SUB-BREAKING WAVE AND ITS NUMERICAL SIMULATION WITH TURBULENT CHARACTERISTICS

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1. Introduction

The understanding of wave breaking is important not only academically but practically for the design of ship hull form. However, the breaking phenomena have not been made completely clear although quite a few experimental and numerical studies have been made. Needless to say, the main difficulty is due to their strong nonlinearity. The phenomena are not so straightforward as expressed in a single word "breaking". Experiments in the uniform flow suggest that the phenomena seem to be related to free-surface flow instability or turbulent flows; the free-surface becomes suddenly unstable as if turbulent transition takes place.

Here recent numerical studies on such fluctuating free-surface flows are presented together with some previous works by authors to have their clearer understandings. The flows are called as "sub-breaking" here to be distinct from overturning breakings.

2. Characteristics of Sub-breaking Waves

Fig. 1 shows the free-surface flows above the submerged hydrofoil at two different oncoming flow velocities, U_0[1]. Although there cannot be seen any symptom at all at the speed U_0= 0.700 m/s, an unstable fluctuation appears at slightly higher speed of U_0=0.718 m/s. The fluctuating suddenly appears to cover all the span in a moment. The appearance is completely different from that of overturning breaking where the wave crest becomes gradually sharp and finally breakdown. It seems similar to transition phenomena.

Fig.2 shows the measured profiles of the mean velocity and the turbulence intensities when a very weak sub-breaking is observed[2], where u and w are the velocity components in the uniform flow and vertical directions respectively. The measurements are carried out by making use of a hot film anemometer and The turbulence quantities are intensive close to the free-surface where the mean velocity sharply defects.

Once the sub-breaking takes place, the averaged momentum equation of the normal component, for example, is given by

$$\frac{\partial W}{\partial t}+U\frac{\partial W}{h\partial s}+W\frac{\partial W}{\partial n}=-\frac{1}{\rho}\frac{\partial p}{\partial n}-n_z g-\frac{\partial \overline{U'W'}}{h\partial s}-\frac{\partial \overline{W'}^2}{\partial n} \qquad (1)$$

where viscous terms are neglected; U, W: the velocity components in the tangential direction (s) and normal direction (n) respectively, t:time, ρ: density, p:pressure, g:gravity acceleration and h: metric coefficient. The last two terms have come from the turbulence production that ontribute to increase the gravity acceleration apparently, for both the terms are experimentally

Fig. 1 Free-surface flows above a submerged foil
left: U_0=0.700m/s (without breaking)
right: U_0=0.718m/s (with sub-breaking)

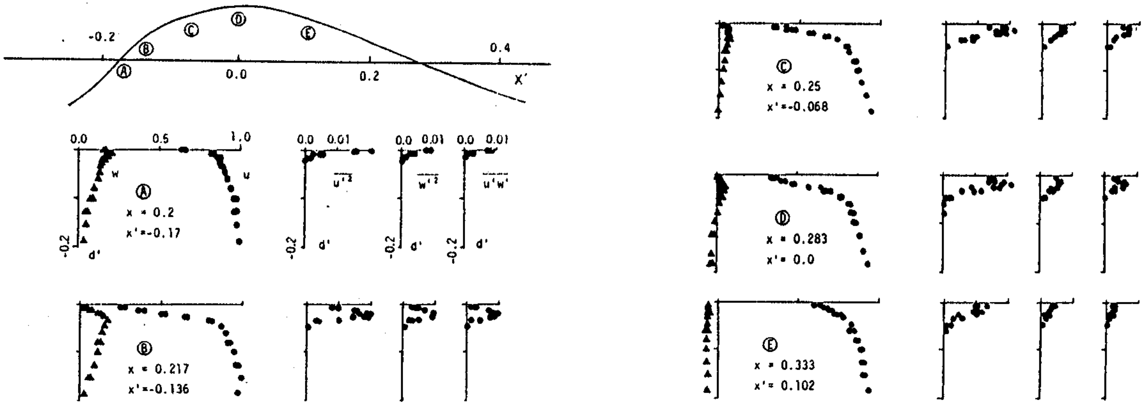

Fig.2 Profiles of measured mean velocity and turbulent quantities above a submerged foil (Rn=7.2x10^4)

positive. This means the production of the fluctuation can reduce the free-surface elevation to sustain the crest without overturning.

3. Instability Analysis

The discussion in the previous section may suggest that the inclusion of the turbulence is important in order for the numerical simulation to be carried out soundly. Needless to say, the turbulent production never appears in the computation unless some numerical disturbance may exist or be introduced and the grid size is small enough to catch the turbulence scale. A possible way of computation is to detect during the computation if the transition takes place and once detected the scheme is switched to that which is capable for the turbulent flow. An instability analysis can be a guideline for the detection.

An inviscid analysis is made for a simple 2-dimensional wave without any stagnant flow such as a wave behind a submerged hydrofoil[2]. The condition is given by

$$\frac{U^2}{(\kappa U^2-g)}\frac{\partial}{\partial x}\frac{(\kappa U^2-g)}{U}>0 \qquad (2)$$

where κ is the curvature of the wave profile. Because (κU^2-g) is negative in most cases, the negative gradient of (κU^2-g)/U with respect to the flow direction gives an indication of the appearance of the fluctuation.

