AN EPISTEMOLOGICAL INNOVATION IN WATER WAVE THEORY: NUMERICAL RESULTS AS A BENCHMARK FOR ANALYTICAL RESULTS

J.A.P.Aranha

Dep.of Naval and Ocean Eng.,EPUSP,S.P.,Brazil

1. INTRODUCTION

The interaction of ocean waves and currents with a floating structure is described by a mathematical model, based on the Potential Theory, that has some inherent difficulties, related with known second order effects and the change of the wave pattern in a moving media, due to the ocean current and the quasi-steady forward speed of the structure. Few analytical results are known and, in most cases, one must resort to a numerical work, an effort that, in spite of its widespread application, it is not free yet of some internal inconsistencies as displayed, for instance, by the fact that different sources, analyzing the same problem, obtain not seldom different answers.

In this scenario one would expect that the few known analytical results would be welcomed since they could, besides their intrinsic qualities, function as a benchmark for the complicated numerical computations that must be done. This expectation, however, has not been fulfilled in at least one case; specifically, a recently proposed formula for wave damping, deduced **exactly** from the accepted set of equations that describes the wave-current interaction and agreeing **exactly** with numerical results obtained from this very same set of equations in several different cases, has been dismissed on the ground that it does not agree with some numerical results obtained for a specific geometry in a specific situation.

The curious aspect in this refusal is that no substantial question about the derivation of the formula has been raised so far; the strongest argument against it is just the stated above, namely, the disagreement between the formula and one set of numerical result, the exact agreement with the remaining ones being considered fortuitous. In this perspective one is facing an epistemological innovation, where a numerical result works as if it were a benchmark for an analytical result. The purpose of the present paper is to describe the environment of this innovation and to discuss, marginaly, its eventual merits; in this way section 2 presents, from a more physical point of view, the derivation of the wave damping formula while section 3 presents some numerical results, apparently considered accidental by the community, and also the numerical benchmark for the theory.

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2. WAVE DAMPING FORMULA: A PHYSICAL DISCUSSION

One considers first a 2D body, being displaced from left to right with a steady forward speed U and exposed to a harmonic wave, propagating in the same direction and with frequency ω;

let $D_U(\omega)$ be the drift force on the body, influenced by the speed U, while $D_0(\omega)$ is the standard drift force (U=0); let also $\{R_U(\omega);T_U(\omega)\}$ be the reflection and transmission coefficients in the first problem while $\{R_0(\omega); T_0(\omega)\}$ be the related coefficients in the second one. To leading order in U the drift force $D_U(\omega)$ can be expressed in terms of $\{R_U(\omega);T_U(\omega)\}$, these coefficients being also related by means of an "energy equation"; these are known results and will not be repeated here; the intention now is to disclose a relation between $D_U(\omega)$ and the standard drift force $D_0(\omega)$.

In the reference system moving with the body one observes a current, from right to left, and waves being propagated in the opposite direction and with the frequency of encounter ω_e. Since only leading order terms in U are considered, the interaction between the current and the body does not produce any new wave, the free surface working then as if it were an impermeable surface. In this circumstance one may infer a relation between the transmission coefficient $T_U(\omega)$ and the standard coefficient $T_0(\omega_e)$. Indeed, one may assume first that the current is initially zero (U=0) and that the incident wave has amplitude A_0; the transmitted wave will then have amplitude $A_0|T_0(\omega_e)|$ and the reflected one $A_0|R_0(\omega_e)|$. If the "ocean current" is now "turned on" and increases slowly from its initial zero value to the final value U the amplitudes of the incident, transmitted and reflected waves change, in accordance with the relative direction between the waves and current; in the final situation the incident wave will have an amplitude A, the amplitude of the transmitted wave will be, by definition, equal to $A|T_U(\omega)|$ and of the reflected one will be $A|R_U(\omega)|$. Since the incident and transmitted waves are propagated in the same direction they change amplitudes by the same factor, or: $A/A_0= A|T_U(\omega)|/ A_0|T_0(\omega_e)|$; from this equality one obtains that $|T_U(\omega)| = |T_0(\omega_e)|$ and using this result in the "energy relation" and in the expression that relates the drift force $D_U(\omega)$ with $\{R_U(\omega);T_U(\omega)\}$ one obtains:

$$D_U(\omega)=\left[1-4\frac{U}{c}\right]D_0(\omega_e)\ . \tag{1}$$

The above result can be proven exactly from the basic set of equations; see Aranha (1994). In 3D a similar result can also be derived. If the forward speed is assumed to be in the x-direction and the incident wave makes an angle β with this direction, the frequency of encounter is defined by

$$\omega_e=\left[1-\frac{U}{c}\cos\beta\right]\omega\ \ ;c=g/\omega\ . \tag{2a}$$

Besides this frequency change the incident wave, originally propagating in the β direction, is also refracted by the "ocean current"; its new direction is given by

$$\beta_1=\beta+2\frac{U}{c}\sin\beta\ . \tag{2b}$$

Let now $\boldsymbol{D}_U(\omega;\beta)$ be the generalized steady force vector in the horizontal plane influenced by the forward speed U and $\boldsymbol{D}_0(\omega;\beta)$ be the standard generalized force; the **i** and **j** components of these vectors correspond to the steady drift force in the horizontal plane while the **k** component is just the yaw moment; following a reasoning similar as the one done for the 2D case one can show that

$$D_U(\omega;\beta)=\left[1-4\frac{U}{c}\cos\beta\right]D_0(\omega_e;\beta_1)\ . \tag{2c}$$

This result, inferred here from some general arguments, can be exactly deduced from the basic set of equations that define the mathematical problem. It should be observed, however, that a yaw moment, caused by the coupling between the steady second order potential and the "incoming" flow, must be added to (2c) since it is of order $O(A^2U)$; this moment, similar in structure to the known Munk's moment and introduced by Grue & Palm (1993), wiill not be considered here.

