A STRIP THEORY IN THE LARGE SHIP MOTIONS

Matao Takagi, Hitachi Zosen Corporation, Osaka, Japan
and
Kimio Saito, Osaka University, Osaka, Japan

1. INTRODUCTION

Needless to say, it is very important to establish the standard of ship operation in the heavy sea in order to avoid the troubles such as a capsizing. And it has been recognized that the simulation program which is able to estimate the ship motions under maneuvering motions is a powerful tool for that.

Hamamoto and others[1),2),3)] made the leading works in this field, where the simulation program is worked out combining the methods in both maneuvrability and seakeeping practically. They have claimed the effectiveness of the above method by comparing the numerical results with the experiments, but they also pointed that there still remained the problems in estimating the hydrodynamic forces and moments relating to the ship motions due to the incident waves.

In this paper, therefore, the authors conduct a mathematical formulations of the hydrodynamic forces and moments under the both maneuvrable and seakeeping motions using the coordinate system of the body axes. Then, they propose a practical method which is able to implement the numerical calculation of the above formulas.

2. COORDINATE SYSTEM

Σ $O\text{-}X_1X_2X_3$ is a coordinate system fixed to the space, where $O\text{-}X_1X_2$ plane is on the still water plane and X_3 axis is vertically upward. And Σ $o\text{-}x_1x_2x_3$ is a coordinate system fixed to the body, where x_1, x_2 and x_3 axes coincide with X_1, X_2 and X_3 axes respectively when the body has no motion and x_3 axis passes through G, the center of gravity of the body. The linear and rotational motions of the body are given by ξ_i(i=1,2,3) and ξ_i(i=4,5,6) respectively. When we need the large rotational motions, we take ξ_i (i=4,5,6) following Euler angle.

3. FORMULATION OF THE PROBLEM

We assume the fluid is incompressible and irrotational. Therefore we have a velocity potential $\Phi(x;t)$ and it satisfies the Laplace equation;

$$\sum_{i=1}^{3} \frac{\partial^2 \Phi}{\partial x_i^{\,2}} = 0 \tag{1}$$

As for the body boundary condition, we have

$$\frac{\partial \Phi}{\partial n} = \sum_{i=1}^{6} \dot{\xi}_i n_i - \frac{\partial \Phi_0}{\partial n}, \tag{2}$$

where Φ: disturbed velocity potential, n: normal on the body toward fluid,
n_i(i=1,2,3): x_i components of n, $n_i = n_{\underline{i-1}}\, x_{\underline{i-2}} - n_{\underline{i-2}}\, x_{\underline{i-1}}$ for i=4,5,6
Φ_0: incident wave potential (3)

In Eq.(3) $\underline{i-2}$ etc. mean that they are less than 3, so, if they are greater than 4, the mode numbers of 3 or 6 must be subtracted from them.

In the problems of unbounded fluid which were treated by Kirchhoff etc. (see Lamb (1932)), we can determine the instataneous solution and introduce the rigorous formulas of the hydrodynamic forces and moments on the body. In our problem where the free surface exists, however, the rigorous solutions cannot be obtained due to nonlinearity of free surface condition. But, since we know that the effect of it is not so important through numerical examples, we assume the following linear free surface conditions;

$$\left(\frac{\partial}{\partial t} - \dot{\xi}_1 \frac{\partial}{\partial x_1}\right)^2 \Phi + g \frac{\partial}{\partial x_3}\Phi = 0 \quad \text{on } x_3=0 \tag{4}$$

Thus we define the mathematical fluid domain as the region below $x_3 = 0$ except for body.

4. HYDRODYNAMIC FORCES AND MOMENTS

Here we assume that we can get Φ which satisfies the conditions of Eqs. (1), (2) and (4). Though in the usual seakeeping theory the pressures on the body are calculated from the linearized Bernouilli's theorem, we introduce the formulas of the hydrodynamic forces and moments on the body without neglecting higher order terms as in the maneuvrable theory. In the problem of unbounded fluid domain this can be done with kinematic energy of fluid following Kirchhoff (See Lamb (1932)). But in our case with free surface we cannot use this method. So, in this paper we will use the full Bernouilli's theorem.

