WAVE RADIATION BY A SUBMERGED CIRCULAR CYLINDER OF FINITE LENGTH

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Introduction

The solutions for the two-dimensional wave radiation and scattering by a submerged, circular cylinder are readily obtained by the method of multipoles. These solutions have proved to be of great utility both in direct application and as benchmark calculations for the verification of panel methods. The present work seeks to extend the available range of semi-analytical solutions by considering horizontal cylinders of finite length. As a first step, a solution procedure is presented for wave radiation by a cylinder placed between parallel vertical walls (see figure 1). This situation will provide insight into the effects of finite length and is directly applicable to a long line of Bristol cylinder wave-power devices. Here forced heave oscillations of the cylinder with radian frequency ω will be considered. The two-dimensional equivalent of this problem has been solved in detail by Evans *et al* (1979).

Formulation

A horizontal, circular cylinder of radius a and length $2l$ is placed symmetrically in a channel of width $2b$, the axis of the cylinder is submerged to a depth d and the water is taken to be infinitely deep. Cartesian coordinates (x, y, z) are chosen with origin in the mean free surface and polar coordinates (r, θ) in the vertical plane perpendicular to the cylinder axis and defined through

$$z = r\sin\theta, \quad y = d + r\cos\theta \tag{1}$$

will also be used. The geometry is sketched in figure 1.

Under the usual assumptions of the time-harmonic, linearised theory of water waves, there exists a time-independent velocity potential $\phi(x, y, z)$ that satisfies Laplace's equation throughout the fluid region, the linearised free-surface condition and the zero-flow condition on the channel wall

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad |z| = b. \tag{2}$$

The boundary conditions to be satisfied on the cylinder surface are

$$\frac{\partial \phi}{\partial r} = \cos\theta \quad \text{on} \quad r = a, \ |z| < l, \tag{3}$$

which describes the heave forcing, and no flow through the cylinder ends so that

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad |z| = l, \ r \leq a. \tag{4}$$

To facilitate the solution of the problem the fluid domain is divided into two regions (labelled I and II in figure 1). Region I is defined by $\{l < |z| < b, \ r \leq a\}$ and is the extension if the cylinder out to the channel walls. Region II is that part of the fluid domain exterior to this extended cylinder. The solution procedure consists of expanding the velocity potential in series form in each of the two regions separately and then imposing appropriate conditions to determine the coefficients in these series. Because of the symmetry of the problem it is necessary to consider only $z \geq 0$.

