A 3-D Panel Method for the Radiation Problem with Forward Speed

Z. J. Zou

Institut für Schiffbau
Universität Hamburg
Germany

Introduction

In the previous paper presented at the last workshop, a 3-D panel method using Rankine singularities was developed to calculate the free surface flow around ships in steady motion at small angles of yaw, Zou (1993). This method is now expanded for calculating other lifting potential flows about three-dimensional surface-piercing bodies such as to apply to the transverse modes of unsteady ship motions. The ship motion problems have been successfully attacked by some authors using three-dimensional Rankine panel methods, e.g. Bertram (1990), Nakos (1990), McCreight (1991). However, theoretical and numerical research in this aspect is still in its initial stage, and more efforts are still needed.

This paper describes a three-dimensional panel method using Rankine singularities for solving the radiation problem with forward speed effects. Emphasis is placed on the transverse modes of ship motions. In addition to the Rankine source distribution on the boundary surface, a Rankine dipole (or vortex) distribution on the ship's centerplane and the symmetry plane downstream of the trailing edge of the ship is used to account for the lifting effect. Correspondingly, a Kutta condition is introduced and imposed at the trailing edge. This method can be used to calculate the radiation forces on ships with forward speed and serves as an alternative to the planar motion mechanism experiments for determining the hydrodynamic coefficients in the ship maneuvering problem.

Formulation

We consider a ship traveling with steady forward speed U on the undisturbed free surface and undergoing small harmonic oscillations $\xi_j(t)$ $(j = 1, 2, \cdots, 6)$ about its mean position in deep or shallow water. The fluid is assumed to be inviscid and incompressible. The water depth is assumed to be constant. The absolute velocity of the fluid is represented by the gradient of a disturbance velocity potential ϕ. The exact boundary value problem in the coordinate system (x, y, z) which moves with the speed U is given by

$$\nabla^2\phi = 0 \qquad \text{in the fluid domain,} \qquad (1)$$

$$\zeta_t + (\nabla\phi - \vec{V}_s)\cdot\nabla\zeta = \phi_z \qquad \text{on the free surface,} \qquad (2)$$

$$\zeta = \frac{1}{g}(\phi_t - \vec{V}_s\cdot\nabla\phi + \frac{1}{2}\nabla\phi\cdot\nabla\phi) \qquad \text{on the free surface,} \qquad (3)$$

$$\nabla\phi\cdot\vec{n} = (\vec{V}_s + \vec{V}_T)\cdot\vec{n} \qquad \text{on the ship's hull,} \qquad (4)$$

$$\phi_z = 0 \qquad \text{on the water bottom,} \qquad (5)$$

$$\nabla\phi = (0, 0, 0) \qquad \text{at infinity,} \qquad (6)$$

where ζ is the elevation of the free surface, g the acceleration of gravity and $\vec{n}$ the unit normal vector on the ship's surface. $\vec{V}_s = (U, 0, 0)$ is the steady speed of the ship, $\vec{V}_T = \dot{\vec{\xi}} + \dot{\vec{\alpha}} \times \vec{r}$ is the velocity due to the oscillatory motions, with $\vec{\xi} = (\xi_1, \xi_2, \xi_3)$, $\vec{\alpha} = (\xi_4, \xi_5, \xi_6)$ and $\vec{r} = (x, y, z)$.

Moreover, the velocity potential should satisfy a radiation condition and, for the transverse modes of ship motions, a Kutta condition at the trailing edge.

To solve this boundary value problem, the disturbance potential ϕ is divided into a steady part $\bar{\phi}(x, y, z)$ due to the forward speed U and unsteady parts $\varphi_j(x, y, z; t)$ due to the harmonic oscillations $\xi_j(t)$, $j = 1, 2, \cdots, 6$. It is assumed that $\varphi_j \ll \bar{\phi}$, so that the steady potential $\bar{\phi}$ and the unsteady potential φ_j can be determined separately.

The determination of the steady potential corresponds to the wave resistance problem. There are several successful numerical procedures, either linear or nonlinear, for solving this problem. In this paper, emphasis will be put on the determination of the unsteady potential.

For the harmonic motions, the unsteady potential is expressed in the form

$$\varphi_j(x, y, z; t) = Re\left\{\hat{\xi}_j\, \hat{\varphi}_j(x, y, z) e^{i\omega t}\right\},$$

where means the complex amplitude.

