Time Domain Ship Motions with a Nonlinear Extension

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1 Introduction

A linear, time-domain, three-dimensional Rankine panel method has been extended to a nonlinear formulation. This linear model, first described in Vada and Nakos (1993) and validated in Nakos, Kring, and Sclavounos (1993), and Kring (1994), provides a foundation from which a rational formulation for the nonlinear problem can evolve. This work considers weakly nonlinear ship wave patterns, since wave-breaking and spray are not included, so that the numerical model designed for the linear problem provides a good template for a nonlinear extension.

The linear problem has been validated through extensive convergence studies over a wide range of vessels and operating conditions, and, more importantly, a thorough numerical analysis delineated the stability properties of the method. The first part of the analysis identified the dispersion properties of a wave propagating over a discrete free surface. The second part examined the equations of motion in order to insure a zero-stable method and provide a region of absolute stability for the numerical integration.

The nonlinear formulation evolved directly from the numerical insight gained in the previous work. In fact, the weak stability analysis, applied to the examination of the linear equations of motions, was adapted from a standard technique for studying nonlinear ordinary differential equations. In the limit as the wave disturbance and numerical time-step size reduce to zero, the nonlinear reduces to the proven linear model. Without this stable basis, a rational nonlinear formulation is difficult to obtain.

The linear foundation is first briefly revisited. After restating the stable form for the numerical modeling of the linear free surface conditions and the equations of motion, a natural, nonlinear form is presented.

2 Linear Foundation

The two vital components to the linear formulation are the discretization of free surface conditions and the identification of stable form for the equations of motion. The discretized free surface and its faithfullness to the continuous formulation were presented at a previous workshop, so only the continuous form of the free surface equations is restated here.

Linear, kinematic, free surface condition:

$$[\frac{\partial}{\partial t} - (\vec{W} - \nabla\Phi)\cdot\nabla]\,\zeta = \frac{\partial^2\Phi}{\partial z^2}\,\zeta + \frac{\partial(\phi+\psi)}{\partial z} \quad \text{on } z = 0. \tag{1}$$

Linear, dynamic, free surface condition:

$$[\frac{\partial}{\partial t} - (\vec{W} - \nabla\Phi)\cdot\nabla)](\phi + \psi) = -g\zeta + (\vec{W}\cdot\nabla\Phi - \frac{1}{2}\nabla\Phi\cdot\nabla\Phi) \text{ on } z = 0. \qquad (2)$$

where the total disturbance potential, Ψ, is decomposed into a basis flow, Φ, a local (or instantaneous) flow, ϕ, and a memory (or wave) flow, ψ. The ship's velocity is $\vec{W}$, and ζ is the linear wave elevation. These conditions provide the template for the nonlinear free surface conditions. The Emplicit-Euler scheme described in Vada and Nakos (1993) provides the discrete form. It is important to restate this form for the conditions as they are the template for the nonlinear method.

The primary point of interest in the decomposition of the problem lies with the separation of the linear perturbation into two components, the instantaneous and memory flows. This decomposition was dictated by the numerical stability analysis conducted by Kring (1994) and is necessary to produce a stable form of the equations of motion. The instantaneous portion of the hydrodynamic solution must be separated analytically and treated implicitly in the equations. In practice, this numerical failure appears in the time derivative of the potential as discussed in last year's workshop by Cao, Beck, and Schultz (1994). A separate boundary value problem could be posed for the time derivative of the potential itself, but a more natural formulation based upon physical reasoning stems from Cummins (1962). In this form, a separate boundary value problem is posed for the pressure relief, or $\phi = 0$, flow. The resulting equations of motion take the form:

$$(M + a_0)\,\ddot{\vec{\xi}}(t) + b_0\,\dot{\vec{\xi}}(t) + (C + c_0)\,\vec{\xi}(t) = \vec{F_m}(\dot{\vec{\xi}}, \vec{\xi}, t) \qquad (3)$$

where, the matrix coefficients, a_0, b_0, and c_0, represent the instantaneous, or infinite frequency, force. The memory force, $\vec{F_m}$, does not depend upon the instantaneous acceleration so that a stable numerical integration can be produced. In practice standard Runge-Kutta or Adams-Bashforth-Moutlon schemes work quite well.

