Modelling the wave envelope of progressive waves

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Consider progressive waves such that the time independent potential satisfies the Helmholtz equation. For example, the travelling wave diffracted from a body. In order to model the wave potential using finite elements it is usual to discretize the domain such that there are ten nodal points per wavelength. However, such a procedure is proving computationally expensive and inpractical. We want to develop a new method in which the discretization of the domain is more sensible; we may express the complex potential ϕ in terms of the real wave envelope A and the real phase p such that $\phi = Ae^{ip}$, and expect that in most regions the functions A and p vary much more gradually over the domain than does the oscillatory potential ϕ. Therefore instead of modelling the potential we may consider modelling the wave envelope and the phase.

The starting point for the research is Astley's [1] [2] [3] wave envelope elements which he uses in the far field where, from the geometry of the problem and the eikonal equation, the phase is known. Astley models the wave envelope by using a new shape function obtained by multiplying the standard Galerkin shape function by the factor e^{ip}, and considers a new weighting function to be the complex conjugate of his shape function. The resulting matrix in the equation to be solved is hermitian. These elements successfully model the far field flow with much fewer nodes.

However, we shall show that over a finite domain the same hermitian matrix can be obtained by considering the standard Galerkin shape and weighting functions for the wave envelope and integrating by parts the element matrix integral. (If an infinte domain is considered and mapped infinite elements used, the line integral from the integration by parts may give an additional contribution.)

An interesting outcome of Astley's formulation is that the transport equations [4, p88] are solved by an antisymmetric matrix. However we can give a trivial proof (not yet found elsewhere) which shows that odd dimensional antisymmetric matrices have zero determinant. This would suggest the peculiar result that if the transport equations were modelled using Astley's formulation we can only consider an even number of unconstrained nodes.

We want to develop the wave envelope method further and consider the whole domain. This is ongoing research and the outcome at the moment is unclear; near the body both the wave envelope A and phase p are unknown. Also, p may not necessarily satisfy the eikonal equation in this region.

Since we have replaced one function, ϕ, by two, A and p, we must first give an estimate for one of the two new functions. We give an estimate for p. (At the moment it is unclear how a good estimate for p will ultimately be obtained although in the short wave limit p can be determined by using the laws of ray tracing in geometrical optics [5]. Our problem would be the simplest case since the 'refractive index' is constant.) This estimate will result in the wave envelope having an imaginary part. From the relative sizes of the real and imaginary parts of the wave envelope a 'better estimate' for ∇p, which occurs in the element matrix, can be obtained. Hence we can iterate until the size of the imaginary part of the wave envelope is negligible.

At the moment, three different types of element matrices are considered, all hermitian. The first is the Astley element matrix. The second is a similar formulation except the eikonal equation is not assumed valid. In the third the phase function is linearised such that the first order term satisfies the eikonal equation.

Our test example is diffraction of plane waves from a cylinder. The diffraction potential has been found in terms of a Bessel function expansion by Macamay and Fuchs [6]. We consider certain values of ka (k is the wavenumber and a is the cylinder radius) and first test the different formulations over finite near field regions against the Bessel function expansion. It is found that the different formulations are more accurate in different regions. It may therefore be necessary to model the flow with different types of elements dependent upon the position in the domain. We then hope to test the method over the whole domain. As yet this has not been done.

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DISCUSSION

Peregrine, D. H.: It is usually found that wave envelope approximations fail because of the problems at points where the amplitude is zero. The phase function is not unique at such points and such points are normally branch points of the phase function where a jump of 2π occurs across the branch cut. Have you investigated these points?

Chadwick, E. & Bettess, P.: No, we have not investigated these points yet. However, it will be interesting to find out how our method deals with such points: the element matrix calculation is determined using the phase gradient ∇p and each iteration gives a better approximation for the phase gradient, from the wave envelope solution. Thus the method determines the phase gradient and wave envelope, rather than the phase and wave envelope. Therefore the discontinuity in the phase should not be a problem in itself for my method. However, the phase gradient is singular at these points and it is unclear how the method will deal with this.

Martin, P. A.: Have you tried any problems where A=0? At points where A=0, the phase p is not defined (in fact, grad p is singular at these points, so-called amphidromic points). For a discussion, see a paper of mine with Dalrymple. (Proc. Roy. Soc. A. , 1994).

Chadwick, E. & Bettess, P.: We shall try as a test the three interacting plane waves discussed in your paper which give amphidromic points and let you know of the results.

Ohkusu, M. I understand your approach is quite effective for treating waves progressing freely, like the ray theory. But our concern is the wave-body interaction. I wonder if you have any idea how to improve computational methods of the wave-body interaction by applying your approach.

Chadwick, E. & Bettess, P.: Our eventual aim is to use the method for wave-body interaction problems. We therefore should use the body boundary condition, which has not yet been done. Also, for large wavenumber, difficulties are foreseen in modelling the shadow region of the body.