Linearizing the unsteady problem about the solution of the steady forward speed problem, we obtain the boundary condition on the mean free surface elevation $z = \bar{\zeta}$

$$\begin{aligned} &-\omega^2 \hat{\varphi}_j + i\omega B \hat{\varphi}_j + 2i\omega \vec{W} \cdot \nabla \hat{\varphi}_j \\ &+[2(\vec{A} - U\nabla\bar{\phi}_x) + B\vec{W}] \cdot \nabla \hat{\varphi}_j + \vec{W} \cdot [(\vec{W} \cdot \nabla)\nabla \hat{\varphi}_j] - g\hat{\varphi}_{jz} = 0, \end{aligned} \tag{7}$$

where

$$\begin{aligned} \vec{W} &= \nabla\bar{\phi} - \vec{V}_s \\ \vec{A} &= \nabla\left(\frac{1}{2}\nabla\bar{\phi} \cdot \nabla\bar{\phi}\right) \\ B &= \frac{\left[\vec{W} \cdot (\vec{A} - U\nabla\bar{\phi}_x) - g\bar{\phi}_z\right]_z}{g - \vec{W} \cdot \nabla\bar{\phi}_z}, \end{aligned}$$

and the boundary condition at the mean position of the ship's surface $\bar{S}_B$

$$\nabla\hat{\varphi}_j \cdot \vec{n} = i\omega n_j + m_j, \tag{8}$$

where

$$\begin{aligned} (n_1, n_2, n_3) &= \vec{n} \\ (n_4, n_5, n_6) &= \vec{r} \times \vec{n} \\ (m_1, m_2, m_3) &= -[(\vec{n} \cdot \nabla)\vec{W}] \\ (m_4, m_5, m_6) &= -[(\vec{n} \cdot \nabla)(\vec{r} \times \vec{W})]. \end{aligned}$$

A three-dimensional Rankine panel method is used to solve this problem. The boundary surface is discretized with quadrilaterals or triangles, and Rankine sources are distributed on these panels. The symmetry ($j = 1, 3, 5$) and the antisymmetry ($j = 2, 4, 6$) of the motions are utilized, so that only half of the boundary surface and the flow field needs to be considered. The source strengths are determined so that the corresponding boundary conditions are satisfied. In this work the calculation will be limited to the case $\tau = \frac{U\omega}{g} > \frac{1}{4}$, so that the radiation condition can be satisfied by a technique like for the steady potential.

Moreover, for the transverse modes of ship motions which involve lifting effects, a dipole distribution on the ship's centerplane and on the symmetry plane downstream of the trailing edge of the ship is used to produce the required circulation, and a Kutta condition is imposed on the trailing edge for determining the dipole strengths. In this work, the Kutta condition is applied indirectly by satisfying a pressure-equality condition (Hess (1974)) in the form

$$p^{(+)} = p^{(-)} \qquad \text{at the trailing edge,} \tag{9}$$

where the superscripts (+) and (-) stand for the right and left side of the trailing edge, respectively. The linearized dynamic pressure due to the oscillatory motions is given by

$$p_j = -\rho(\varphi_{jt} + \vec{W} \cdot \nabla\varphi_j) \qquad \text{on } \bar{S}_B. \tag{10}$$

From (9) and (10) follows the pressure-equality Kutta condition for the antisymmetric modes of ship motions $j = 2, 4, 6$

$$i\omega\hat{\varphi}_j^{(+)} - U\hat{\varphi}_{jx}^{(+)} + \nabla\bar{\phi}^{(+)} \cdot \nabla\hat{\varphi}_j^{(+)} = 0 \qquad \text{at the trailing edge,} \tag{11}$$

where

$$\bar{\phi}_x^{(+)} = \bar{\phi}_x^{(-)}, \quad \bar{\phi}_y^{(+)} = -\bar{\phi}_y^{(-)}, \quad \bar{\phi}_z^{(+)} = \bar{\phi}_z^{(-)};$$

$$\hat{\varphi}_j^{(+)} = -\hat{\varphi}_j^{(-)}, \quad \hat{\varphi}_{jx}^{(+)} = -\hat{\varphi}_{jx}^{(-)}, \quad \hat{\varphi}_{jy}^{(+)} = \hat{\varphi}_{jy}^{(-)}, \quad \hat{\varphi}_{jx}^{(+)} = -\hat{\varphi}_{jx}^{(-)}.$$

For the symmetric modes of ship motions, the pressure-equality Kutta condition is satisfied automatically. Hence the dipole strengths can be set to zero.

