Nonlinear effects in ship wave pattern predictions

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Conventional methods to predict the steady wave pattern of a ship linearise the free surface boundary conditions. Ref. [1] presented estimates of the magnitude of the higher-order terms neglected in this linearisation, obtained "a posteriori" from linearised solutions. Methods to solve the fully *nonlinear* wave resistance problem have been developed since, now permitting a direct evaluation of the validity of linearised methods and of the importance of separate nonlinear effects. This study aims at explaining those effects.

The Raised-Panel Iterative Dawson method

The code RAPID [2], [3] solves the problem of a steady free surface potential flow around a surface-piercing body by an iterative procedure. Each iteration solves a problem linearised with respect to a previous approximation of the wave surface and velocity field. A panel method is used with Rankine source panels on the hull and at a distance *above* the wave surface. Starting from any initial approximation (e.g. a flat free surface and uniform flow) a number of iterations is carried out. Next, the position of the free surface panels is adapted to the new free surface shape in order to retain a proper conditioning and accuracy, the hull paneling is adapted accordingly, and trim and sinkage are adjusted to the new equilibrium. Then the calculation is restarted for the next few iterations. The method usually converges in 5 to 15 iterations. The recent extension with an iterative matrix solver has further reduced the total CPU time for the whole sequence of iterations to little more than that required for most linearised methods: about 9 CPU sec per iteration for a case with 3000 panels, 28 sec for 5000 panels, on a CRAY C98. Extensive validations have shown that the predicted wave patterns are quite close to those measured and are consistently much better than linearised predictions. The method is now in routine use in commercial projects at MARIN and proves most useful.

Nature of nonlinear effects

From many calculations for practical cases it appears that

> **nonlinear effects are much larger, and occur in other cases, than had been anticipated. Diverging waves are much more affected than transverse waves. There is qualitatively little difference between a slow-ship linearised method and a fully nonlinear method for the wave pattern of full-block ships at a usual speed. But there is often a surprisingly large effect of nonlinearities on the wave pattern of faster, more slender ships (ferries, containerships etc.).**

An example is the prediction for DTRC model 5415, a high-speed transom stern destroyer, at Fn = 0.25. At the "Wake-off" workshop at DTRC [4] all linearised methods substantially underestimated the amplitude of the bow wave system away from the hull, and so does our linearised DAWSON code (Fig.1). But the RAPID prediction is in generally good agreement with the experimental data. These large differences for such a slender vessel at moderate speed are puzzling.

To analyse these and other nonlinear effects, in the following we only consider nonlinearities in the free surface conditions (FSC's), not those in the hull conditions. A slow-ship linearised FSC can be written as:

$$Fn^2(\Phi_x^2\varphi'_{xx} + 2\Phi_x\Phi_z\varphi'_{xz} + \Phi_z^2\varphi'_{zz}) + \varphi'_y = RHS$$

imposed on the still water plane $y = 0$. Here Φ is the double-body flow potential and φ' is a wave-like perturbation. The left hand side has a form pertinent to *refraction of short waves by a slowly varying flow field.* Replacing the latter by a uniform flow we obtain the Kelvin FSC, which therefore contains no refraction effect. A fully nonlinear method, on the other hand,

- includes refraction by the *actual* flow field around the hull, rather than by the double-body flow;
- includes all "higher-order" terms in the wave perturbation (but no order assumptions are actually made);
- imposes the FSC at the *actual* wave surface, not at the still water surface. This gives rise to what we shall call the **"transfer effect"**.

The first two items can only be distinguished for short waves. To study the relative importance of these contributions we shall compare wave patterns predicted by:

- A Neumann-Kelvin solution (labeled "NK" in the following), found as the first iteration of RAPID ; *(no transfer, no refraction)*
- Dawson's slow-ship linearised method; *(no transfer, refraction by double-body flow)*
- The fully nonlinear RAPID code; *(transfer, full refraction and other nonlinear terms)*
- A "No Transfer" method (labeled "NT"), found by applying RAPID with the free surface collocation point movement switched off. Thus the nonlinear FSC is iteratively imposed on the undisturbed free surface and transfer effects are absent. *(no transfer, full refraction and other nonlinear terms)*

To minimise the effect of numerical errors on the comparison, all methods have been implemented using raised panels, and the same panelings have been used. The preliminary abstract for this workshop showed application to a few practical cases. We here consider a more systematic study, for three "wall-sided" models with parabolic waterlines (fore and aft symmetric), with length/beam ratio's of 12, 8 and 4, respectively, all at $Fn = 0.25$.

