Abstract for the 9^{th} International Workshop on Water Waves and Floating Bodies, Kuju, Oita, Japan, April, 1994.

TRANSIENT WAVES NEAR A CIRCULAR CYLINDER IN CLOSED AND OPEN DOMAINS

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Introduction. The hydrodynamic interaction of waves and bodies is a subject of much recent and continuing interest. If one is restricted to the case of inviscid-fluid motion, boundary-integral formulations remain the most attractive approach. Under this category, there are two basic approaches, one is based on the availability of a time-dependent free-surface Green function, whose numerical evaluation is non-trivial (see e.g., [1] and [2]); the other is based on the so-called or Rankine-source formulation, where the free-space source is distributed on the free-surface (see e.g., [3] and [4]). This latter approach normally leads to a larger number of unknowns and requires the use of an approximate open-boundary condition.

If only relatively simple geometries are under consideration, such as circular or elliptical cylindrical structures, which are typical for ocean-related applications, it would be worthwhile to pursue a pseudo-spectral formulation. During the past two decades, various spectral methods have been extensively studied and applied to the numerical solution of several fluid dynamics problems (see e.g., [5]). Although the methods are restricted by their inability to handle general geometry, their efficiency and accuracy are outstanding. In the present paper, a pseudo-spectral formulation is developed for the solution of unsteady three-dimensional inviscid free-surface flow around circular structures. Specifically, a Poisson-equation solver is first examined on the basis of a spectral formulation. Although a Laplace-equation solver would have sufficed for the particular types of problem considered here, successful treatment of the Poisson equation would allow one to handle the solution of the pressure equation, frequently encountered in the solution of the Navier-Stokes equations for a viscous fluid.

Formulation and Solution Method. Under the assumptions of incompressible fluid and irrotational flow, the Laplace equation for the velocity potential is to be solved, with the following boundary conditions: a pair of dynamic and kinematic linearized free-surface conditions, the Neumann body condition, and an appropriate far-field condition for the closure of the problem. To complete the problem, initial conditions should also be specified.

The Laplace equation is solved here, as a special application of a Poisson solver, by a pseudo-spectral formulation similar to Tan [6]. The Poisson-equation solver developed for such wave-motion problems is adapted to cylindrical or elliptical coordinates, using Chebyshev polynomials in the vertical direction and Fourier modes in the circumferential direction. A diagonalization technique is used to decouple the Chebyshev and Fourier modes, before the resulting ordinary differential equations in the radial direction are solved by a finite-difference approach. The method developed can treat both homogeneous and inhomogeneous boundary conditions of the Dirichlet, Neumann, or mixed type, which encompass the typical free-surface flow problems one encounters.

Analytical Validation. Before discussing more general and complex problems, we establish the accuracy and convergence characteristics of the procedure, by testing it against two problems for which analytical solutions can be obtained.

In the first place, we solve the Poisson equation with known analytical solutions for different types of boundary conditions. Once the solution is obtained, the pointwise maximum errors of the solution is computed over the entire domain and for a range of grid resolution (Fig. 1). It shows that the error decays *exponentially* as the grid resolution increases.

To test the accuracy of our algorithm in the handling of wave-related problems, we solve, next, an axisymmetric Cauchy-Poisson wave problem. In this problem, an initial axisymmetric wave form is given in the closed domain, with the wave motion allowed to evolve according to physical laws. The numerical results are compared with analytical solutions derived by us. Excellent agreement is found at every instant of time.

Computational Methods	$\omega(r_i/g)^{1/2} = \pi/4$		$\omega(r_i/g)^{1/2} = \pi/2$	
	$\mu_{xx}/\rho\pi r_i^2 d$	$\lambda_{xx}/\omega\rho\pi r_i^2 d$	$\nu_{xx}/\rho\pi r_i^2 d$	$\lambda_{xx}/\omega\rho\pi r_i^2 d$
Yeung [7]	0.40159	0.45051	0.46771	0.04650
Present Method	0.41047	0.44689	0.47149	0.04198

Table 1: Comparison of added mass and damping coefficient computed by the present method and those in Yeung [7].

As an example of this simulation, we show the form of the free surface at $T = 100.0$ (Fig. 2a), and the RMS error in free-surface elevation plotted as a function of the time-step (Fig. 2b). The error is bounded at 0.5% when a nondimensional time step of 0.1 is used, but reduces drastically to a mere 0.05% if the time step is halved. These results lend credence to the potential and effectiveness of the method.