The Bernouilli's theorem in the coordinate system fixed to the body is shown in Lamb (1932) as follows:

$$\frac{p}{\rho} = -\frac{\partial\Phi}{\partial t} + \sum_{j=1}^{3}(\dot{\xi}_j - \dot{\xi}_{\underline{j+5}}x_{\underline{j+1}} + \dot{\xi}_{\underline{j+4}}x_{\underline{j+2}})\frac{\partial\Phi}{\partial x_j} - \frac{1}{2}\sum_{j=1}^{3}\left(\frac{\partial\Phi}{\partial x_j}\right)^2 - gX_3, \tag{5}$$

where p and ρ are pressure and density of fluid, respectively. The suffix $\underline{j+5}$ etc. must be less than 6. So, when it exceeds 6, the mode number 3 must be subtracted from them.

Now let us put

$$J_i \equiv \frac{1}{2}\iint_{S_H}\sum_{j=1}^{3}\Phi_{x_j}^2 n_i dS, \quad (i=1,2,\cdots,6) \tag{6}$$

where S_H is the wetted surface of the body below $X_3 = 0$.

(i) the cases of i=1,2,3

In Eq.(6) we apply the Gauss' integral formula to the j direction and then partially integrate them considering irrotationality and Eq.(1). Thus we get

$$J_i = \iint_{S_H}\Phi_{x_i}\sum_{j=1}^{3}\Phi_{x_j} n_j dS \ . \tag{7}$$

Then substituting Eq.(2) into Eq. (7), we get

$$J_i = \iint_{S_H}\Phi_{x_i}\left\{\sum_{j=1}^{3}(\dot{\xi}_j - \dot{\xi}_{\underline{j+5}}\, x_{\underline{j+1}} + \dot{\xi}_{\underline{j+4}}\, x_{\underline{j+2}})n_j - \frac{\partial\Phi_0}{\partial n}\right\}dS \tag{8}$$

Hence, if we put the forces or moments to i-direction as F_i except for static and Froude-Kriloff Forces, we get

$$F_i/\rho = -\iint_{S_H}\frac{p}{\rho}n_i dS = \iint_{S_H}\frac{\partial\Phi}{\partial t}n_i dS +$$

$$+ \iint_{S_H} \sum_{j=1}^{3} (\dot{\xi}_j - \dot{\xi}_{\underline{j+5}} x_{\underline{j+1}} + \dot{\xi}_{\underline{j+4}} x_{\underline{j+2}})(\Phi_{x_i} n_j - \Phi_{x_j} n_i) dS - \iint_{S_H} \Phi_{x_i} \frac{\partial \Phi_0}{\partial n} dS . \quad (9)$$

Here, applying Stokes theorem to the 2nd integral terms in Eq.(9) we get

$$F_i/\rho = \iint_{S_H} \frac{\partial \Phi}{\partial t} n_i dS - \sum_{j=1}^{3} \dot{\xi}_j (ixj) \oint_C \Phi dx_{k \neq i,j} +$$

$$+ \dot{\xi}_{\underline{i+4}} \iint_{S_H} n_{\underline{i+2}} \Phi dS - \dot{\xi}_{\underline{i+5}} \iint_{S_H} n_{\underline{i+1}} \Phi dS - \dot{\xi}_{\underline{i+4}} \oint_C \Phi\, x_i dx_{\underline{i+1}} - \dot{\xi}_{\underline{i+5}} \oint_C \Phi\, x_i dx_{\underline{i+2}} +$$

$$+ \dot{\xi}_{\underline{i+3}} [\oint_C (\Phi\, x_{\underline{i+1}} dx_{\underline{i+1}} + \Phi\, x_{\underline{i-1}} dx_{\underline{i+2}})] - \iint_{S_H} \Phi_{x_i} \frac{\partial \Phi_0}{\partial n} dS , \quad (10)$$

where (ixj) is ±1 and follows the law of unit vector product and c is the line of intersection between the free surface and body surface.

