Step-response functions in ship hydrodynamics

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Abstract

In this paper step-response functions are used to calculate the hydrodynamic coefficients and drift forces for a two-dimensional problem of a cylinder of infinite length. The equation of motion will be written in a general form containing a convolution integral of the step-response function. The response function is written as a finite sum of Laguerre polynomials in order to make the function both smooth and asymptotically correct. The effect of forward speed on the response function will be studied.

Introduction

In the past much attention has been paid to the calculation of both the hydrodynamic coefficients and the drift forces of a ship. Most of these calculations were done under the assumption that the incoming wave was sinusoidal. This assumption was made in order to simplify both the equation of motion and the numerical algorithm, and gave rise to frequency-domain analyses.
However, it is clear from elementary water-wave knowledge that ocean waves are far from sinusoidal. Using the frequency domain, it is assumed that the waves can be decomposed in several harmonic waves. For each frequency separate calculations have to be carried out. The forces on the object can then be calculated using the Fourier-transform of the incoming wave. Inverse Fourier-transforming then gives the time history of the actual force.
The time domain, however, allows us to simulate the full wave itself, thus needing only one calculation per wave. Furthermore it allows us to calculate the hydrodynamic coefficients for all frequencies in one single calculation. Thus it is very useful to drop the assumption of harmonic waves and to study general time signals instead.

Prins [2] developed an algorithm to solve the time-dependent equations using a boundary-integral method. The absorbing-boundary condition used in that study only absorbs waves of one specific frequency. Thus a more general condition is needed in order to absorb the outgoing waves. This method has been developed by Sierevogel and will also be presented at this workshop. Furthermore the equation of motion has to be modified. In this equation, the added mass and damping coefficients depend on frequency; they are replaced by a memory integral over the step-response function. Then the hydrodynamic coefficients can be calculated using this response function. The results found will be compared with the results of Prins [2] and with Vugts [3].

Mathematical model

The mathematical model used in this study is the same as used by Prins [2], based on potential theory and linearization around the double-body potential. However, because we want to study general time signals, the absorbing-boundary condition and the equation of motion have to be adapted. A boundary condition which absorbs general time signals has been developed by Sierevogel and can be applied close to the ship. The equation of motion has to be rewritten according to Ogilvie [1]. He showed that this equation should be written as an integro-differential equation:

$$(\boldsymbol{M} + \overline{\boldsymbol{A}}) \frac{\partial^2 \vec{Y}}{\partial t^2} + \overline{\boldsymbol{B}} \frac{\partial \vec{Y}}{\partial t} + \overline{\boldsymbol{C}} \vec{Y} +$$

$$+\int_0^t \boldsymbol{K}(t-s)\frac{\partial \vec{Y}}{\partial s}(s)ds = \begin{pmatrix} \vec{F}_{\text{inc}} \\ \vec{M}_{\text{inc}} \end{pmatrix} \quad . \tag{1}$$

The matrix function $\boldsymbol{K}(t)$ is the step-response matrix for the hull, which is not dependent of frequency. The matrices $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{B}}$ only depend on the geometry of the hull and the forward speed. Note that they do not depend on frequency. The added mass and damping as defined in the frequency domain can be calculated using

$$\boldsymbol{A}(\omega) = \overline{\boldsymbol{A}} - \frac{1}{\omega}\int_0^\infty \boldsymbol{K}(t)\sin(\omega t)dt \quad , \tag{2}$$

and

$$\boldsymbol{B}(\omega) = \overline{\boldsymbol{B}} + \int_0^\infty \boldsymbol{K}(t)\cos(\omega t)dt \quad . \tag{3}$$

This means that once the hull-dependent matrices $\boldsymbol{K}(t)$, $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{B}}$ are known, the added mass and damping can be calculated for every relevant frequency.

To calculate the hydrodynamic coefficients we have to prescribe a forced motion of the ship. This displacement function must contain enough information for all frequencies of interest and must be twice continuous differentiable at $t = 0$. A displacement function which meets these demands, is given by

$$f(t) = \begin{cases} 0 & t < 0 \\ \left(\sum_{i=1}^{M} \frac{\sin(\omega_i t)}{\omega_i}\right)\left(1 - e^{-ct}\right)^3 & t \geq 0 \end{cases} \quad . \tag{4}$$

The sine functions are scaled in order to ensure that all frequencies equally contribute to the velocity. The function is damped by the last factor in order to make it continuously differentiable in $t = 0$. The constant c is positive. Tests showed that three frequencies are enough to have sufficient information throughout the frequency-domain of interest.