Results and Discussion. Two problems of practical interests are treated in this section. We first consider a vertical cylinder with given periodic swaying velocity, starting its motion in still water. The solution of this problem is sought by using the method mentioned above for two frequencies of oscillation. The hydrodynamic force acting on the cylinder is shown in Fig. 3 as a function of time. It is evident that a harmonic steady state is reached after the body has undergone only 2 or 3 periods of oscillation. From the force time history, it is possible to evaluate the added-mass and damping coefficients and compare them with known analytical results [7]. Table 1 shows such a comparison for a grid density of $L = 150$, $M = 48$, and $N = 24$ (see caption of Fig. 4 for definition), when solved as a transient three-dimensional problem. The agreement is seen to be excellent and the envelope of the force curve in Fig. 3 confirms the method has excellent stability characteristics.

We solve, as a second example, a more complex Cauchy-Poisson problem with a non-axisymmetric initial wave elevation (see Fig. 3a). This problem has physical features not so commonly understood. It is of interest to note the intricate interaction between the waves and the body (Figs. 3b-3d), resulting in the formation of small-amplitude short waves dispersed over the domain, and yet retaining the same total energy as the initial wave pulse. The detailed flow features are well captured by a radial grid dimension of merely 48 points. Fig. 4 shows that the error in total energy is no more than 2% even after a relatively long simulation.

Conclusions. A very effective pseudo-spectral Poisson solver has been developed and applied to several transient free-surface wave problems associated with wave-body interaction. The method is found capable of providing three-dimensional results of high accuracy. Excellent convergence characteristics and relatively low computational requirements are also attractive properties of this method. The limitation is that only relatively simple geometry can be studied at this point. However, by introducing a hybrid formulation and numerical mapping, treatment of more complicated body geometry is believed to be possible. Future extension of this work to consider nonlinear boundary conditions and to account for effects of fluid viscosity also appears promising.

References

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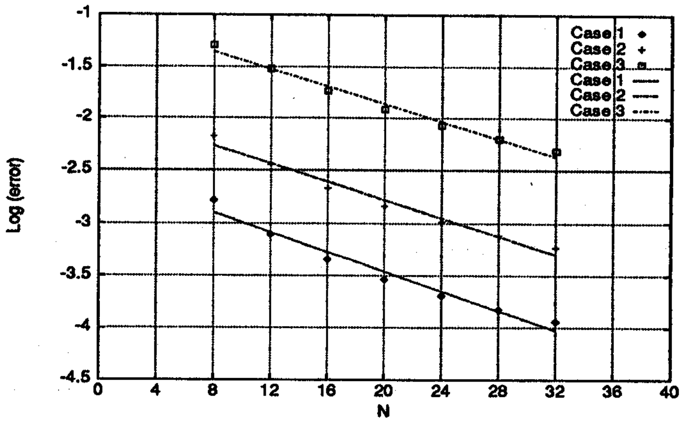

Figure 1: Logarithmic maximum pointwise errors vs. grid resolution for various boundary conditions (Case 1: Dirichlet Conditions on R and Z directions; Case 2: Neumann Conditions on R and Dirichlet on Z; Case 3: Mixed Conditions on R and Z).