Wave profiles along the hull

Fig.2 shows the hull wave profiles predicted by different methods for the three models. The wave heights are scaled with the beam, so a thin-ship method would give coincident lines. The methods considered here very nearly do so for the two more slender models. But large deviations occur when going to $L/B = 4$: Both linearised methods predict a decrease of the bow wave height and a strong reduction of wave amplitude along the hull, while in the nonlinear method the bow wave height slightly *increases* and waves along the hull remain significant. Fig.3 shows that for $L/B = 4$ the (nonlinear) NT method gives almost the same bow wave height as both linearised methods. This is a recurrent feature for all cases studied and indicates that

> **the well-known underestimation of the bow wave height by all linearised methods is merely a transfer effect, a consequence of imposing the FSC at the still water surface.**

This underestimation, which is 30 % at $L/B = 4$ for the slow-ship method, appears to decrease only slowly for increasing slenderness: for $L/B = 12$, there is still 13 % difference.

Furthermore we see that the bow wave crest remains at the same longitudinal position in NK, moves slightly forward in DAWSON , and moves significantly forward and becomes shorter and more pointed in RAPID . Method NT fully agrees with RAPID in this respect, indicating that

> **the difference in bow wave length and position is a consequence of refraction or other nonlinear contributions.**

This is easy to explain in the case of low Fn, when the waves generated are short relative to the length scale of the flow around the hull. We may then assume that the bow wave only feels the local velocity (which is reduced in the vicinity of the stagnation point), not the undisturbed incoming velocity. A smaller propagation speed then suffices to make the bow wave stationary relative to the ship, and this corresponds with a smaller bow wave length. The fuller the bow, the larger the local wavelength reduction. Method NT fully incorporates this effect and shows the proper behaviour. The slow-ship method approximates the refracting velocity field by the double-body flow, which has less flow retardation around the bow and causes a slightly too small shift and wavelength reduction. In the NK method the refraction is entirely absent, so the bow wave is too long and does not respond to increasing bow fullness. Although in this case we cannot assume that the waves are short, the nonlinear effects still appear to show precisely the same trends.

Wave pattern away from the hull

Fig. 4 shows longitudinal wave cuts at $0.20L$ off the centreline. The vertical scales are proportional to the beam. It can be observed that for increasing fullness the waves move forward, except in the NK method in which the phases do not respond to the fullness at all; thus a substantial phase lag results. At least for $L/B = 12$ and 8 method NT has the same phase as the fully nonlinear prediction, so again it is refraction that causes these phase differences; and method NT again has a stronger refraction than the slow-ship method. As refraction effects are cumulative, they are more pronounced at a distance from the hull.

Already for L/B = 12 the nonlinear method predicts a 30 % larger bow wave amplitude at z/L = 0.20 than the slow-ship method, and this increases to 90 % for L/B = 4. The NT result shows that this is partly due to transfer, partly to refraction. These wave amplitude differences are much larger than the differences in bow wave height along the hull, so methods predicting higher bow waves additionally appear to give a further amplification (or a slower decay) of the waves while they propagate through the ship's near field! Here seems to be a key to the drastic amplitude differences so often observed.

If we again suppose that waves are short, we may apply ray theory. This tells that the refraction by a flow field around the bow stagnation point increases the crest angle of diverging waves (makes them more "longitudinal") while the waves move away from the hull. For double-body flow refraction this effect on the crest lines has been calculated from ray theory by Hermans and Brandsma [5], and shows good qualitative agreement with what we observe in the RAPID predictions. At the same time refraction increases the amplitude of these waves because they meet an increasingly opposing flow upon leaving the retarded flow area around the bow. A larger flow retardation (so a higher bow wave) leads to a stronger refraction, and thereby a larger wave amplification. [1] This matches our observation that amplitude differences between the methods initially increase with distance from the hull.

The drastic underestimation of diverging bow waves by linearised methods for relatively slender vessels therefore most likely is due to:

- **the neglect of the transfer effect, causing a too small bow wave height along the hull, so a too small initial height of the wave system;**
- **the insufficient wave amplification by refraction in the near field, due to neglecting the refraction altogether (NK methods) or using a base flow with reduced velocity gradients (or perhaps improper phasing or length scale of these gradients) (slow-ship methods).**

Why these effects act mainly on diverging rather than transverse waves is still not completely understood, but agrees with what was found in [5].