3 Nonlinear Extension

A solution scheme has been developed that parallels the linear method very closely by design. The essential difference comes with the realization that this is a weakly nonlinear problem. A decomposition of the flow is used such that all nonlinear terms are treated explicitly and a quasi-linear problem can be solved for each time step. The formulation reduces to the exact problem as the time-step size goes to zero. This results in a time-faithfull nonlinear simulation. Accuracy, with respect to the exact conditions, is ensured in the continuous formulation since the time-step size is relatively very small. The time-step sizes generally used, as suggested by the linear numerical stability problem, easily justifies this weakly nonlinear decomposition.

The exact form for the free surface conditions follows, where the wave elevation is assumed to be single-valued, so there are no breaking waves or spray.

Kinematic:

$$[\frac{\partial}{\partial t} - (\vec{W} - \nabla\Psi)\cdot\nabla)](z - \zeta(x,y,t)) = 0 \text{ on } z = \zeta(x,y,t). \qquad (4)$$

Dynamic:

$$(\frac{\partial}{\partial t} - \vec{W} \cdot \nabla)\,\Psi + \frac{1}{2}\nabla\Psi \cdot \nabla\Psi + g\zeta = 0 \quad \text{on } z = \zeta(x, y, t). \tag{5}$$

In order to solve this problem an explicit decomposition of the total disturbance potential, Ψ, and the wave elevation, ζ, is posed,

$$\begin{aligned} \Psi(\vec{x}, t) &= \Psi(\vec{x}, t - \Delta t) + \phi(\vec{x}, t) + \psi(\vec{x}, t) \\ \zeta(x, y, t) &= \zeta(x, y, t - \Delta t) + \eta(x, y, t) \end{aligned} \tag{6}$$

here, notating the previous solution as $\Psi_p = \Psi(\vec{x}, t - \Delta t)$, and $\zeta_p = \zeta(x, y, t - \Delta t)$, the new variables ϕ, ψ, and η are seen as perturbations in the solution from one time step to another. These temporal perturbations will be treated as in the linear problem, except that the previous solution provides an explicit forcing to the problem, and the Taylor expansion is about the previous wave elevation rather than the $z = 0$ plane. A boundary value problem is solved for the instantaneous and memory flows separately. The conditions now take the form,

Kinematic:

$$\begin{aligned} &[\frac{\partial}{\partial t} - (\vec{W} - \nabla\Psi_p) \cdot \nabla]\,\eta = \\ &-[\frac{\partial}{\partial t} - (\vec{W} - \nabla\Psi_p) \cdot \nabla]\,\zeta_p - \eta\frac{\partial}{\partial z}(\nabla\Psi_p \cdot \nabla\zeta_p + \frac{\partial\Psi_p}{\partial z}) + \frac{\partial(\Psi_p + \phi + \psi)}{\partial z} \quad \text{on } z = \zeta_p. \end{aligned} \tag{7}$$

Dynamic:

$$\begin{aligned} &[\frac{\partial}{\partial t} - (\vec{W} - \nabla\Psi_p) \cdot \nabla)]\,(\phi + \psi) = \\ &-g(\zeta_p + \eta) - [1 - \eta\,\frac{\partial}{\partial z}]\,((\frac{\partial}{\partial t} - \vec{W} \cdot \nabla)\Psi_p + \frac{1}{2}\nabla\Psi_p \cdot \nabla\Psi_p) \quad \text{on } z = \zeta_p. \end{aligned} \tag{8}$$

This looks similar to the linear conditions by design. This not only provides a time faithfull expression for the nonlinear conditions but a form that has been thoroughly tested in its linear limit. The linear basis flow is essentially replaced here by the solution from the previous time step.

The equations of motion must take a slightly modified form. The pressure relief problem from which, ϕ, is obtained is taken about the elevated free-surface so the instantaneous forces becomes time-dependent and nonlinear. Also, the hydrostatic restoring force, $\vec{F}_c$ is treated nonlinearly.

$$(M + a_0(t))\,\ddot{\vec{\xi}}(t) + b_0(t)\,\dot{\vec{\xi}}(t) + c_0(t))\,\vec{\xi}(t) = \vec{F}_m(\dot{\vec{\xi}}, \vec{\xi}, t) + \vec{F}_c(\vec{\xi}, t) \tag{9}$$

The important feature in this form is that the instantaneous acceleration terms are still treated implicitly in the numerical integration.

Preliminary results for a submerged spheroid look very encouraging and a sample run with the new code is illustrated in figure 1 with the linear result included for comparison. This case shows a small wave distrubance, the maximum wave elevation is only three percent of the body length, so that the nonlinear and linear computations are not significantly different. This demonstrates that the nonlinear method reduces to the linear method, and work is currently being conducted to extend the disturbance to more nonlinear regimes.