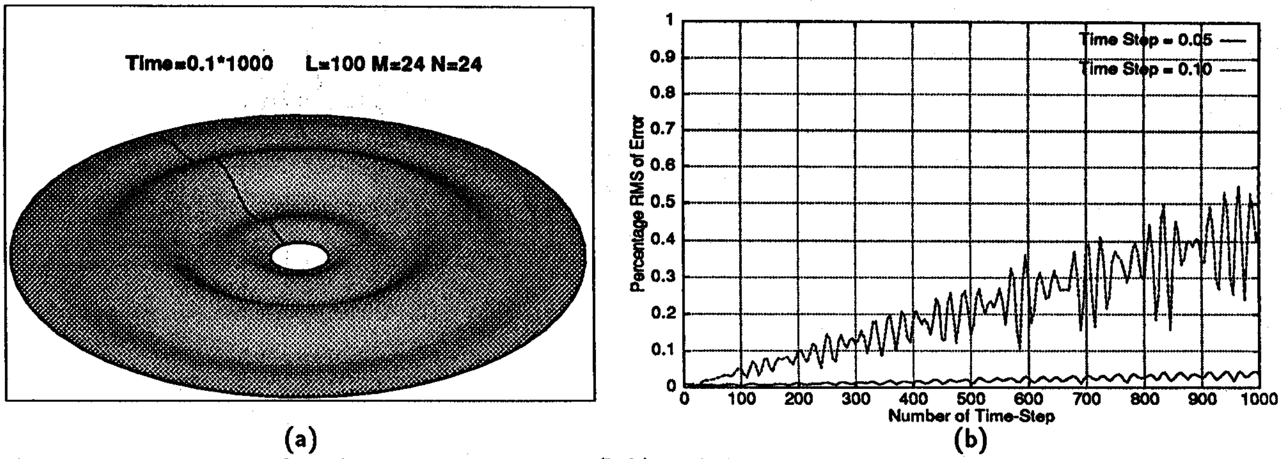

Figure 2: Free-surface elevation at $Time$ = 100.0 (left) and the root mean square of errors of free-surface elevation as a function of time-steps (right) in the axisymmetric Cauchy-Poisson wave problem.

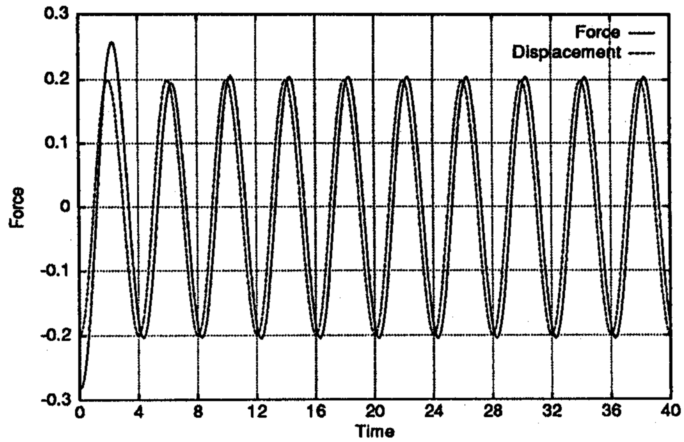

Figure 3: Horizontal force on the cylinder as function of time.

