WAVE ABSORPTION IN A 2D NUMERICAL WAVE BASIN BY COUPLING TWO METHODS

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The absorption of outgoing waves in the numerical time-domain simulation of unsteady free surface hydrodynamics is still an open problem. A good review of the different absorbing techniques available for surface waves can be found in Romate [2]. The aim of the present study was to develop a method of outgoing waves absorption which does not require any spectral knowledge of the incident wave train.

In the frequency domain, the well known Sommerfeld relation links the local dynamic pressure and the local normal (horizontal) velocity along a fictitious vertical truncating boundary through a simple multiplicative coefficient: the phase velocity. This coefficient depends only on the wave frequency. Thus the tuning of the absorbing relation requires a spectral knowledge of the incident wave. Unfortunately, the Sommerfeld condition has no counterpart in the time domain. If such a relation would exist, it should necessarily be the inverse Fourier transform of the previous one and, consequently, involve a complex velocity impulse response function which should represent the time-domain counterpart of the real frequency dependent phase velocity. Such a function is meaningless in the time domain.

Nevertheless, a lot of people use the so-called Orlanski condition [1] which is a simple transposition of the Sommerfeld relation to the time domain:

$$\frac{\partial \Phi}{\partial t}(y,t) = c(t)\frac{\partial \Phi}{\partial x}(y,t) \qquad (1)$$

The coefficient c is generally considered as a function of t only (thus independent of y), and is interpreted as an *instantaneous phase velocity*. It must be estimated numerically during the computations from the results (pressure and velocity) of the previous time steps. When a BEM solver is used, this estimation is made difficult by the fact that these quantities are known only on the boundary itself and the free surface, but not in the fluid domain.

THE PISTON WAVE-ABSORBER.

At the last Workshop in Newfoundland [3] we shown how a translating piston driven by the hydrodynamic force signal *F(t)* may (partly) absorb an incident wave train. The instantaneous velocity of the piston *V(t)* is given by the sum of two convolution products: the first one involving the velocity *V(t)* itself (an autoregressive feedforward control loop), the second one involving the force signal *F(t)* (an exogenous feedback control loop). The transfer functions of these control loops were derived in the frequency domain, then transposed in the time domain by a Fourier transform, and finally approximated to obtain a *realisable* absorbing condition on the piston motion.

In low frequency waves, the fluid moves like a piston, and in the limit ω -> 0 the *total* absorption boundary condition becomes, in nondimensionalized notations :

$$\Phi_n(y,t) = V(t) = F(t) \qquad (2)$$

in which one recognizes the vertically integrated form of the Sommerfeld-Orlanski relation.

The implementation of this simplified form of control law (2) in our non-linear numerical wave basin [4], results in an absorption coefficient curve plotted on fig1 (hollow squares). Results in the linear case are practically the same (see Clément [7]). The amplitude absorption coefficient is defined by one minus the ratio of the amplitude of a monochromatic wave packet before and after its reflection at the absorbing end. It was measured by *"numerical experiment"* in our numerical wave basin (CANAL1.2) by simulating in the time domain the absorption of monochromatic wave packets, in the same way we would have proceeded in a physical flume. The absorption coefficient starts from 1 in the low frequency range, then it decreases rapidly whith increasing frequency. This behaviour is a natural consequence of the above arguments. At last, it is important to notice that the boundary condition (2) is independent of the frequency of the incident wave train.

ABSORBING LAYER (or "BEACH")

Another widely used method for absorbing the outgoing waves at the end of such numerical wave basins is the well known absorbing layer or "beach". It was introduced by Baker & al. [5] and then adopted by a lot of further authors.

Extra terms are added to the RHS of the free surface evolution ODE(s) :

$$\frac{D\Phi}{Dt} = -y + \frac{\left(\vec{\nabla}\Phi\right)^2}{2} - \nu(x).f(\Phi) \qquad (3)$$

$$\frac{D\vec{x}}{Dt} = \vec{\nabla}\Phi - \nu(x).g(\vec{x})$$

at $M(x,y)$ belonging to the free-surface.

