# Wave Force on Floating Platform on the Water of Varying Depth

M. Ohkusu and Y.Imai

Research Institute for Applied Mechanics, Kyushu University, Japan

## Introduction

The location of huge floating platforms like a floating airport which was once designed in Japan will not be offshore but close to the shore where the water is not deep. When the size of the platforms is so huge like the designed Japanese airport (a few thousands meters long) as comparable to the horizontal scale of depth variation of the sea bottom, the bottom topography will affect wave forces on the platforms. We will present a numerical method based on linear theory to predict wave forces on such a large floating platform supported by numerous floats and located on the sea of sloping bottom close to the shore.

## Refraction and Diffraction

We assume a platform supported by numerous floats of vertical and truncated circular cylinder shown by shaded circles in Fig.1. The platform is floating on the free surface with its long side paralell to the shoreline (the $y$ axis ). For simplicity the number of floats in the lines paralell to the shore is assumed to be large enough to be considered infinite. The sea bottom has a slope from the shore of zero depth ($x = 0$) to offshore of large depth. The bottom contour is paralell to the $y$ axis.

The length scale of the platform is assumed to be of the same order of the horizontal scale of depth variation of the sea, while the size of the cylindrical floats is very small. Therefore waves propagating distances as long as the whole platform's dimension will be refracted under the effect of the bottom slope. On the other hand the wave motion within the distance comparable to the size of each cylindrical float will be considered to be the wave motion on the constant water depth.

When waves with the crest line paralell to the $y$ axis are incident from offshore, they will be refracted due to the sloping bottom. Their height and length will change during propagating on the distance of the platform's scale. They are diffracted by the floats and the interaction between them will occur. If the waves come close to the shore to become steeper than some critical value, they will break.

Refraction on the sloping bottom whose slope is much smaller than the wave slope will be described by the mild slope equation (Mei (1988))

$$\nabla(CC_g\nabla\zeta) + \omega^2\frac{C_g}{C}\zeta = 0 \tag{1}$$

where $\zeta(x, y)e^{i\omega t}$ is the wave elevation and the operator $\nabla$ implies to take the gradient in the horizontal plane. $C$ and $C_g$ are the local phase velocity and the group velocity of waves. They are determined by the wave frequency $\omega$ and the local wave number $k$ at the reference depth $h$. The local wave number satisfies the relation

$$k_0 = \frac{\omega^2}{g} = k\tanh kh \tag{2}$$

The mild slope equation can not describe the localized effect of scattering and radiation of the waves by the floats. Our assumption is that the size of each float is very small relatively to the scale of the bottom slope and the water depth can be considered to be constant in the region of the float size and close to it. The solution of the diffraction waves in this inner region must match with the outer solution, a solution of the miled slope equation in the region shown by $\Omega$ in Fig.1, valid away from each float. In the diffraction problem the inner solution of wave elevation at close to a truncated circular cylinder will be written in the form

$$\zeta_I(r,\theta)e^{i\omega t} = \sum_{m=0}^{\infty} F_m \left[ J_m(kr) B_{m0} H_m^{(2)}(kr) + \sum_{p=1}^{\infty} B_{mp} K_m(\alpha_p r) \right] \cos m\theta e^{i\omega t} \tag{3}$$

where $(r,\theta)$ is the polar coordinate with the the origin at the center of the circular cross section of the cylinder. $F_m$ are constants to be determined later. $B_{mp}$ are determined such that the solution (3) satisfies the condition of zero normal velocity on the surface of the truncated circular cylinder. $k$ is the local wave number at the center of the cylinder. $i\alpha_p (p = 1, 2, \ldots)$ are the solutions of imaginary number of equation (2). We henceforth omit the time factor $e^{i\omega t}$.

Upon letting $r \to \infty$, the outer limit of the solution (3) will be

$$\zeta_I \sim \sum_{m=0}^{\infty} F_m [J_m(kr) + B_{m0} H_m^{(2)}(kr)] \cos m\theta \tag{4}$$

.Equation (4) will be matched with the solution of the mild slope equation in the region $\Omega$. The unknowns $F_m$ are determined by this matching. The mild slope equation is solved numerically as stated later and eq.(4) will be imposed as a boundary condition on $Pj (j = 1, 2, \ldots)$ shown in Fig.1.

For the radiation problem this boundary condition for the outer solution should include a particular solution satisfying the inhomogeneous condition for the normal velocity on the float surface.

The assumption of an infinite number of the floats in the $y$ direction will lead to the wave motion periodical to the $y$ direction. The boundary condition on two chain lines in Fig.1 with the spacing $B$ is the zero normal fluid velocity.

**Boundary Conditions Ashore and Offshore**

We need the boundary conditions both ashore and offsore for the mild slope equation (1). Derivation of the shore condition is not straightforward.

