The solution is obtained by a numerical method. ϕ and ϕ_n on Γ_F determine ϕ_y and ϕ_z on the free surface at $t = t_1$. The value of ϕ and ζ_1 in the next instant $t_2 = t_1 + \Delta t$ are obtained by integration of (9)(10). Thus the step by step integration determines the fluid motion in the subsequent time step. The same procedure is repeated at other sections $x = x_2, x_3, \dots$.

Outer problem

The fluid motion in the far field is substantially three dimensional, but the linearized free surface condition is applicable there, because of decay of the disturbance. The linearized free surface condition with respect to the coordinate system is well known, and is written as

$$\zeta_t - \phi_z = 0 \quad (11)$$

$$\phi_t + g\zeta = 0 \quad (12)$$

on $z = 0$ instead of $z = \zeta$. Eliminating ζ between (11) and (12), we have

$$\phi_{tt} + g\phi_z = 0 \quad \text{at } z = 0 \quad (13)$$

The disturbance by a slender ship is expressed by the fluid motion generated by a source distribution along the x -axis. The free surface motion associated with the source distribution of time dependent density $\sigma(X, t)$ moving along the x -axis with uniform speed U is given by

$$\begin{aligned} \phi = & \frac{1}{\pi} \int_{-\infty}^t d\tau \iint_{-\infty}^{\infty} \sin \left[g^{1/2} (k^2 + m^2)^{1/4} (t - \tau) \right] g^{1/2} (k^2 + m^2)^{-1/4} dk dm \\ & \times \int \sigma(\xi, \tau) \exp \left[z(k^2 + m^2)^{1/2} - ik(x - \xi - U\tau) - imy \right] d\xi \end{aligned} \quad (14)$$

The inner expression in the near field of the above is obtained by putting $x = Lx', y = By', z = Bz', k = k'/L, m = m'/B$. Taking account of $B = \varepsilon L$, and omitting terms of $O(\varepsilon^2)$, we obtain the inner expression for ϕ as

$$\phi_i = 4 \int_{-\infty}^t \sigma(x - U\tau, \tau) d\tau \int_0^{\infty} \sin \left[(gm)^{1/2} (t - \tau) \right] \cos(my) e^{mz} (g/m)^{1/2} dm \quad (15)$$

It is readily shown that (15) satisfies the two-dimensional Laplace equation, and corresponds to the two-dimensional part of the inner solution ϕ_1 which is obtained numerically in the inner problem. However there is another term corresponding to the function $g(x)$ in the inner solution, which represents the genuine three-dimensional effect. It is given by the value of outer solution (14) along the x -axis, such as

$$g(x) = g(x, t) = (\phi - \phi_i) \quad \text{at } y = 0, z = 0. \quad (16)$$

The source density $\sigma(x, t)$ in (14) and (15) should be determined by matching with the numerical solution of the boundary value problem in the near field.

Determination of the function $g(x)$

Let us consider the wave pattern associated with ϕ_i given by (15)

$$\zeta = -(1/g) \partial \phi_i / \partial t \Big|_{z=0} \quad (17)$$

Transforming the x -coordinate to the moving coordinate X by the relation $x = X + Ut$, and the integral variable by $t - \tau = u$, we have

$$\zeta = -4 \int_0^{\infty} \sigma(X + Uu, t - u) du \int_0^{\infty} \cos(\sqrt{gm}u) \cos(my) dm \quad (18)$$

Taking the double Fourier transform with respect to t and X , one obtains

$$\zeta^{**}(k, y, \omega) = \int_{-\infty}^{\infty} e^{ikX} dX \int_{-\infty}^{\infty} e^{i\omega t} \zeta(x, y, t) dt = \sigma^{**}(k, \omega) A(k, y, \omega) \quad (19)$$

where $\sigma^{**}(k, \omega)$ is the double Fourier transform of $\sigma(X, t)$,

$$A(k, y, \omega) = -(4i/g) (\omega - Uk) \left[\cos(\kappa y) \text{Ci}(\kappa y) + \sin(\kappa y) (\pi/2 + \text{Si}(\kappa y)) - \pi i \cos(\kappa y) \text{sgn}(\omega - Uk) \right] \quad (20)$$

$\kappa = (\omega - Uk)^2/g$, and Ci, Si are integral cosine and integral sine respectively. Thus the double Fourier transform of the source density is expressed in terms of the double Fourier transform of the wave profile.

$$\sigma^{**}(k, \omega) = \zeta^{**}(k, y, \omega) / A(k, y, \omega) \quad (21)$$

The free surface elevation ζ given by (18) corresponds to the outer expression of the elevation ζ_1 obtained as a numerical solution of the fully nonlinear boundary value problem in the inner problem. Thus the double Fourier transform of the source density is determined by the solution of the inner problem.