Sum representation using Laguerre polynomials

The most straightforward way of calculating the step-response function is by using a least-squares method on the different forces calculated from the equation of motion and from potential theory. However, the system of equations will be under determined, thus yielding possible sources of errors. If we would allow every possible value of $\boldsymbol{K}$ at any time t_i without demanding some sort of continuity, the result for $\boldsymbol{K}(t)$ might include delta pulses superimposed upon the wanted function. These delta pulses would alter the result for $\overline{\boldsymbol{A}}$ considerably. Thus we have to require that $\boldsymbol{K}(t)$ is a smooth function of time.
An elegant way of imposing continuity in time upon the step-response function is expressing the function in terms of a sum of orthogonal polynomials. To avoid infinite sums, the orthogonal polynomials should have the same behaviour as the function we want to calculate: a damped, oscillatory function. This requirement is fulfilled by the generalized Laguerre polynomials. Thus we may write

$$K_{ij}(t) = \sum_{p=1}^{N} c_p L_p^{'}(\sigma(t)) + \varepsilon(t) \quad , \tag{5}$$

with

$$L_p^{'}(x) = e^{-\frac{x}{2}} L_p(x) \quad ,$$

and

$$\sigma(t) = \frac{\sigma_1 t}{\sigma_2 - t} \quad ; \quad \sigma_1, \sigma_2 > 0 \quad .$$

The function $\varepsilon(t)$ is assumed to be small and to decay with increasing N. The scale function σ is introduced in order to match the time scale of the Laguerre polynomials with the time scale of the step-response function. The coefficients c_p can be determined using a least-squares fit on the equation of motion (1). The number of terms in the sum-representation of $K_{ij}(t)$ is much smaller than the number of time steps, thus yielding an over-determined system of equations.

To improve the result for the step-response function some analytical knowledge about the behaviour of the function has been used in the least-squares fit. Because the damping for zero frequency is zero, we can conclude from (3) that

$$\int_0^\infty K_{ij}(t)dt + \bar{b}_{ij} = 0 \quad . \tag{6}$$

Furthermore we can invert the cosine transform of the step-response function:

$$K_{ij}(t) = \frac{2}{\pi}\int_0^\infty \left(b_{ij}(\omega) - \bar{b}_{ij}\right)\cos(\omega t)d\omega \quad .$$

From this we can conclude that

$$\frac{\partial K_{ij}}{\partial t}(0) = 0 \quad . \tag{7}$$

Imposing the latter condition improves the results close to $t = 0$ considerably, which is a sensitive part of the step-response function. Equation (6) improves the results for $t \rightarrow \infty$, which are not part of the fit. However, the asymptotical behaviour is necessary for the calculation of the added mass and damping coefficients.

Results

The results presented in this section are calculated using the displacement function as discussed in the previous section. The time step used was $\Delta t = .004$; the time integration lasted 1000 steps. The grid size was chosen such that even the shortest wave was represented accurately. The absorbing boundary was adjusted according to the guidelines of Sierevogel based on the lowest frequency.

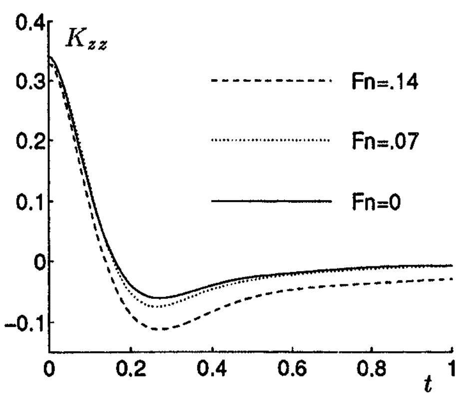

Figure 1: Step-response function $K_{zz}(t)$ for three velocities.

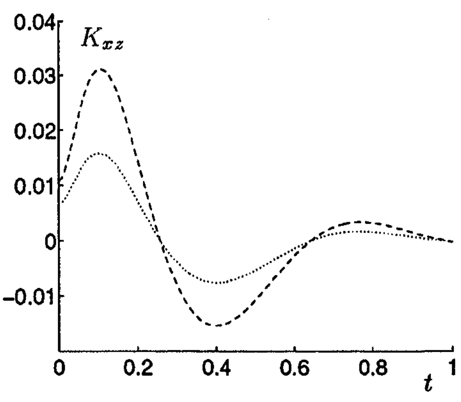

Figure 2: Step-response function $K_{xz}(t)$ for three velocities.

Figures 1 and 2 show the step-response functions in heave for the $z-$ and $x-$direction for $Fn =$ 0, .07, and .14. The value of $K_{zz}(0)$ depends only little on the velocity U. However, the function seems to steepen for small t, and to deaden less quickly for large t. This means that the influence of a disturbance is much longer perceptible for higher velocities. The function $K_{xz}(t)$ seems to depend linearly on the velocity. For zero forward speed, this function is of course zero. The response function for $Fn = .14$ is 'exactly' twice the function for $Fn = .07$. This result has been obtained despite the fact that second-order terms in the velocity were taken into account in the free-surface. It implies that the coupled hydrodynamic coefficients depend linearly on the forward speed.

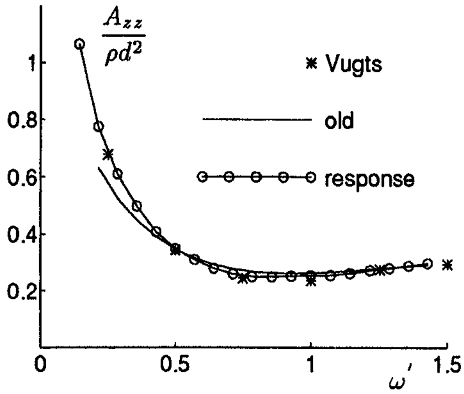

Figure 3: Added mass coefficient in heave, $Fn = 0$.