Waves will become steeper as they come close to the shore and break when their height is over a critical value. This effect must be considered in the shore boundary condition. Floating platforms will not be located so close to the shore and we may assume it is not in the midst of the breaking waves. Consequently we may not need to predict the pressure field of breaking waves that is anyway difficult. The effect we need to model will be reduction of the amplitude of the reflected waves from the shore. With this idea we derive a mathematical expression of the reflected waves from the shore accounting for the effect of sloping bottom and empirically prescribed reflection coefficient less than unity resulting from wave energy dissipation by breaking.

We consider first the case of complete reflection from the shore. At shallow region close to the shore the mild slope equation (1) reduces to

$$\frac{\partial^2 \zeta}{\partial t^2} - g\left[\frac{\partial}{\partial x}\left(sx\frac{\partial \zeta}{\partial x}\right) + \frac{\partial}{\partial y}\left(sx\frac{\partial \zeta}{\partial y}\right)\right] = 0 \tag{5}$$

Diffraction waves due to the floats disposed as shown in Fig.1 are generally short crested. So we seek the solution of the linear shallow water equation (5) in the form of

$$\zeta_n = \eta_n(x)\cos(\frac{2n\pi}{B}y)e^{i\omega t} \tag{6}$$

where $s$ is the bottom slope and therefore $sx$ is the local water depth $h(x)$. The solution $\eta_n(x)$ is given by

$$\eta_n(x) = W_{\kappa\mu}\left(\frac{4\pi n}{B}x\right) / \sqrt{\frac{2\pi}{B}x} \quad \text{where} \quad \kappa = \frac{\omega^2}{g}\frac{4\pi n s}{B}, \quad \mu = 0 \tag{7}$$

$W_{\kappa\mu}(\xi)$ is the Whittaker's function as a solution of the equation

$$\frac{d^2W_{\kappa\mu}}{d\xi^2} + \left(-\frac{1}{4} + \frac{\kappa}{\xi} + \frac{\frac{1}{4}-\mu^2}{\xi^2}\right) W_{\kappa\mu} = 0 \tag{8}$$

For $n = 0$, the solution will be a well-known one

$$\eta_0(x) = J_0\left(2\omega\sqrt{\frac{x}{gs}}\right) \tag{9}$$

We can show a superposition of those functions

$$\zeta_R = A_0 J_0\left(2\omega\sqrt{\frac{x}{gs}}\right) + \sum_{n=1}^{\infty} A_n \frac{W_{\kappa(n)\mu}\left(\frac{4\pi n}{B}x\right)}{\sqrt{\frac{2\pi}{B}x}} \cos\left(\frac{2\pi n}{B}\right) \tag{10}$$

is a mathematical expression of the completely reflected waves in the linear theory. The first term is known to match with the nonlinear solution at very close to the shore. Our conjecture is that it may be true with the terms periodical in the $y$ direction too. In this equation $A_n(n = 0, 1, 2, \ldots)$ are constants and $\kappa(n)$ implies that $\kappa$ is a function of $n$ as given in equation (7 ).

At large distances from the shore but when

$$\frac{4\pi n}{B}x < 4\kappa(N) \tag{11}$$

is satsified, the asymptotic espression of euation (10) will be (Abramowitz & Stegun (1964))

$$\zeta_R \sim A_0\left(\frac{\omega^2}{gs}\pi^2 x\right)^{-\frac{1}{4}} \cos\left(2\omega\sqrt{\frac{x}{sg}} - \frac{\pi}{4}\right) + \sum_{n=1}^{N} A_n\sqrt{2n}\left(\frac{\omega^2}{gs}x\right)^{-\frac{1}{4}} \cos\left(2\omega\sqrt{\frac{x}{sg}} - \kappa(n) + \frac{\pi}{4}\right) + O(e^{-x}) \tag{12}$$

This is evidently a standing wave when the incident waves are completely reflected at the shore. Furthermore it agree with our intuitive description of waves propagating in a channel with the sloping bottom. A wave mode of sinusoidal variation in the $x$ direction as well as of the periodical behaviour $\cos\left(\frac{2\pi n}{kB}ky\right)$ in the $y$ direction can exist only when $kB > 2n\pi$ is satisfied. Local wave number $k$ will become larger when the water depth becomes shallower. The consequence is that the maximum of $n$ that can exist will decrease when the waves are away from the shore. The condition $kB > 2n\pi$ determining the max $n$ at a distance $x$ is identical to eq(11) when the local wave number at the shallow water

$$k \sim \frac{\omega}{\sqrt{gsx}} \tag{13}$$

is substituted.