4. Numerical Simulation

Several computations of free-surface flows have been carried out by taking into account the turbulence where the critical condition suggests its appearance[3][4].

Fig.3 shows the comparison of the computed wave profiles between with and without the k-equation turbulence model [3]. A slight difference is observed by the inclusion of the turbulence. The mean velocity profiles and the turbulence quantities are shown in Fig.4. Although the flow conditions are not the same, they resemble qualitatively those measured shown in Fig.2.

However, whenever any model is invoked for the turbulence flow simulation, there remains always an ambiguity about the constants used. It was the case especially for the

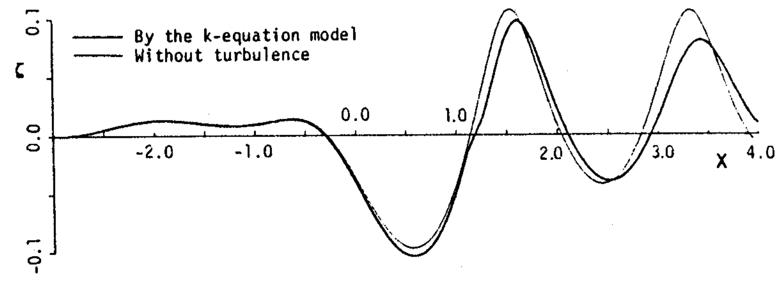

Fig. 3 Comparison of computed wave profiles with and without turbulence model

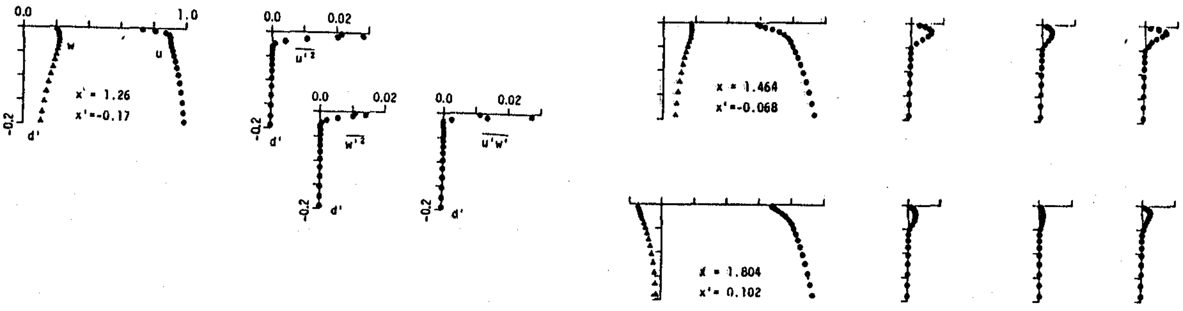

Fig. 4 Profiles of computed mean velocity and turbulent quantities by k-equation model (Rn=2000)

free-surface flows where little experimets have been carried out. For this reason a direct simulation is carried out on a fine grid system although the computed flow is a simpler one with a pressure distribution acting on the free-surface [4]. Fig.5 shows the development of the wave elevation; the wave crest has developed too sharp to continue the computation by the present scheme without breakdown.

An artificial disturbance, given by

$$w = A \sin k_1 x \cdot \sin k_2 t \quad , \tag{3}$$

is introduced at the time step of T=4.0 into the w-components only to generate the fluctuation. In (3), A, k_1 and k_2 are the constants small enough for the grid size and time step. The disturbance is introduced for a certain range just ahead the crest and taken away at T=6.0 after 2000 time-steps,

Fig.6 shows the development of the free-surface elevation after the introduction of disturbance. The elevation of the first crest has reduced to maintain without breaking and wave train is well computed. Fig.7 shows the profiles of the mean velocity and the turbulence at T=20. Although the velocity defect is not so sharp as observed in Fig.2, the turbulence quantities are well simulated.

By the way, when the flow is sub-critical for the appearance of the fluctuation, the disturbance disappears and the flow does not change al all even if an external disturbance has been introduced.

5. Concluding Remarks

A discussion is made on the breaking at an early stage characterized by its fluctuation that is called sub-breaking to be distinct from general term of "breaking". We can mention following remarks as a conclusion;

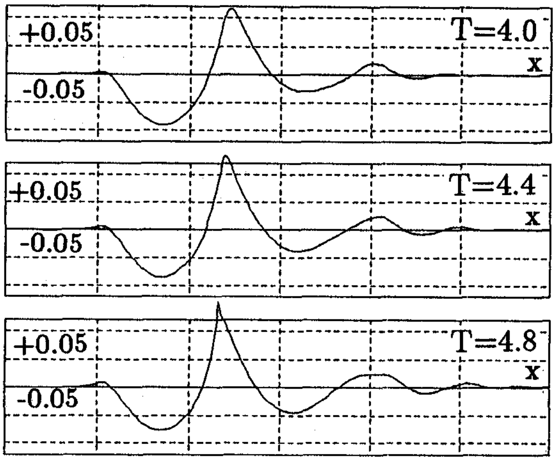

Fig. 5 Computed wave profiles just before breaking (without turbulence)