Of course the use of ray theory here is far from mathematically rigorous, as the basic assumption of short waves is obviously violated. The quantitative results of ray theory therefore cannot be used; but again the trends it indicates appear to be in good agreement with what we observe from our calculations, and help us to understand what happens; an understanding that is necessary to be able to optimize hull form designs based on a calculated flow field and wave pattern.

References

[1] Raven,H.C., "Adequacy of Free-Surface Conditions for the Wave-Resistance Problem", *18th Symp. Naval Hydrodynamics.*, Ann Arbor, U.S.A.,1990.

[2] Raven, H.C., "A Practical Nonlinear Method for Calculating Ship Wavemaking and Wave Resistance", *19th Symp. Naval Hydrodynamics*, Seoul, South-Korea, 1992.

[3] Raven, H.C., "Nonlinear Ship Wave Calculations using the RAPID Method", *6th Int. Conf. on Numerical Ship Hydrodynamics*, Iowa City, USA, 1993.

[4] Lindenmuth, W.T., Ratcliffe, T.J., and Reed, A.M., "Comparative accuracy of numerical Kelvin wake code predictions — "Wake-off"," *DTRC report 91/004*, Bethesda, USA, 1991.

[5] Hermans, A.J., and Brandsma, F.J., "Nonlinear ship waves at low Froude number", *Jnl. Ship Res.* Vol. 33-3, 1989.

[1] That for L/B = 4 method NT also has a too small amplitude and a phase lag (Fig.4) agrees with this explanation: this method also underestimates the bow wave height, and its refraction effect thus is insufficient. Taking the flow field at y = 0 predicted by the full nonlinear method as the base flow appears to eliminate the phase lag and increase the amplitude.

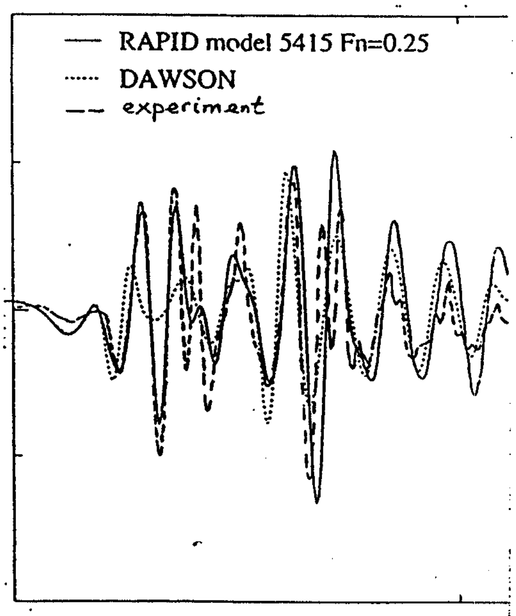

Fig. 1. DTRC model 5415, Fn=0.25. Longitudinal cut at z/L = 0.324.

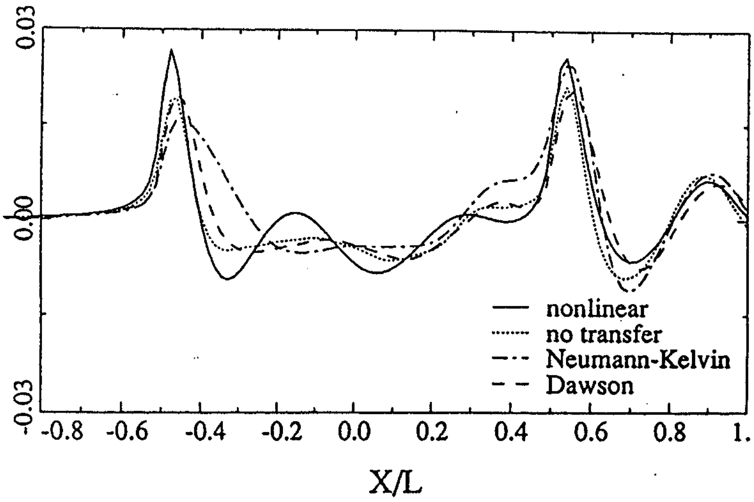

Fig. 3. Hull wave profile for parabolic model with L/B=4.