(ii) the cases of i=4,5,6

In the same way as in the case of (i), we get

$$F_i/\rho = \iint_{S_H} \frac{\partial \Phi}{\partial t} n_i dS - \iint_{S_H} \sum_{j=1}^{3} \dot{\xi}_j \{\delta_{\underline{i+2},j} \cdot n_{\underline{i+1},j} - \delta_{\underline{i+1},j} \cdot n_{\underline{i+2},j}\} \Phi\, dS +$$

$$- \dot{\xi}_{\underline{i}} \oint_C \{(x_{\underline{i+1}} \Phi) dx_{\underline{i+1}} + (x_{\underline{i+2}} \Phi) dx_{\underline{i+2}}\} + \dot{\xi}_{\underline{i+1}} \oint_C \{(x_{\underline{i+1}} \Phi) dx_{\underline{i}} + \dot{\xi}_{\underline{i+2}} \oint_C (x_{\underline{i+2}} \Phi) dx_{\underline{i}} +$$

$$+ \dot{\xi}_{\underline{i+1}} \iint_{S_H} n_{\underline{i+2}} \Phi dS - \dot{\xi}_{\underline{i+2}} \iint_{S_H} n_{\underline{i+1}} \Phi dS +$$

$$+ \oint_C (\dot{\xi}_{\underline{i+2}} x_{\underline{i+1}} - \dot{\xi}_{\underline{i+1}} x_{\underline{i+2}}) \Phi (x_{\underline{i}} dx_{\underline{i}} + x_{\underline{i+1}} dx_{\underline{i+1}} + x_{\underline{i+2}} dx_{\underline{i+2}}) +$$

$$- \iint_{S_H} (x_{\underline{i-2}} \Phi_{x_{\underline{i-1}}} - x_{\underline{i-1}} \Phi_{x_{\underline{i-2}}}) \frac{\partial \Phi_0}{\partial n} dS , \quad (11)$$

where δ_{ij} is Kronecker's δ.

As for the nonlinear effect we will need a numerical investigation from the engineering view points.

5. VELOCITY POTENTIAL

The conditions satisfied by the velocity potentials are shown in section 3. But from the engineering view point they should be modified considering various effects such as nonlinearity and viscosity. Here the authors propose a method to estimate them. From Eq. (2) we can write as

$$\Phi = \sum_{i=1}^{7} \Phi_i \quad (12)$$

where Φ_i (i=1,2,···,6) are due to $\dot{\xi}_i$ and Φ_7 is due to $\partial\Phi_0/\partial n$.

First of all, as for Φ_1, an approximate value is enough. Because the effects of Φ_1 on large ship motions are considered to be small. So we propose to take the values in $\omega_e = 0$ of a hemispheroid which roughly corresponds to the ship.

Next, for Φ_i (i=2,3,4,··,6) the strip method is used. Namely we use the velocity potential of the two-dimensional problem with linearized free surface condition and the instantaneous wettted hull surface of the ship.

But, since the free surface condition belongs to a transient problem, the solutions will be expressed in terms of the impulse response. If $V^{(k)}$ is resultant velocity of the section in k-mode of motion (k=2: sway, k=3: heave, k=4:roll), the velocity potential of k-mode on a hull section will be given as

$$\Delta\Phi^{(k)} = V^{(k)}\Lambda_k + \int_0^\infty \Gamma_k(\tau)\ V^{(k)}(t-\tau)d\tau\ , \tag{13}$$

where Λ_k is the velocity potential for $\omega_e=\infty$ in k-mode and Γ_k is a function of memory effect. And they are assumed to be obtained under the hull form at $\tau=t$. Generally $V^{(k)}$ can be expressed as