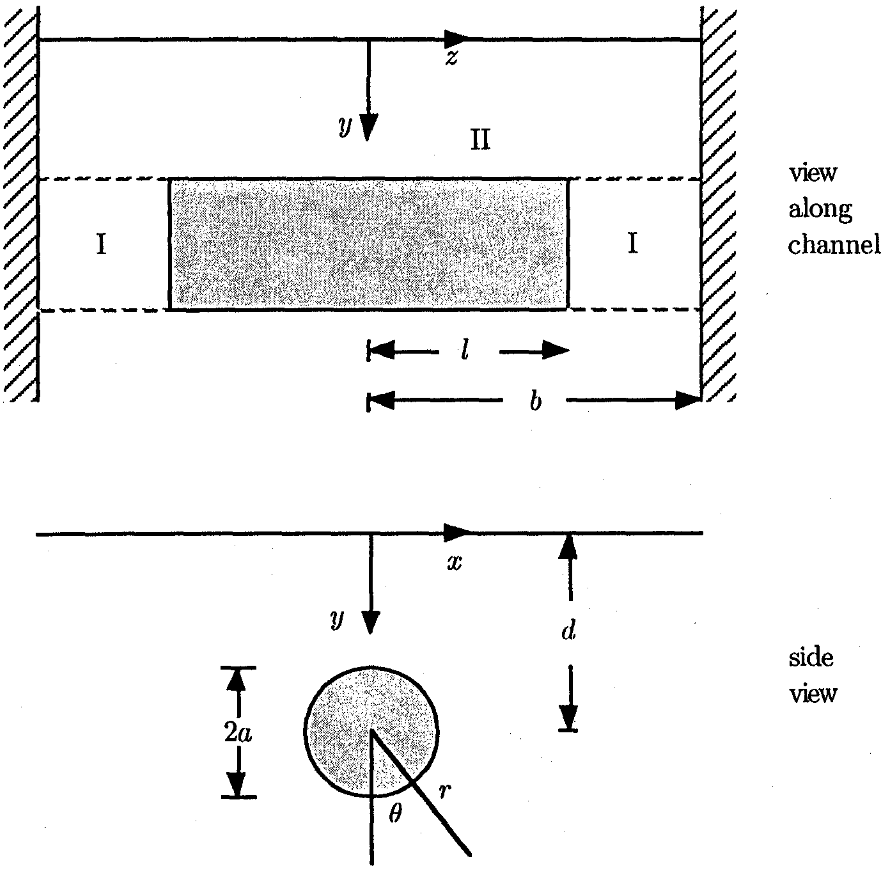

Figure 1: Definition sketches.

Separation of variables in terms of the cylindrical polar coordinates (r, θ, z) leads to the region I solution

$$\phi_1 = \sum_{m=0}^{\infty} \cos m\theta \sum_{n=0}^{\infty} A_{mn} F_{mn}(r) f_n(z). \tag{5}$$

Here, the orthonormal eigenfunctions

$$f_n(z) = \epsilon_n^{1/2} \cos p_n(z-l), \ \ p_n = n\pi/(b-l), \tag{6}$$

satisfy the condition (4) on the cylinder end and the zero-flow condition through the channel wall (2), $\epsilon_0 = 1$, $\epsilon_n = 2$ for $n \geq 1$ and

$$F_{mn}(r) = \begin{cases} (r/a)^m & \text{if } n = 0, \\ I_m(p_n r) & \text{if } n \geq 1, \end{cases} \tag{7}$$

where I_m denotes a modified Bessel function of the first kind and order m.

Region II consists of the whole fluid domain apart from a horizontal cylinder of radius a completely spanning the channel and the potential may be expressed in terms of a multipole expansion

$$\phi_2 = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B_{mn} \phi_{mn}(r, \theta) g_n(z). \tag{8}$$

Here, the orthonormal eigenfunctions

$$g_n(z) = \epsilon_n^{1/2} \cos q_n z, \ q_n = n\pi/b, \tag{9}$$

satisfy the channel wall boundary condition (2) and have the required symmetry about the channel centre line. Each multipole potential ϕ_{mn} individually satisfies the free surface condition and the radiation condition of outgoing waves. For $n = 0$ the ϕ_{mn} are the two-dimensional multipoles used by Evans *et al* (1979) and for non-zero n are given by

$$\phi_{mn} = K_m(q_n r) \cos m\theta + (-1)^{m+1} \int_0^\infty \frac{K+\beta}{(K-\beta)\beta} e^{-\beta(y+d)} \cos tx \cosh m\mu \, dt \tag{10}$$

where K_m denotes a modified Bessel function of the second kind and order m,

$$\beta = (q_n^2 + t^2)^{1/2} \quad \text{and} \quad \cosh \mu = \beta / q_n. \tag{11}$$

The path of integration in (10) runs beneath the pole so as to satisfy the radiation condition. The multipole potentials may be expanded in polar coordinates as

$$\phi_{mn} = K_m(q_n r) \cos m\theta + \sum_{s=0}^{\infty} \alpha_{mns} I_s(q_n r) \cos s\theta, \quad n \geq 1, \tag{12}$$

where the α_{mns} are known coefficients. Similar expansions for the two-dimensional multipoles when $n = 0$ are given by Evans *et al* (1979). Application of the cylinder boundary condition (3) and continuity of ϕ and its normal derivative on $r = a, \ l < z < b$ leads to a set of linear equations for the unknown coefficients $\{A_{mn}, B_{mn}; m, n = 0, 1, 2 \ldots\}$ which may be solved by truncation. Added mass and damping coefficients then follow in the usual way by integration of the pressure over the cylinder surface.

Results

A selection of results for heave added mass and damping coefficients are presented in figure 2. The mass of fluid displaced by the finite length cylinder is used to obtain non-dimensional coefficients. The results of the present theory show the typical 'spiky' behaviour associated with channels; for a channel of half-width b resonant behaviour is expected when Kb is close to $n\pi$ for any positive integer n. In each of the figures comparison is made with the corresponding two-dimensional result.