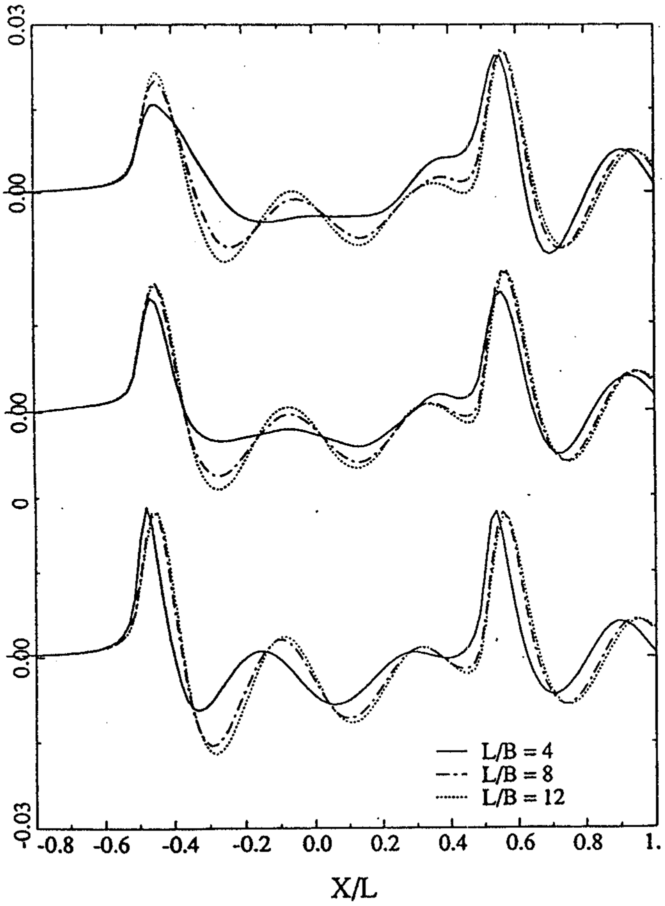

Fig. 2. Hull wave profiles for parabolic models; scaled with the beam.. Top: Neumann-Kelvin; middle: Dawson; bottom: nonlinear.

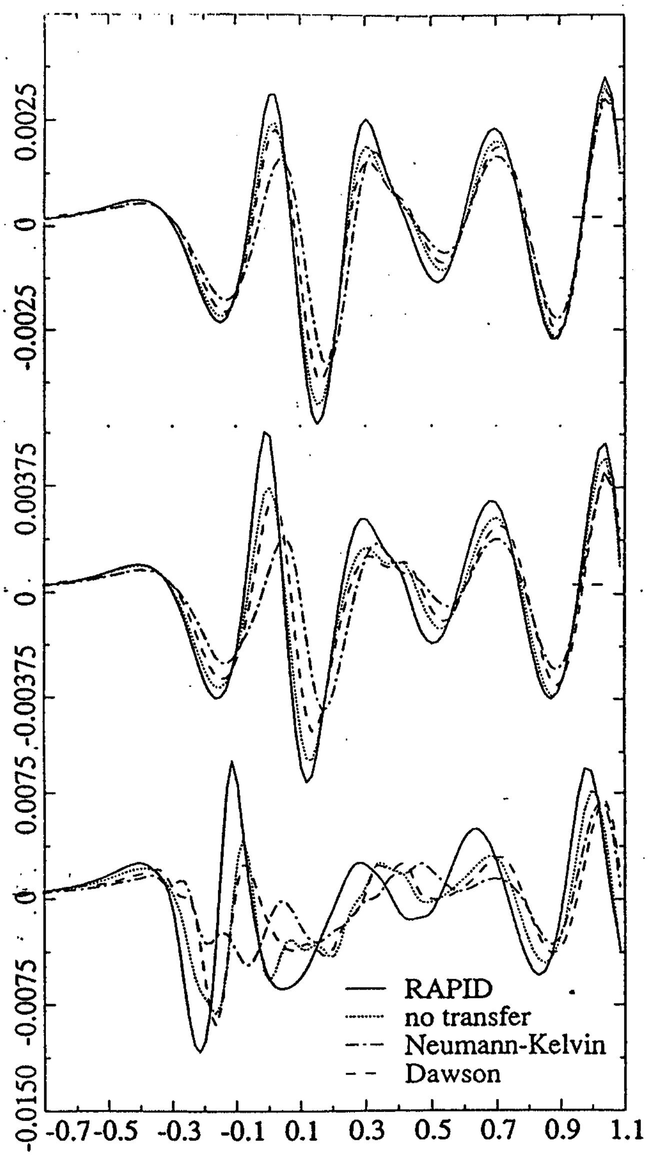

Fig. 4. Longitudinal wave cuts at z/L=0.20, for parabolic models. Top: L/B=12; middle: L/B=8; bottom: L/B=4.