It is straightforward to obtain a linear expression of the incident waves periodical in the $y$ direction and propagating on the sloping bottom toward the shore. It is

$$\zeta_I = \frac{A_0}{2}\left[J_0\left(2\omega\sqrt{\frac{x}{gs}}\right) + iY_0\left(2\omega\sqrt{\frac{x}{gs}}\right)\right] + \sum_{n=1}^{\infty}\frac{A_n}{2}\,\frac{W_{\kappa\mu}\left(\frac{4\pi n}{B}x\right) + ie^{i\kappa\pi}M_{\kappa\mu}\left(\frac{4\pi n}{B}x\right)}{\sqrt{\frac{2\pi}{B}x}} \cos\left(\frac{2\pi ny}{B}\right) \tag{14}$$

where $M_{\kappa\mu}(\xi)$ is another Whittaker's function. Naturally this expression is singular at the shore $x = 0$.

The superposition of equations (10) and (14) at a small $x$ value will give the shore condition for the mild slope equation in the form

$$\zeta = C_1\zeta_I + C_2\zeta_R \tag{15}$$

The value of $C_2/C_1$ is prescribed such that the reflection coefficient from the shore, which is predicted by some empirical formula as a function of the slope of the sea bottom (for example see Battjes (1988)), is equal to $C_2/C_1/(1 + C_2/C_1)$.

The offshore condition is readily derived assuming the place is far away from the platform and the water depth is very deep. It is

$$\zeta = e^{ik_0 x} + \sum_{j=1}^{j_{max}} D_j e^{-k_0 x \cos\chi} \cos\left(k_0 y \frac{2\pi(j-1)}{k_0 B}\right) \tag{16}$$

where the first term is the incident wave and

$$\cos\chi = \sqrt{1 - \left(\frac{2\pi(j-1)}{k_0 B}\right)^2}, \qquad j_{max} = \left[\frac{k_0 B}{2\pi}\right] + 1 . \tag{17}$$

## Numerical Computation

It is a well known procedure to replace a boundary value problem of the mild slope equation with the stationarity of a functional of the wave elevation. The boundary conditions (4), (15) and (16), and the condition of zero normal fluid velocity on the chain lines in Fig.1 are incorporated in this functional. The region of the free surface $\Omega$ is divided into a number of panels and the functional is expressed with unknown $\zeta$ on those panels (linear variation assumed on the panel).

Numerical results for wave forces of the first order and the second order ( steady forces) computed by integrating the pressure on the floats' surface will be presented.

## References


[1] Mei,C.C.(1988): *The Applied Dynamics of Ocean Surface Waves* 2nd Printing, World Scientific

[2] Battjes, J.A.(1988): Surf Zone Dynamics, Annual Review of Fluid Mechanics, vol.20

[3] Abramowitz,M. and Stegun,I.(1964): *Handbook of Mathematical Functions, with Formulas, Graphs and Mathematical Tables* Dover


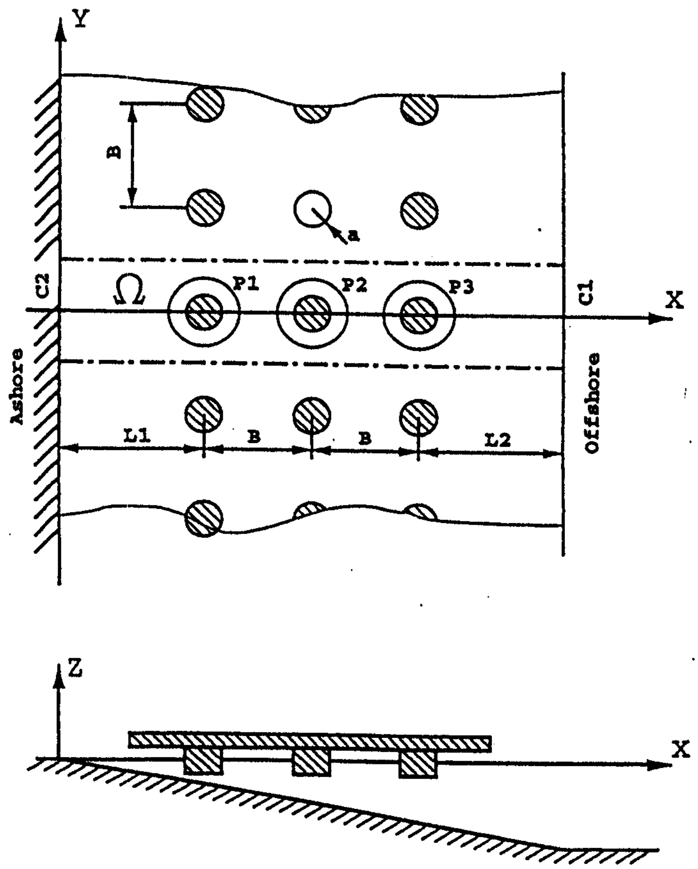


**Fig.1 Coordinate System**