4 References

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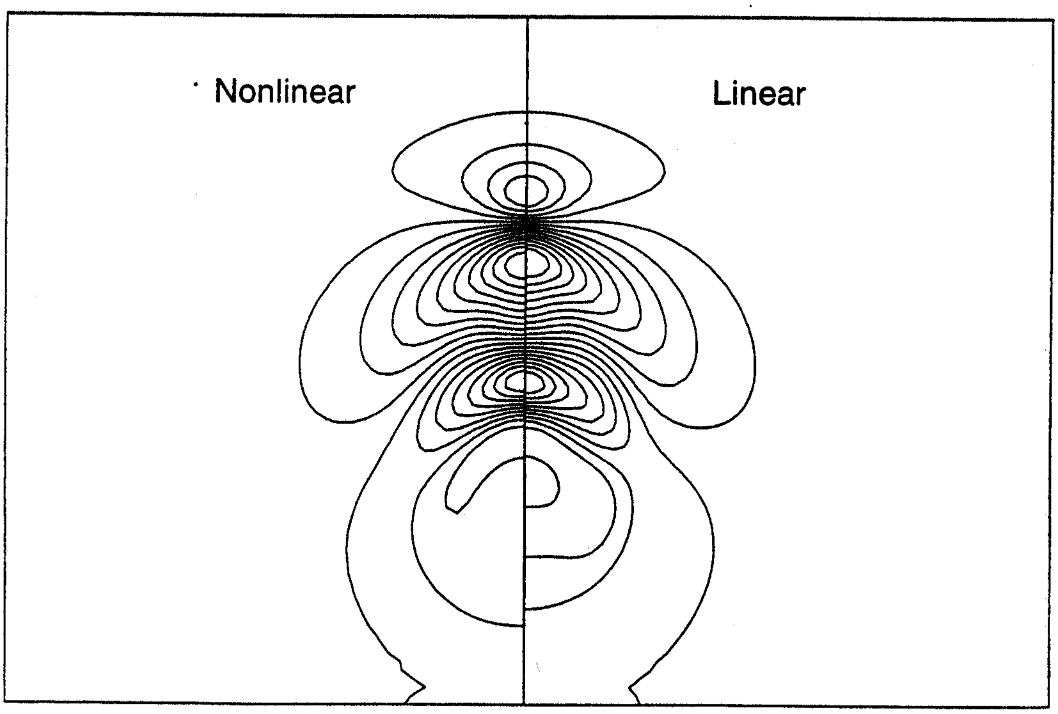

Figure 1. Nonlinear vs. linear wave contours for a submerged spheroid after travelling one ship length from rest.

DISCUSSION

Raven. H. C. : Thanks for this interesting paper. Although of course your linear time-domain method has not been set up for steady wave making calculations, you showed some results for that problem. Could you give an indication of the true (dimensionless) time required before the results have become approximately steady? And how many time steps does this take (if only the steady result is of interest)? How does the CPU time required for such a calculation compare with that of the original SWAN wave making code?

Kring, D. , Huang, Y. & Sclavounos, P. D.: Our time-domain code has been set up to solve the complete ship motions problem of course, but in the absence of any ambient waves, we recover the steady wave behaviour. The start-up transients all decay after the ship has travelled approximately six ship-lengths for all cases we have studied. This translates into a few hundred time steps. The stability analysis has allowed us to optimize the numerical problem, so that the entire unsteady simulation time is no larger than the original SWAN steady code.

Faltinsen, O.: My question consists of three parts. What happens to the short wavelengths? Can you simulate following seas? Can you treat the $\tau = 1/4$ problem?

Kring, D. Huang, Y. & Sclavounos, P. D.: The Rankine panel method can not accurately represent wavelengths at the size of the Nyquist wavelength or smaller. In fact, the stability analysis shows that energy at these scales possesses an unphysical group velocity. Fortunately, we have precise knowledge of the short wavelength behaviour so we can effectively filter them. The resultant loss of energy is very small as shown by convergence of the method. We can simulate all wave headings, including encounter regimes both below and above $\tau = 1/4$. We can even produce convergent solutions that are independent of domain size exactly at $\tau = 1/4$. The $\tau = 1/4$ disturbance is not wavelike but our numerical beach has proven to be quite effective. The truncation error due to domain size can be made arbitrarily small given sufficiently large domains. Practically, we have found no case for which we could not make a sufficiently large domain to avoid significant truncation errors.