Now let us take the double Fourier transform of $g(x, t) = g(X + Ut, t)$ such as

$$g^{**}(k, \omega) = \int_{-\infty}^{\infty} e^{ikX} dX \int_{-\infty}^{\infty} g(X + Ut, t) e^{i\omega t} dt \quad (22)$$

After some reduction, one obtains from (14)(15)(16),

$$g^{**}(k, \omega) = B(k, \omega) \sigma^{**}(k, \omega) \quad (23)$$

$$\begin{aligned} B(k, \omega) &= 4[\ln 2 + \ln |\kappa/k| + \cos^{-1}(\kappa/|k|)/\sqrt{(k/\kappa)^2 - 1} - \pi i \operatorname{sgn}(\omega - Uk)] & |k| > \kappa \\ &= 4[\ln 2 + \ln |\kappa/k| + \cosh^{-1}(\kappa/|k|)/\sqrt{1 - (k/\kappa)^2} + \pi i(1/\sqrt{1 - (k/\kappa)^2} - 1) \operatorname{sgn}(\omega - Uk)] & |k| < \kappa \end{aligned} \quad (24)$$

From (19) we have

$$g^{**}(k, \omega) = \zeta^{**}(k, y, \omega) B(k, \omega)/A(k, y, \omega) \quad (25)$$

Thus $g^{**}(k, \omega)$ is determined from the wave profile which is obtained by the numerical solution in the near field, and $B(k, \omega)/A(k, y, \omega)$ is the transfer function which relates $g(x, t)$ to $\zeta_1(x, y, t)$.

Problems for the implementation of this method

There are problems to be resolved for the implementation of this theory as a method of computation of the nonlinear fluid motion around the hull.

- (1) To develop the stable and robust numerical scheme to enable the computation to continue in long duration.
 - (2) A rational method of truncation correction for the Fourier transform of the longitudinal wave profile.
- This is achieved by the extrapolation using the linear solution for the wave profile. It is obtained from the solution of the Cauchy-Poisson problem in two dimensions with the initial condition by the transverse wave profile behind the hull.

Linear approximation

If the amplitude of motion is small, the linearized solution can be employed as the inner solution, which is written in the form

$$\begin{aligned} \phi_1 &= \frac{1}{4\pi} \int_{\Gamma_H} \left(\frac{\partial \phi}{\partial n} - \phi \frac{\partial}{\partial n} \right) \ln \frac{(y-y')^2 + (z-z')^2}{(y-y')^2 + (z+z')^2} ds + \frac{1}{2\pi} \int_{-\infty}^t d\tau \int_{-\infty}^{\infty} \sin[|gm|^{1/2}(t-\tau)] |g/m|^{1/2} e^{|m|z - imy} dm \\ &\times \int_{\Gamma_H(x, \tau)} \left[\frac{\partial \phi}{\partial n} - \phi(n_z |m| + in_y m) \right] e^{|m|z' + imy'} ds \end{aligned} \quad (26)$$

Then the function $g(x)$ is determined directly from the double Fourier transform,

$$g^{**}(k, \omega) = B(k, \omega) M^{**}(k, \omega) \quad (27)$$

where

$$M^{**}(k, \omega) = \int_{-\infty}^{\infty} e^{ikX} dX \int_{-\infty}^{\infty} M(X + Ut, t) e^{i\omega t} dt \quad (28)$$

$$M(x, t) = \frac{1}{4\pi} \int_{\Gamma_H} \left(\frac{\partial \phi}{\partial n} - \kappa n_z \phi \right) ds \quad (29)$$

and $B(k, \omega)$ is given by (24). However the solution of the boundary value problem with the linearized inner solution is not simpler than that of the original nonlinear problem, so the linear approximation is not necessarily recommended.

Conclusions

A method of analysis of the nonlinear fluid motion by a ship making large amplitude motion at finite forward speed is proposed.

The numerical solution of the two-dimensional fully nonlinear boundary value problem in the near field is matched with the linear solution for the three-dimensional motion in the far field, and the term representing the genuine three-dimensional effect is determined therefrom.

The nonlinear free surface phenomena, such as the bow wave breaking, generation of spray and deck wetness, can be analysed by this method.