# The Slender Body Approximation of a Ship in Following Sea

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## 1. Introduction

The analysis which will be developed here intends an application of the slender body theory to the case of a ship running in the following sea. When the encounter wave frequency tends to zero, it is known that the prediction of the wave bending moment from strip theory will diverge. And the broaching phenomena of high speed small ships may occur under this circumstances.

To study the physics of following sea problem, theoretical formulation using slender body approximation is developed here. In addition to the slenderness ratio of the ship hull form, the encounter wave frequency and the incident wave angle are also assumed to be small in this formulation. Velocity potentials for steady forward motion, radiation and diffraction problems are perturbed to formulate the linear boundary value problems.

The flow field around the ship is found to be characterized mainly by the steady motion parameter ν. And the inner expansion of the Kelvin wave source potential is modified for diffraction problem in accordance with Tuck[1] and Reed[2].

## 2. Perturbation expansion[3]

Assuming the small disturbances, we define our discussion within the limit of linear theory. The plane-progressive wave potential is expressed in the moving reference frame direction fixed on the ship.

$$\Phi_w = \frac{ig}{\omega_o}\zeta_w \exp\{K_o z + iK_o(x\cos\chi - y\sin\chi)\}e^{i\omega t}$$
$$\omega = \omega_o - K_o U\cos\chi \tag{2.1}$$

Here $K_o$ is the wave number, $\omega_o$ is the radiation frequency in the space-fixed reference frame, $\omega$ is the frequency of encounter, and $\chi$ denotes the angle of the wave propagation relative to the direction of the ship's forward motion.

The total potential consists of the steady and unsteady components. With the restriction that the unsteady motions are sinusoidal in time with the frequency of encounter $\omega$, the unsteady component of the velocity potential can be expressed as

$$\begin{aligned}\Phi(x,y,z;t) &= \Phi_o(x,y,z) + \Phi_1(x,y,z,t)\\ &= (Ux+\phi_o) + \{\textstyle\sum l_j\phi_j + (\phi_w+\phi_d)\zeta_w\}e^{i\omega t}\end{aligned} \tag{2.2}$$

Here $\phi_w$ is the incident-wave potential of unit amplitude, and $\phi_d$ is the scattered potential. The components $\phi_j$ are the radiation potentials due to motion of the ship. Each components are governed by the hull boundary conditions as follows.

$$[H]\quad \begin{cases}\dfrac{\partial\phi_o}{\partial n} = -Un_1\\[2mm] \dfrac{\partial\phi_j}{\partial n} = i\omega n_j + Um_j\\[2mm] \dfrac{\partial\phi_d}{\partial n} = -\dfrac{\partial\phi_w}{\partial n}\end{cases} \tag{2.3}$$

The slender body approximation is now introduced. Firstly we set the order of magnitudes for U, $\omega_o$ and $\omega$ such that

$$U = O(1),\quad \omega_o = O(1),\quad \omega = O(\varepsilon) \tag{2.4}$$

From the hull boundary conditions, the leading orders of each velocity potentials can be estimated

$$\phi_o = O(\varepsilon^2),\quad \phi_j = O(\varepsilon),\quad \phi_d = O(\varepsilon),\quad \phi_w = O(1) \tag{2.5}$$

Secondly we assume the magnitude of the incident wave angle $\chi$ is also small. Then the scattered potential is expanded in ascending power series of $\varepsilon$ and $\chi$, and the leading order term of the scattered potential with respect to $\varepsilon$ can be divided into symmetric and asymmetric components, if $\chi = O(\beta) \quad \beta \ll 1$

$$\begin{aligned}\phi_d &= \varepsilon\phi_d^{(1)} + \varepsilon^2\phi_d^{(2)} + \cdots \\ &= \varepsilon\left(\phi_d^{(1,0)} + \chi\phi_d^{(1,1)} + \cdots\right) + \varepsilon^2\left(\phi_d^{(2,0)} + \chi\phi_d^{(2,1)} + \cdots\right) + \cdots \\ &= \varepsilon\left(\phi_{dS} + \chi\phi_{dA} + O(\beta^2)\cdots\right) + O(\varepsilon^2)\end{aligned} \tag{2.6}$$

## 3. Steady forward motion potential

In this section the linear boundary value formulations for the steady forward motion of the ship are given according to Tuck and Reed. The boundary value problem in the far field and in the near field are derived from an asymptotic analysis which is valid to leading order in the body slenderness. When we set $\phi_o = U\varphi_o$