The extra term in the dynamic relation above dissipates energy with the motion of the free-surface. It is generally taken equal to the potential Φ itself. Nevertheless, Cao & al.[6], at the last Workshop shown that this choice leads to an energy absorption rate which may eventually become negative in such a way that the beach may put energy into the fluid instead of extracting it. They proposed to use a term proportional to Φ_n in order to ensure a positive damping. Both methods have been tested, in linear and nonlinear simulations, and they gave approximately the same results. Nevertheless we have observed in these computations that the formulation based on a potential extra term needs to be coupled with the modified kinematic relation (3,b) to give its best, whereas the formulation in normal velocity give good results without modifying the original kinematic ODE. Thus, we finally retain: $f(\Phi) = \Phi_n$; $g(\vec{x}) \equiv 0$.

The function $\nu(x)$ is non-zero in the damping part of the free surface, and zero elsewhere. The matching function is cubic in x in order to ensure a smooth transition between the free-surface and the "beach". After numerical optimization (Clément [7]), the magnitude of $|\nu(x)|$ was set to 0.2 . It has been shown in previous works that the efficiency of the beach increases with its length to wavelength ratio, and that its amplitude must be tuned to the incident wave frequency to maintain a good absorption efficiency. A major goal of the present study was to derive an absorbing strategy *independent of the incident wave frequency*. Thus, the beach length was kept constant and equal to twice the water depth in all the reported numerical simulations.

The results of this second approach are plotted on Fig.1 (hollow circles). As it could be expected, the behaviour is exactly opposite than using the piston method: for a given fixed length the damping efficiency of the beach is better and better as the frequency increases. (the upper scale of the figures shows the beach-length to wavelength ratio).

The nonlinear simulations were performed with a wave steepness of 5% in a wave basin of 6 water depth length. Once again, results of linear and nonlinear computations are very close to each other.

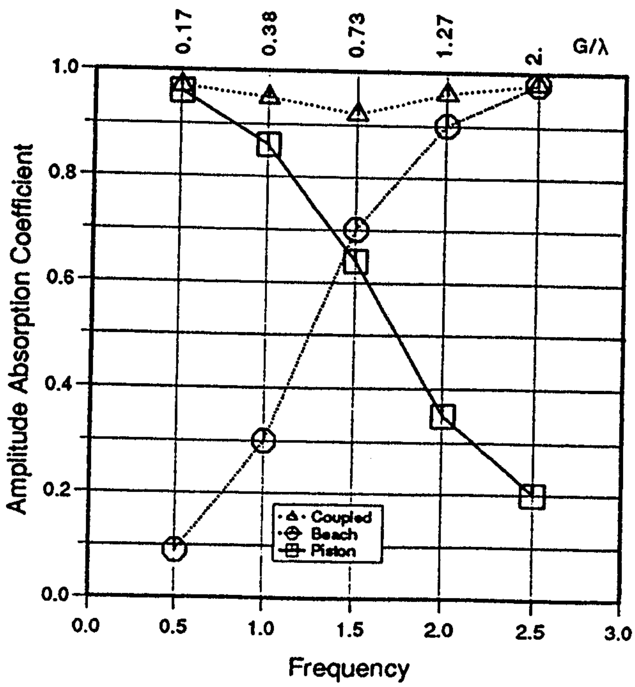

Fig.1. Amplitude Absorption Coefficients comparison of the three methods in nonlinear simulations

COUPLING PISTON AND BEACH

Looking at the curves relative to these two absorption techniques, the idea of coupling came very naturally. One method being efficient for low frequencies behaves like a *high-pass water waves filter*, while the other (the "beach") being a *low-pass* filter.

The methods being independent from each other, their implementation was numerically straightforward; the extra numerical burden implied is negligible with regard to the global algorithm.