In a numerical procedure, the Kutta condition is imposed on the collocation points on the ship surface adjacent to the trailing edge. Correspondingly, the semi-infinite dipole sheet is divided horizontally into strips, the number of which is equal to the number of the panels on the ship surface adjacent to the trailing edge. The dipole strength on each strip is assumed to change proportionally from zero at the leading edge to a unknown value at the trailing edge. This dipole distribution is equivalent to a system of horseshoe vortices, the vertical bound vorticity of which is independent of x between the leading edge and the trailing edge. For example, on the l-th strip we have

$$\hat{\gamma}(x, z_l) = \hat{\gamma}_l = \text{constant} \qquad X_a(z_l) < x < X_f(z_l),\ y = 0 \tag{12}$$

where X_a and X_f are the x-coordinates at the trailing edge and the leading edge, respectively.

In contrast to the steady flow, the dipole strength downstream of the trailing edge is not constant, which means, the strength of the vertical vortices in the wake is no longer zero on the horizontal strip. For the harmonic oscillatory motions, it follows from the condition of continuity of pressure across the free dipole sheet and Kelvin's theorem

$$\hat{\gamma}(x, z_l) = \frac{i\omega}{U} e^{i\omega\frac{X_a(z_l)-x}{U}} \int_{X_a(z_l)}^{X_f(z_l)} \hat{\gamma}(\xi, z_l)\, d\xi \qquad -\infty < x \le X_a(z_l),\ y = 0. \tag{13}$$

Hence there is only one unknown $\hat{\gamma}_l$ for the l-th strip.

From (7), (8) and (11) we can determine the complex amplitude of the unsteady potential, $\hat{\varphi}_j$. The complex force amplitude on the ship in mode k due to a harmonic oscillation of unit amplitude in mode j is given by

$$T_{kj} = -\rho \iint_{\bar{S}_B} (i\omega\hat{\varphi}_j + \vec{W} \cdot \nabla\hat{\varphi}_j) n_k dS. \tag{14}$$

From (14) follow the added-mass a_{kj} and the damping coefficient b_{kj}

$$\begin{aligned} a_{kj} &= \frac{1}{\omega^2} Re\{T_{kj}\}, \\ b_{kj} &= -\frac{1}{\omega} Im\{T_{kj}\}. \end{aligned}$$

In comparison with the steady forward speed problem, now not only the induced velocities, but also the unsteady potential and the second derivatives of the steady potential in the m-terms need to be calculated for collocation points on the ship's surface. The potential due to the source and dipole distributions can be analytically or numerically calculated. On the other hand, the calculation of the second derivatives of the steady potential due to the source distribution on the ship's surface may cause significant numerical difficulties. This numerical problem is still to be solved.

Status

This work is still in progress.

References

[1] Bertram,V. (1990): *Ship Motions by a Rankine Source Method.* Ship Technology Research, Vol.37, pp. 143-152

[2] Hess,J.L. (1974): *The problem of three-dimensional lifting potential flow and its solution by means of surface singularity distribution.* Comput. Methods Appl. Mech. Eng. 4, pp. 283-319

[3] McCreight,W.R. (1991): *Ship Motions with Nonlinear High Speed Effects.* Sixth International Workshop on Water Waves and Floating Bodies, Woods Hole, USA

[4] Nakos,D.E. (1990): *Ship Wave Patterns and Motions by a Three Dimensional Rankine Panel Method.* PhD. thesis, MIT, USA

[5] Zou,Z.J. (1993): *A 3D Panel Method for Free-Surface Flows around Ships at Small Angles of Yaw.* Eighth International Workshop on Water Waves and Floating Bodies, St.John's, Canada

DISCUSSION

Newman J.N.: I wonder if it will be good to impose the Kutta condition at the stern, since this may overpredict some maneuvering coefficients.

Zou Z.J.: For calculating the lifting potential flow, it is important to impose a Kutta condition to determine the circulation. Under the assumption that no separation appears ahead of the stern, as in the case of small transverse motions, the Kutta condition can be imposed along the trailing edge of the hull.

Bingham H.: I believe that your F.S.B.C. is applied on the steady wave pattern, $\overline{\zeta}$. Are your panels also distributed on this wavy surface?

Zou Z.J.: Yes. The F.S.B.C. is satisfied at the collocation points on the mean free surface $z = \overline{\zeta}$. But the source distribution on the free surface is raised to a horizontal plane above the free surface and replaced by point sources.