Far Field

$$\begin{aligned}&[L] \quad \Delta\varphi_o = 0 \\ &[F] \quad \frac{\partial^2\varphi_o}{\partial x^2} + v\frac{\partial\varphi_o}{\partial z} = 0 \quad on \quad z = 0 \\ &[B] \quad \lim_{z\to-\infty} \nabla\varphi_o = 0 \\ &[R] \quad radiation \ \ condition\end{aligned} \tag{3.1}$$

Near Field

$$\begin{aligned}&[L] \quad \Delta^{2D}\varphi_o = 0 \\ &[F] \quad \frac{\partial\varphi_o}{\partial z} = 0 \quad on \quad z = 0 \\ &[H] \quad \frac{\partial\varphi_o}{\partial n} = -n_1 \quad on \quad \overline{H}\end{aligned} \tag{3.2}$$

The solution of the steady motion potential in the far field corresponds to the line distribution of the steady wave sources. The asymptotic expression of the far field solution can be reduced[4), 5)]

$$\varphi_o \approx \frac{1}{\pi}\sigma(x)\ln r - \frac{1}{2\pi}\int_L d\xi\sigma'(\xi)\ln(2|x-\xi|)sgn(x-\xi) - \frac{1}{4}\int_L d\xi\sigma'(\xi)\left[\mathbf{H}_o(v(x-\xi)) + \{2 + sgn(x-\xi)\}Y_o(v|x-\xi|)\right] \tag{3.3}$$

The outer expansion of the inner solution can be expressed in the form

$$\varphi_o \approx \frac{1}{\pi}a(x)\ln r + b(x) \tag{3.4}$$

The inner and outer solutions are matched to determine the unknown source strength of the outer solution and the function b(x) in the inner solution.

$$\begin{aligned}a(x) &= \sigma(x) = S'(x) \\ b(x) &= -\frac{1}{2\pi}\int_L d\xi\sigma'(\xi)\ln(2|x-\xi|)sgn(x-\xi) - \frac{1}{4}\int_L d\xi\sigma'(\xi)\left[\mathbf{H}_o(v(x-\xi)) + \{2 + sgn(x-\xi)\}Y_o(v|x-\xi|)\right]\end{aligned} \tag{3.5}$$

## 4. Diffraction potential

### 4.1 Symmetric component

The scattered potential in the far field satisfies the same boundary conditions as the steady forward motion potential. This means that the symmetric component of the scattered waves behaves like the steady waves in the far field. Thus the same asymptotic expression as the steady motion potential can be used again.

On the contrary, the near field boundary conditions must include the effects of the incident wave and the steady forward motion. However the leading-order free-surface condition for the diffraction problem still remains the rigid-wall boundary condition.

Now introducing new potentials defined as $\phi_{DS} = \phi_{dS} + \phi_{oz}e^{iK_o x}$ and $\phi_{7S} = \phi_{eS} + \phi_{oz}$ , The boundary value problem in the near field reduced to

$$\begin{aligned}&[L] \quad \Delta^{2D}\phi_{7S} = 0 \\ &[F] \quad \frac{\partial}{\partial z}\phi_{7S} = 0 \quad on \quad z = 0 \\ &[H] \quad \frac{\partial}{\partial n}\phi_{7S} = -i\omega_o e^{K_o z}n_3 - Um_3 \quad on \quad \overline{H}\end{aligned} \tag{4.1}$$

Here we define a new function which coincides with the symmetric component in the far field[6].

$$\phi_{DS} = \phi_{dS} + \phi_{oz} e^{iK_o x} = \phi_{7S} e^{iK_o x} \tag{4.2}$$

The inner solution takes the form

$$\phi_{DS} = \phi_S^{2D}(y,z) e^{iK_o x} + f(x) \tag{4.3}$$

where f(x) is an arbitrary function of x expressing the three-dimensional effect. Similar matching procedure as in the case of the steady forward motion potential employed again, and we can determine the source strength using the hull boundary condition ,in the form

$$a_o(x) = \sigma(x) = \int_{C(x)} \frac{\partial}{\partial n} \phi_{7S} ds$$

$$f(x) = -\frac{1}{2\pi} \int_L d\xi \sigma'(\xi) \ln(2|x-\xi|) sgn(x-\xi) - \frac{1}{4} \int_L d\xi \sigma'(\xi) [\mathbf{H}_o(\nu(x-\xi)) + \{2 + sgn(x-\xi)\} Y_o(\nu|x-\xi|)] \tag{4.4}$$