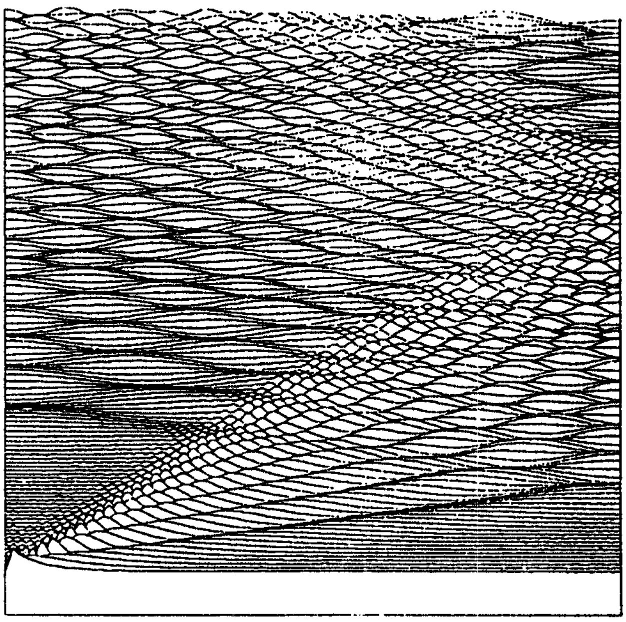

Fig.2 : impulsive wavemaker motion. L=10
reference case : no absorption

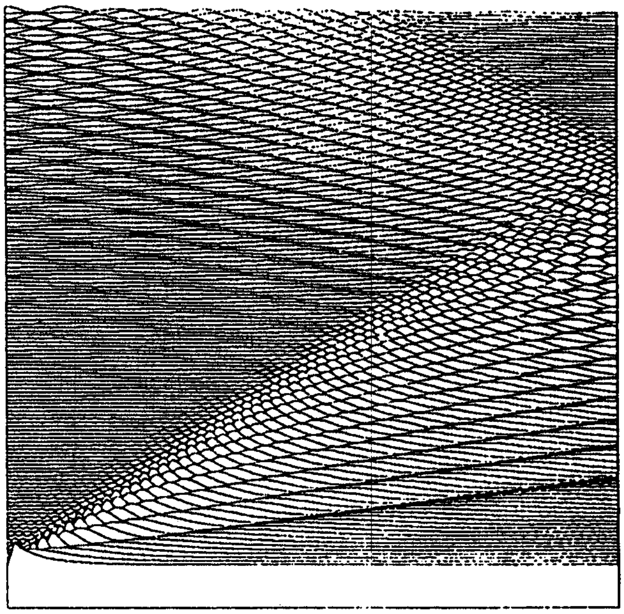

Fig.3 : impulsive wavemaker motion. L=10
Piston-like absorbing boundary condition (2)

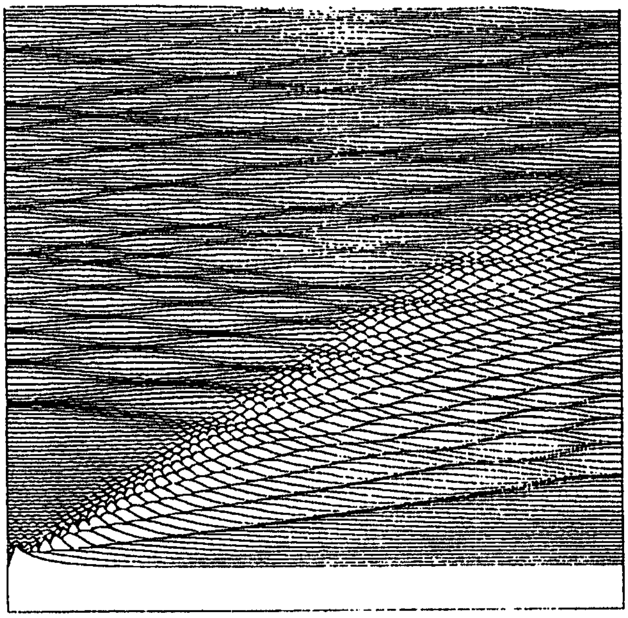

Fig.4 : impulsive wavemaker motion. L=10
Modified free-surface condition (3): beach length=2