$$V^{(k)} = \sum_{i=2}^{6} a_{ik}\dot{\xi}_i \quad (k=2,3,4)\ . \tag{14}$$

Therefore the velocity potential due to $\dot{\xi}_i$ on the hull section will be given as

$$\Delta\Phi_i = \dot{\xi}_i\ a_{ik}\Lambda_k + \int_0^\infty \Gamma_k(\tau)\dot{\xi}_i(t-\tau)a_{ik}d\tau \tag{15}$$

and Φ_i will be given as

$$\Phi_i = \dot{\xi}_i\ \bar{\Lambda}_k^{(i)} + \int_0^\infty \bar{\Gamma}_k^{(i)}(\tau)\dot{\xi}_i(t-\tau)d\tau\ , \tag{16}$$

where

$$\bar{\Lambda}_k^{(i)} = \int_L a_{ik}\Lambda_k d\ell\ , \qquad \bar{\Gamma}_k^{(i)}(\tau) = \int_L a_{ik}\Gamma_k(\tau)d\ell\ . \tag{17}$$

$\bar{\Lambda}_k^{(i)}$ and $\bar{\Gamma}_k^{(i)}$ can be expressed in terms of the solution of frequency domain. But they are desirable to be corrected three-dimensionally. At this time Maruo's Interpolation-Theory (1978) is simple and effective, but some correction might be needed for the values around $\omega_e = 0$.

As for diffraction forces and moments, we can use Haskind relation extended by Wehausen (1967) for the linear part. Namely,

$$F_i^{(7.1)}/\rho = \iint_{S_H} \frac{\partial\Phi_7}{\partial t} n_i dS = \iint_{S_H} \frac{\partial^2\Phi_o}{\partial t\partial n}\bar{\Lambda}_k^{(i)} dS - \int_{-\infty}^{t} d\tau \iint_{S_H} \frac{\partial^2\Phi_o}{\partial\tau\partial n}\bar{\Gamma}_k^{(i)}(t-\tau)dS \tag{18}$$

where $F_i^{(7.1)}$ is the diffraction forces or moments due to the linear term. For the last terms in Eq. (10) and (11) are difficult to be calculated unfortunately due to the differential terms of Φ.

Lastly, needless to say, the static and Froude-Kriloff forces and moments should be calculated as correctly as possible and this is not so difficult.

REFERENCES

1)Hamamoto, M., Akiyoshi, T., (1988) : Study on Ship Motions and Capsizing in Following Seas (1st Report), J. of Soc. of Naval Arch., Vol.163.

2)Hamamoto, M., Shirai, T., (1989) : Study on Ship Motions and Capsizing in Following Seas (2nd Report), J. of Soc. of Naval Arch.,Vol.165.

3)Hamamoto, M., Kim, Y.S., Uwatoko, K., (1991) : Study on Ship Motions and Capsizing in Following Sea(Final Report),J. of Soc. of Naval Arch.,Vol.170.

4)Lamb, H.(1932): On the Motion of Solids Through a Liquid: Dynamical Theory, Chapter IV, Hydrodynamics, Dover Publications.

5)Maruo, H., Tokura, J. (1978) : Prediction of Hydordynamic Forces and Moments Acting on Ships in Heaving and Pitching Oscillations by Means of an Improvement of the Slender Ship Theory, J. of Soc. of Naval Arch., Vol.143.

DISCUSSION

Newman J.N.: You mentioned a three-dimensional correction derived by Maruo, but for horizontal modes (sway and yaw) this correction based on slender-body theory is of higher-order, as compared to vertical modes.

Takagi & Saito: In the present study, the case of following waves is considered, in which the frequency of encounter becomes very low and the strip theory gives infinite values of heave added mass. Therefore some three-dimensional correction for them is needed and we propose here to use Maruo's interpolation theory since it gives relatively simple correction for strip theory. And the correction is applied only to vertical modes.