4.2 Asymmetric component

The asymmetric component of the scattered potential in the far field can be expressed as a line dipole distribution along the x-axis. The asymptotic form is reduced to

$$\phi_{dA} \approx \frac{\gamma(x)}{\pi} \frac{\partial}{\partial y} \ln r + \frac{1}{8} \nu^2 y \int_L d\xi \gamma'(\xi) \left[ \frac{2}{\nu(x-\xi)} \mathbf{H}_1(\nu(x-\xi)) + \{2 + sgn(x-\xi)\} \{Y_2(\nu|x-\xi|) + Y_o(\nu|x-\xi|)\} \right] \tag{4.5}$$

The boundary value problem in the near field can be treated as the same way as the symmetric component.

Defining the new potentials $\phi_{dA} = \phi_{eA} e^{iK_o x}$ and $\phi_{7A} = \phi_{eA} - iK_o y \phi_{oz}$ , the final boundary conditions are reduced such as

$$\begin{aligned} &[L] \quad \Delta^{2D} \phi_{7A} = 0 \\ &[F] \quad \frac{\partial}{\partial z} \phi_{7A} = 0 \quad on \quad z = 0 \\ &[H] \quad \frac{\partial}{\partial n} \phi_{7A} = -i\omega_o \frac{\partial}{\partial n} (e^{K_o z} y) - iK_o \frac{\partial}{\partial n} (\phi_{oz} y) \quad on \quad \overline{H} \end{aligned} \tag{4.6}$$

Definition of the new function which coincides with the asymmetric component in the far field, and the expression of the solution in the near field

$$\phi_{DA} = \phi_{dA} - iK_o y \phi_{oz} e = \phi_{7A} e^{iK_o x} \tag{4.7}$$

$$\phi_{DA} = \phi_A^{2D}(y,z) e^{iK_o x} + y g(x) \tag{4.8}$$

Results of the matching procedure.

$$b_o(x) e^{iK_o x} = \gamma(x)$$

$$g(x) = \frac{\nu^2}{8\pi} \int_L d\xi \gamma'(\xi) \left[ \frac{2}{\nu(x-\xi)} \mathbf{H}_1(\nu(x-\xi)) + \{2 + sgn(x-\xi)\} \{Y_2(\nu|x-\xi|) + Y_o(\nu|x-\xi|)\} \right] \tag{4.9}$$

Coming from the asymmetric feature of the near field potential, we can not determine the strength of the dipole distribution directly from the hull boundary condition in this case. Considering the normal derivative of the inner solution, we can obtain an integral equation to determine the dipoles.

## 5. Forces acting on the ship

From the theorem due to Tuck[7], the periodical hydrodynamic forces is given in the form

$$F_i = \rho \iint_{S_H} (i\omega n_i - Um_i) \phi ds \tag{5.1}$$

Substituting the diffraction potential, we can obtain the wave exiting force for i=1 or 5, it follows that

$$E_i = \rho \iint_{S_H} (i\omega n_i - Um_i)(\phi_{wS} + \phi_{dS}) ds \approx \rho \iint_{S_H} \{ i\omega n_i \phi_{wS} - Um_i (\phi_{wS} + \phi_{dS}) \} ds \tag{5.2}$$

and the radiation force can be expressed in the form

$$F_{ij} = \rho \iint_{S_H} (i\omega n_i - Um_i) \phi_j ds \approx \rho \iint_{S_H} (-Um_i) \phi_j ds \tag{5.3}$$

Here the yaw exciting moment which will cause the broaching can be estimated, using the asymmetric potential defined $\phi_A = (\phi_{wA} + \phi_{wA})\chi$ , it follows that

$$E_6 = \rho \iint_{S_H} (i\omega n_6 - Um_6)(\phi_{wA} + \phi_{dA}) \chi ds \approx \rho \iint_{S_H} (-Um_6)(\phi_{wA} + \phi_{dA}) \chi ds \tag{5.4}$$

The final results show that the interaction between the steady forward motion and the incident wave plays significant role in the following sea problem.

Reference


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