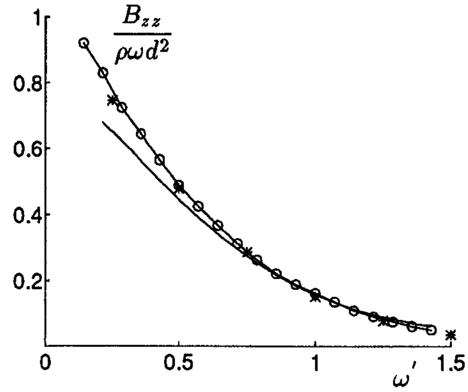

Figure 4: Added damping coefficient in heave, $Fn = 0$.

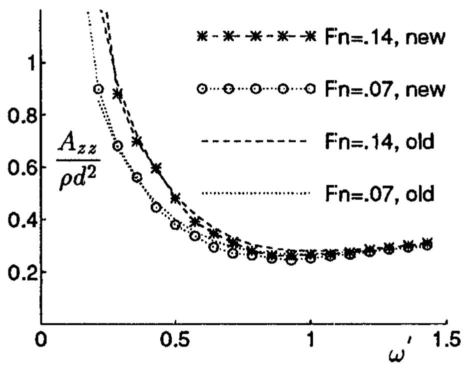

Figure 5: Added mass coefficient in heave.

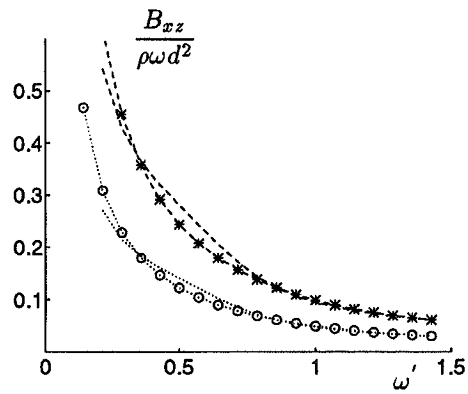

Figure 6: Coupled added damping coefficient in heave.

In figures 3 and 4 the added mass and damping coefficients are given and compared with calculations of Prins [2] and Vugts [3]. The agreement with Vugts for the new results is much better than for the old results. Figures 5 and 6 show the added mass and coupled added damping coefficients calculated using the above step-response functions. The drawn lines represent the old calculations; the circles and asterisk the new ones. Both the added mass and the coupled added damping coefficients agree very well with the old calculations.

References

[1] Ogilvie, T.F., *Recent progress towards the understanding and prediction of ship motions*, Proceedings of the 5th Symposium on Naval Hydrodynamics,1964

[2] Prins, H.J. and Hermans, A.J., *Time-domain calculations of drift forces on floating two-dimensional object in current and waves*, JSR, Vol. 38, No. 2

[3] Vugts, J.H., *The hydrodynamic coefficients for swaying, heaving and rolling cylinders in a free surface*, Report nr. 194, Delft University of Technology, The Netherlands, 1968

DISCUSSION

Bingham, H. B.: This is an interesting technique and appears to produce very nice results. As I understand it you solve a Volterra integral equation, where the kernel is the body motion $x_j(t)$, to get the step-response function $K_{jk}(t)$. Can you show that this integral equation has a unique solution in general, or at least for the motion that you are using?

Prins, H. J. & Hermans, A. J.: In general this integral equation does not have a unique solution. Only signals with infinite frequency-domain support will render a unique solution for $K(t)$. For signals with finite support, the solution for $K(t)$ will only be unique within this support in the frequency domain. This means our results for $K(t)$ may differ from the true K by some very slow and very fast oscillations. However, for the results we are interested in, the added mass and damping, this is of no harm within the support of the forcing signal. Extrapolation outside this range is questionable, and will be the subject of investigation in the future.

Clément, A.: You mentioned possible numerical difficulties which may occur when applying an actual step motion to compute the step response function. We use actual step motions in our numerical basin (i.e.: $V(t) = 1/\Delta t$ during Δt) without special trouble. Could you explain why it is not possible in your approach?

Prins, H. J. & Hermans, A. J.: It is indeed possible to use such an approximation for the delta-pulse in the velocity. The forces for this function can easily be calculated using our potential theory program. However, in order to establish the step-response function, we have to know the time-derivative of the velocity function, see(1). So our displacement function has to be twice differentiable for all t. If not, the response function is poorly approximated.

Newman, J. N.: The time signal input which you used has two spectral peaks and a "basin" at intermediate frequencies where the energy is much smaller. Why did you use this particular input?

Prins, H. J. & Hermans, A. J.: This function is based upon a harmonic system: 3 harmonic waves and a damping factor: the last distributing the energy over different frequencies. We made this choice, because it was easy to implement. We realise that better choices could have been made, but it appeared that this function contains enough energy for the relevant frequencies.