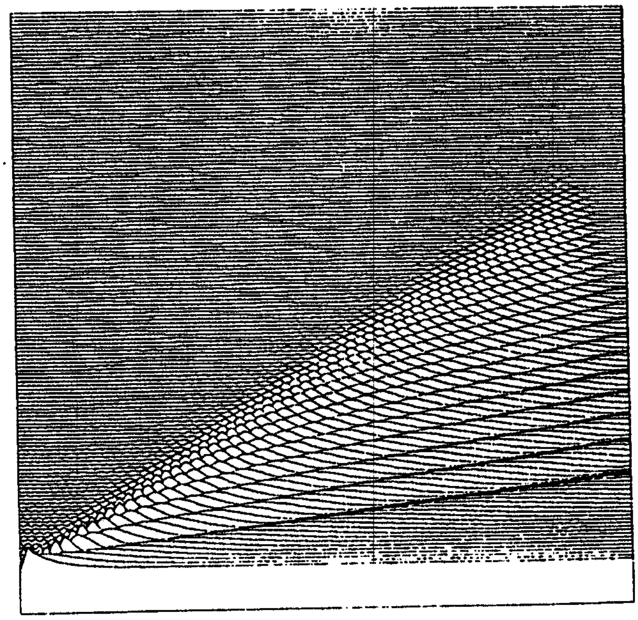

Fig.5 : impulsive wavemaker motion. L=10
Coupled Piston+Beach methods

Results of the coupling are marked by triangles on Fig.1. The global behaviour of the coefficient is conform to our expectations. The coupling benefits from the behaviour of the two methods in the medium frequency range. In the worst case ($\omega \approx 1.5$), where neither the numerical beach nor the piston boundary condition were able to absorb more than 70%, the coupling results in a coefficient never smaller than 0.93 (in terms of amplitude ratios, that is 0.995 for the corresponding absorption coefficient in energy). These results were obtained without any tuning, with a fixed "beach" length of $2h$.

In order to illustrate the ability of the coupling method to works *blindly* with regard to the incident wave frequency, it was submitted to a severe test. In a basin of length equal to $10h$, the left piston end was given at t=0 an impulsive velocity in such a way that all the frequencies were present in the spectrum of the wave train. At the opposite end of the basin, the three absorption strategies presented herein were successively used; a fourth case with no absorption at all (i.e: a steady wall) was also made as a reference. (Fig.2).

Results of these four simulations are given as waterfall view, on Fig. 2 to 5. The vertical axis is the time axis, the wavemaker is located at the left edge, the wave-absorber at the right edge. At first we observe the spreading of the wave packet with increasing time resulting from the dispersive nature of the free surface boundary conditions. Then, successive reflections of this system of waves occur upon alternatively the left and right end of the domain.

On fig. 3, the piston-like absorbing boundary condition (2) was applied on the right end of the basin. The high-pass character of this absorbing device is highlighted by comparing fig.3 to the previous simulation (fig.2) : the long leading waves are now cancelled while the shorter are partially, then completely reflected back in the computational fluid domain. The simulation using the numerical beach alone is illustrated by fig.4 where, now, the opposite low-pass filtering behaviour of this device may be observed. Finally the high damping efficiency obtained by coupling these two methods is clearly illustrated by fig.5 without further comments.

CONCLUSION

Coupling the "numerical beach" and the piston-like absorbing condition (2) seems to give better results than any other previous one used alone (Orlanski, beach, piston,..) for the absorption of outgoing waves of unknown frequency in the time-domain at the boundary of numerical wave basins. The most interesting features of this method are the following:

- no tuning with regard to the incident waves, neither before nor during simulation.
- fixed beach length, due to the fact that long waves are handled by the piston,
- it is as easy to implement in linear as in non-linear algorithms, and performs as well in both cases,
- its efficiency is very high in the whole frequency range of interest.

Then, we recommend to use it instead of the *"single beach"* or Orlanski classical approaches in linear or nonlinear numerical wave basin. It saves a lot of computing time and memory requirement in the present 2D non-linear algorithm; this latter point will be of crucial interest in 3D numerical wave basins now under development.

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REFERENCES

[1] I. ORLANSKI: *A Simple Boundary Condition for Unbounded Hyperbolic Flows. Journal of Computational Physics;* 21, pp251-269; 1976.

[2] J.E. ROMATE *Absorbing Boundary Conditions for Free Surface Waves* Journal of Computational Physics. 1992.

[3] C. MAISONDIEU, A. CLEMENT: *A realizable force feedback-feedforward control loop for a piston wave-absorber* 1993 - 8th Int. Workshop on Water Waves and Floating Bodies; St John's Newfoundland.