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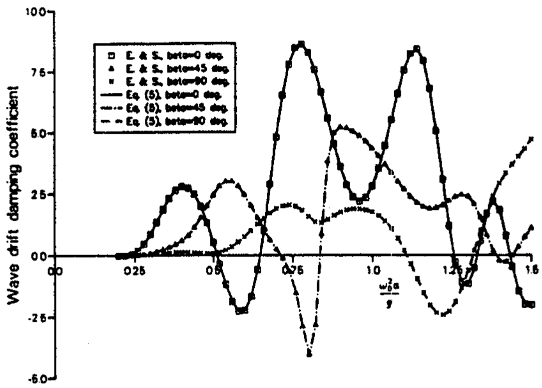
3. NUMERICAL RESULTS AS A VIRTUAL REALITY: ACCIDENTS AND COUNTER-EXAMPLE

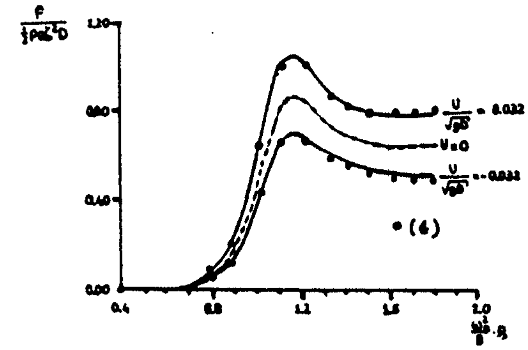
In the orthodox, perhaps old fashioned, epistemology one would follow the mathematical derivation that lead to (2c) to check if it had some flaw; if no mistake were detected then, necessarily, all numerical results must conform to it. In this perspective one would display some numerical results just to make more visual and appealing the analytical derivation. The heterodox epistemology seems to follow a different route; if one single numerical set of results is observed to be different from the analytical one this latter result has, by necessity, to be abandoned; the eventual matching between both in other cases must be understood as accident. It is as if the numerical work, with its overpowering ability to deal with mathematical problems, would have the possibility to create a new reality in this world, a sort of "virtual reality" that, although yet expressionist in its lay out has, subjacent to it, the promise to become a faithful portrait of the reality it aims at.

We will not dispute here on the different kinds of possible epistemologies, not even on the distance between the unquestionable promises of the numerical reality and its present status; instead we can offer facts that can be judged as they stand.

Fig. (1a), extracted from Clark & Malenica & Molin (1992), shows the comparision between (2c) and numerical results obtained for an array of cylinders fixed in waves; these authors have derived, apparently by a trial and error method, expression (2c) for the **i** component of $D_U(\omega;\beta)$ and have reported a "less good agreement" when the body was free to respond to waves. It was then said that apparently (2c) does not work in this situation. We then used a numerical result obtained by Faltinsen (1994) for a hemisphere free to surge and heave and compared it to (2c), see Fig.(1b), and the agreement is perfect. To show that this is not restricted to special geometries we compared (2c) to results obtained by Nossen & Grue & Palm (1992) for an offshore platform with a ring-like pontoon, four columns and free to surge, see Fig.(1c); the agreement is again very good and the difference in the high frequency range is likely due to expected discretization problems. The case where a sensible discrepancy has been observed between (2c) and the numerical results is for a vertical cylinder (or an array) free to respond to waves; Fig.(1d) shows, exactly for this case, a comparision made by Faltinsen (1994) for the stedy heave force using two different numerical approaches; the disagreement is clear and suggests that the counter-example used to dismiss (2c) does not seem to be numerically robust yet for this purpose.



(a)



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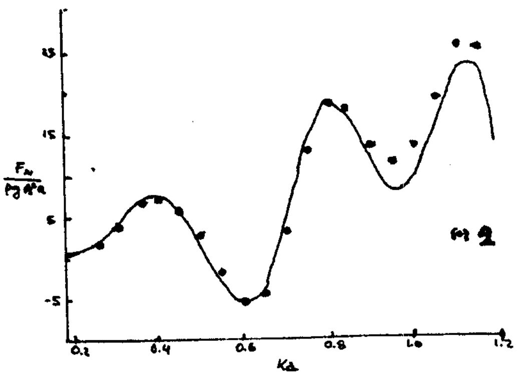

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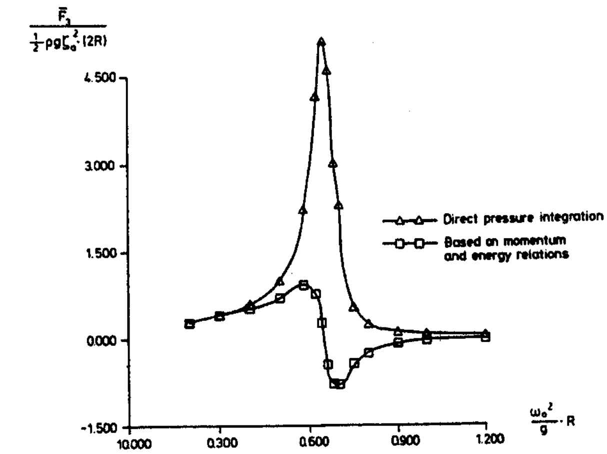

(d)

FIGURE (1): Numerical results (see section 3)