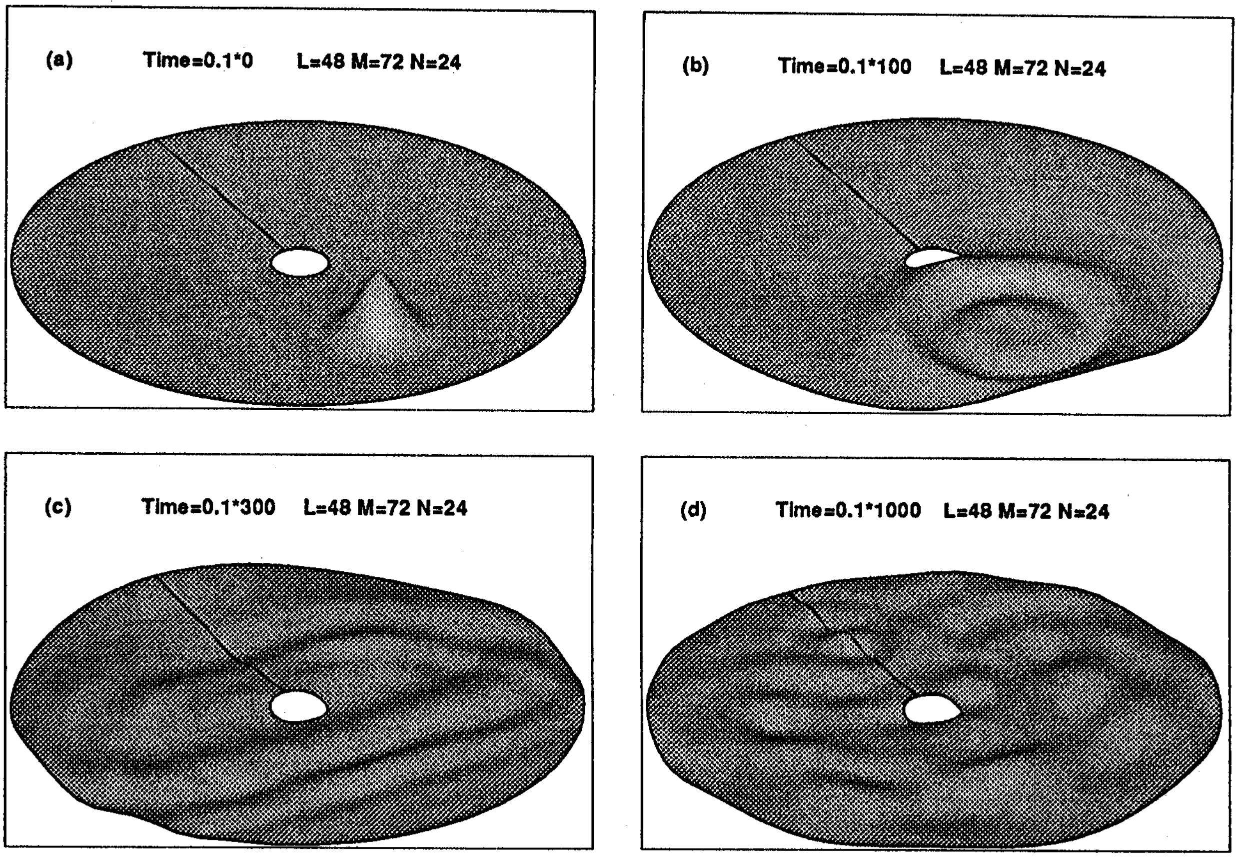

Figure 4: Instantaneous free-suface elevation plots for the Cauchy-Poisson wave problem, L, M, N representing resolution in the radial, circumferential, and vertical directions.

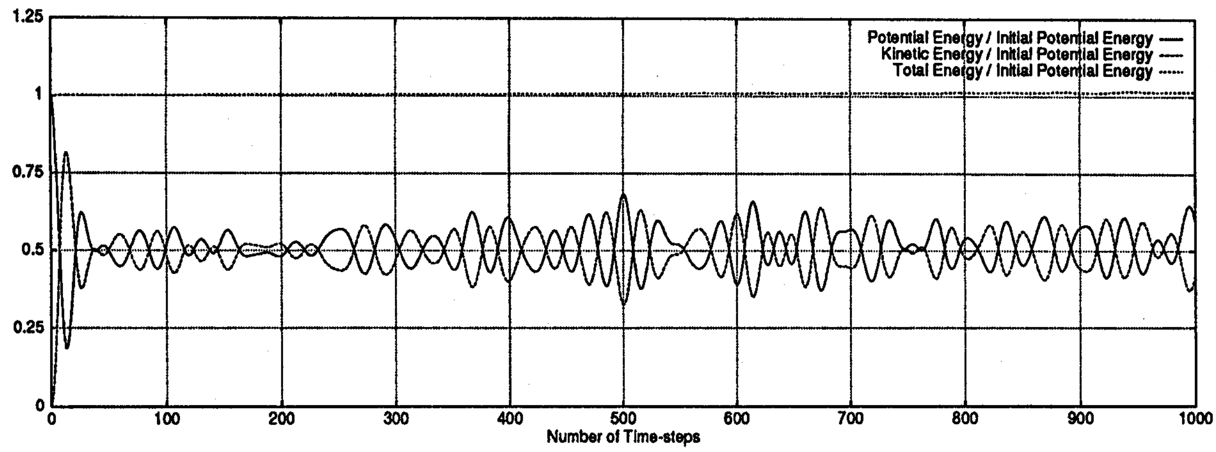

Figure 5: Energy balance in the Cauchy-Poisson wave problem in Fig. 4.

DISCUSSION

Roberts A.J.: So far your analysis just involves the linear free-surface conditions. Are you planning to include nonlinear effects?

Yeung R.W.: This is indeed our plan. Because of the high accuracy of the method and its excellent convergence characteristics, we expect not a great deal of difficulty in its extension.

Linton C.M.: The results shown in Table 1 attributed to Yeung are for a truncated cylinder. Are your computations for truncated cylinders also?

Yeung R.W.: The case of bottom-touching cylinder can be deduced from the truncated case of Yeung (1981), as a special limit, very easily. In fact, a simple and exact solution exists, since there would not be any fluid region under the cylinder.