[4] A. CLEMENT *The Diffraction of a solitary wave by a free surface piercing cylinder.* 6th Workshop on Water Waves and floating Bodies. Woods Hole - Massachusetts. 1991

[5] G.R. BAKER, D.I. MEIRON, S.A. ORSZAG: Application of a Generalized Vortex Method to non-linear Free Surface flows. 3rd Int. Conf. Numer. Ship Hydrod. Paris 1981.

[6] Y. CAO, R.F BECK, W. SCHULTZ: *An absorbing Beach for Numerical Simulations of Nonlinear Waves in a Wave Tank* 1993 -8th Workshop on Water Waves and floating Bodies. St John's; Newfoundland.

[7] A. CLEMENT: *Coupling of Two Absorbing Boundary Conditions for 2D Time-domain Simulations of Free Surface Gravity Waves.* (1994) (submitted).

DISCUSSION

Yeung, R. W.: I thought you had neglected to mention that a completely general and perfect-absorption open boundary condition is to let ϕ and ϕ_n be relatable in a memory or convolution manner. Within the context of linearized time-dependent problems, we presented a "shell" method (see Yeung & Cermelli, 1993, 8th Workshop in Newfoundland) that can efficiently handle the memory by a one-time computation only. This idea has been validated in 3-D recently for a number of impulsive response functions by Hamilton (1994). The matching was exact and there was no parameter to adjust or optimize.

Clément, A. & Domgin, J. F.: The major criticism I formulate about Orlanski's condition lies in the fact that it is local in time, instead of accounting for the flow history through convolution integrals as should be the case if one considers it as the inverse Fourier transform of the Sommerfeld condition. Thus I agree with you on that point. The "shell function" matching technique you devised with Cermelli is an excellent illustration of this point of view. But using the absorbing shell function method in our numerical basin should require:

1. a matching between the non-linear inner solution with a linear solution outside, which could bring some numerical problems, especially in 2D models;

2. some efficient and robust algorithm to compute the time-domain Green-function in finite water depth. Such algorithms are still under development but are not yet available (to my knowledge!).

On the contrary, our technique cannot be applied in infinite water depths.

Kim, Y. W.: In the application of Orlanski's method, the generation of short waves can be predicted if we apply Fourier analysis to the discrete wave equation. To minimise this generation, Chan (2nd Int. Conf. on Numerical Ship Hydrodynamics) applied a 3 point filtering scheme that has a very strong damping effect. Have you ever tried this kind of scheme?

Clément, A. & Domgin, J. F.: No, we have not. I am not sure this filtering scheme is mass-conservative, whereas our method is as long as no dissipative term is added to the kinematic free surface condition, but only to the dynamic one.

Kim, Y. W.: For the damping that you applied, let's consider the linear case, i.e.

$$\phi_{tt} - \nu g \phi_{tz} + g\,\phi_z = 0$$

The dispersion relation of this condition is:

$$\omega^2 = \frac{i\nu kg \pm \sqrt{4kg - (\nu kg)^2}}{2}$$

continued/ ...

so there is another term inside the root, and it means that the linear dispersion relation is not valid any more. In numerical computation, this change will cause the generation of another kind of wave. In addition, when you select the damping coefficient adopting the above condition, you can know the approximate size of damping zone if you apply the basic theory of vibration. (see the paper by Faltinsen, 1978).

Clément, A. & Domgin, J. F.: Of course the dispersion relation is modified by the addition of the extra-term proportional to ϕ_z, and another kind of wave may exist in this case but we were here mainly concerned by the efficiency of the absorption in the non-linear computation. In that context, an argument is simply that this term is nothing but a local pressure over the free surface; the nature and sign of which ensures a negative work, removing energy from the fluid at each time-step. About the length of the numerical beach, we kept it constant in all our computations, because one of the most important constraints we adopted at the beginning of this work was to develop an absorption method completely independent of the nature (i.e. spectral content) of the incident wave train.

Grue, J.: Can the absorbing boundary condition you are describing also absorb a steady Stokes drift?

Clément, A. & Domgin, J. F.: The piston-like absorbing condition on the vertical surface is equivalent to a physical piston reacting to the hydro-dynamic force with the velocity $v(t) = F(t)$. Then no mass flux is possible across this surface, and we will have a return current in the numerical basin, as in the physical one.