For all the results presented here, the depth of submergence d is fixed as $1.5a$ and the channel width is five times the length of the cylinder so that, away from the resonances, the channel walls might be expected to have relatively little effect. For the first set of results, the cylinder has a length 10 times its diameter and the added mass and damping curves both follow the two-dimensional results quite closely For the second set of results, the length of the cylinder is only twice its diameter and the trend of the added mass and damping curves is consistently below the prediction of the two-dimensional theory. It is interesting to note that the damping curves converge for quite moderate Ka whereas the high frequency limit of the added mass for the finite cylinder appears to be below the two-dimensional value.

Reference

Evans, D.V., Jeffrey, D.C., Salter, S.H. & Taylor, J.R.M. 1979 Submerged cylinder wave energy device: theory and experiment. *Applied Ocean Research* 1, 3-11.

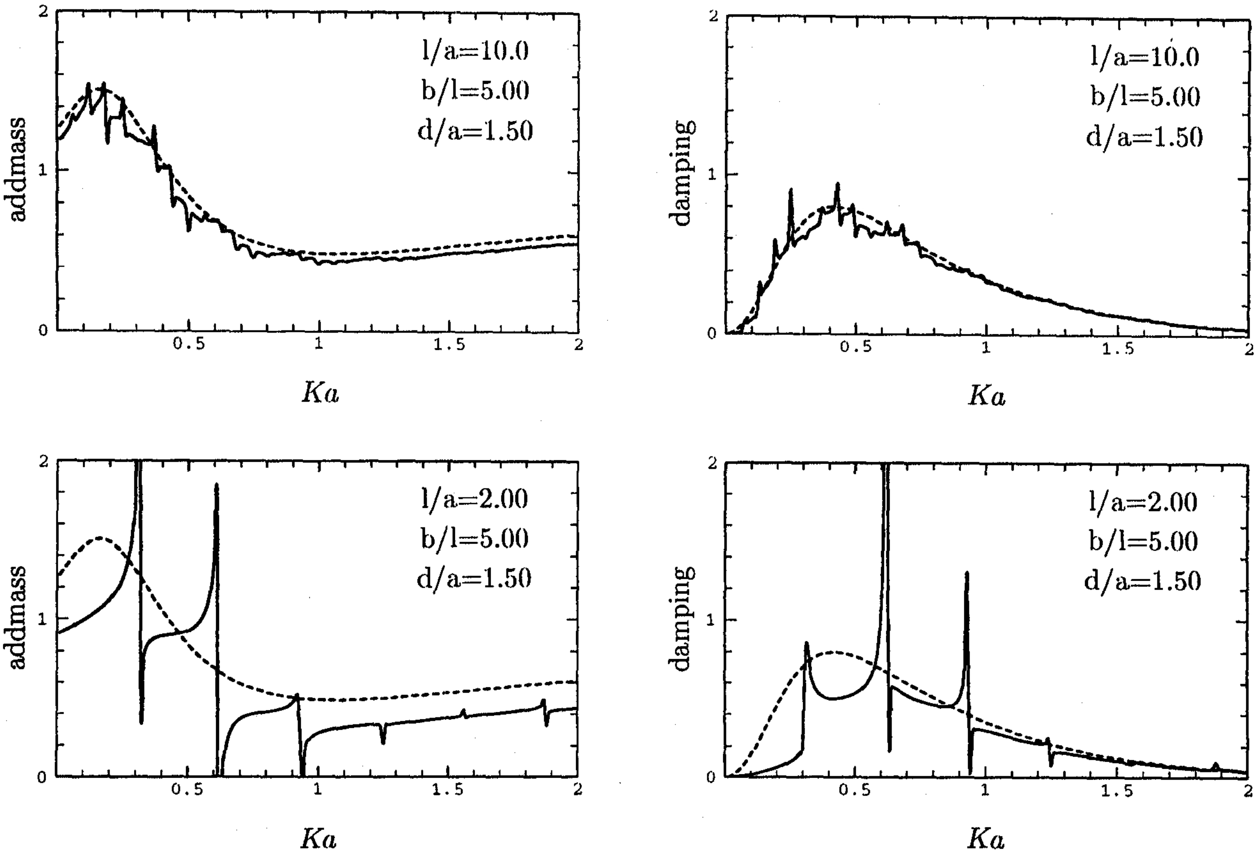

Figure 2: Heave added mass and damping coefficients for a submerged circular cylinder. Finite length cylinder in channel(———), two-dimensional (– – – – –).