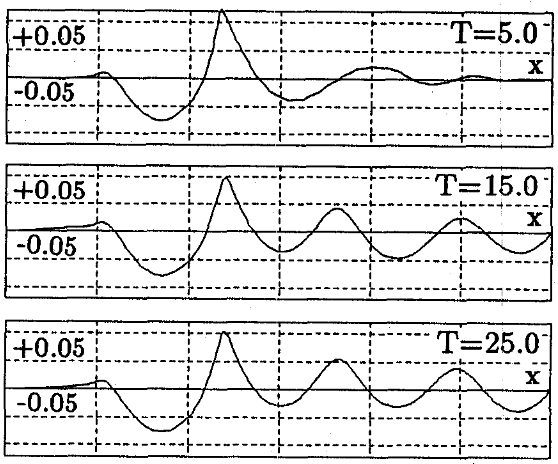

Fig. 6 Computed wave profiles including turbulence by direct simulation

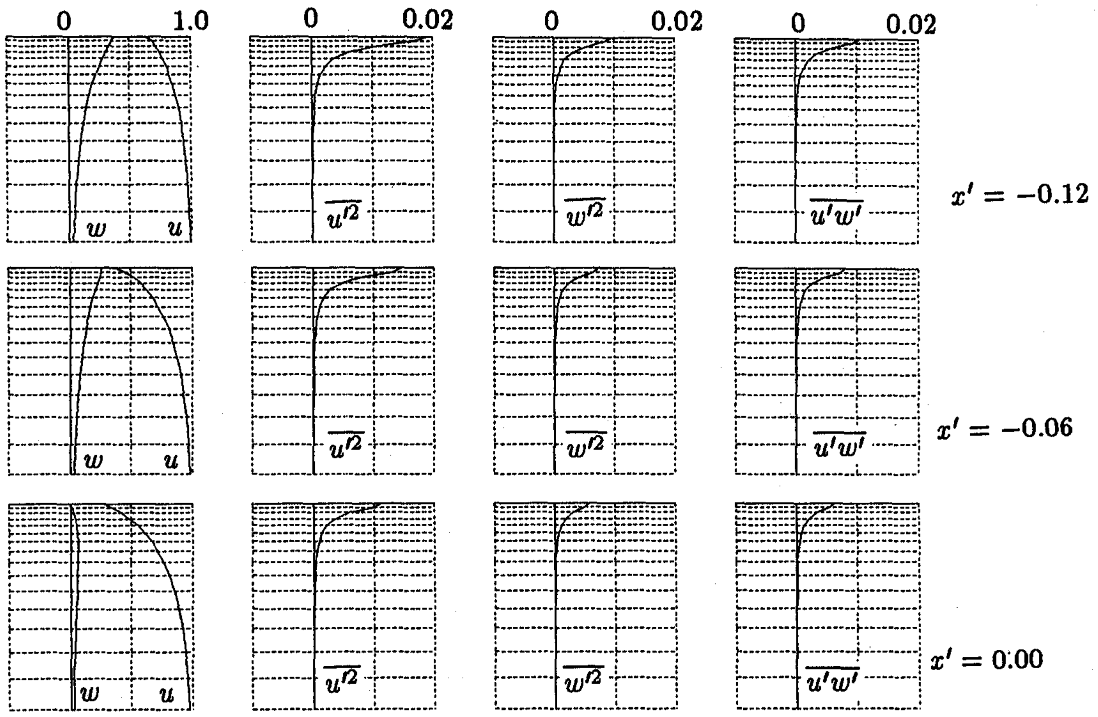

Fig.7 Computed profiles of mean velocity and turbulent quantities by direct simulation while sub-breaking is taking place (Rn=2000, T=20)

—The turbulence terms appeared in the momentum equations to increase the gravity acceleration apparently.
—The production of the turbulence may dissipate the accumulated energy around the wave crest to maintain the crest without overturning.
—It is important to pay a special attention to the appearance of the turbulence in the numerical simulation. Otherwise, any computations may mislead us even if they show an overturning breaking.
—A direct numerical simulation including the turbulence provided an acceptable wave profles and flows.
—The simulated results should be compared with the measured to improve the model of the computation in future to confirm the simulation by the present method.

References

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DISCUSSION

Tulin M.: How have you verified your stability criterion, eq(2)? If you apply it to Stokes waves, as the steepness increases, are the Stokes waves unstable for same steepness?

Mori K.: We have applied (2) to the measured flows; the LHS remains almost constant for the flow which is completely free from the breaking, but (1) is satisfied for that where the breaking is to appear. However, we have not applied (1) to Stokes waves.

Palm E.: I wondered about your use of the k-equation. Aren't there some constants which you can choose somewhat freely and thereby make the answer uncertain?

Mori K.: There is room for improvement in the modeling of free-surface turbulent flow, including the choice of the constants. Our present purpose is to make clear how the turbulence affects the wave profiles and so on; this is the reason why we are trying to simulate directly.