THIRD-ORDER DIFFRACTION OF SURFACE WAVES BY A TIME-DOMAIN RANKINE PANEL METHOD

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1. Abstract

A Rankine panel method developed for the treatment of the forward-speed seakeeping problem of ships is here extended to the solution of the zero-speed linear, second- and third-order diffraction of regular or random surface waves by floating bodies of arbitrary shape. The objective of this study is the development of a three-dimensional computational method for the simulation of the springing and ringing loading mechanisms of offshore structures in regular random waves.

2. The Rankine Panel Method SWAN

As with most methods employing the Rankine source as the fundamental singularity in Green's theorem, SWAN distributes panels over the body surface and part of the free surface. The unknown velocity potential over each panel is approximated by a bi-quadartic spline which allows the flow velocities and their first gradients are obtained as part of the solution. Details may be found in [1].

An essential attribute of the method is the use of a dissipative beach as the means to enforce the radiation condition. The proper selection of the dissipation mechanism and beach size allow the absorption of most of the radiated and diffracted wave energy. Figure 1 illustrates the typical computational grid around a cylinder, where the outer annulus coincides with the beach. Computations will be presented demonstrating the effectiveness of this method of enforcing the radiation condition in the time domain.

Another feature of SWAN is the use of two unknown state variables, the velocity potential φ and wave elevation ζ. This choice has the important property that it requires the computation of up to second gradients of any of the two state variables in the third-order problem, which may be handled easily by the quadratic spline approximation of φ and ζ over the free surface mesh.

3. Free Surface Conditions

The nonlinear dynamic and kinematic free surface conditions take the familiar form

$$g\zeta + \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla\phi \cdot \nabla\phi \right)_{z=\zeta} = 0 \tag{1}$$

$$\frac{\partial \zeta}{\partial t} + \nabla\phi \cdot \nabla\zeta = \frac{\partial \phi}{\partial z}, \qquad z = \zeta \tag{2}$$

The linear, second- and third-order free-surface conditions follow upon introduction in (1) and (2) of the perturbation expansions

$$\phi = \phi_1 + \phi_2 + \phi_3 + \dots$$

$$\zeta = \zeta_1 + \zeta_2 + \zeta_3 + \dots$$

FIRST ORDER

$$g\zeta_1 + \frac{\partial \phi_1}{\partial t} = 0, \qquad z = 0$$

$$\frac{\partial \zeta_1}{\partial t} - \frac{\partial \phi_1}{\partial z} = 0, \qquad z = 0$$

SECOND ORDER

$$g\zeta_2 + \frac{\partial \phi_2}{\partial t} = -\frac{1}{2} \nabla\phi_1 \cdot \nabla\phi_1 - \nabla\phi_1 - \zeta_1 \frac{\partial^2 \zeta_1}{\partial t^2}$$

$$\frac{\partial \zeta_2}{\partial t} - \frac{\partial \phi_2}{\partial z} = -\nabla\phi_1 . \nabla\zeta_1 - \zeta_1 \left(\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2} \right)$$

Due to lack of space the third order condition is not reproduced here but as in the second order problem up to second derivatives of φ and ζ appear in the right-hand-side forcing terms.

4. Body Boundary Condition

In the diffraction problem, the body boundary condition takes the simple form

$$\frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_I}{\partial n}$$

to all orders, where φ_D and φ_I are the incident and diffraction velocity potentials, respectively. The linear, second- and third-order incident wave potential in a monochromatic or random environment may be obtained by the method developed in [2].

5. Computational Issues

The solution of the free-surface problems formulated in Section 3 proceeds in a direct manner. All free-surface conditions are linear and forced by lower-order solutions of the two state variables. Since panels are distributed over the free surface, all desired spatial and temporal gradients of φ and ζ become readily available as part of the solution. Moreover, the selection of a common panel mesh over the body and the free surface to all orders, permits the computation of the influence coefficient matrix once for the linear, second- and third-order solutions and all time steps.

As expected, the appropriate size of the beach was found to increase monotonically with the spatial wavelength which must be resolved accurately. Therefore, identifying a priori the range of wavelengths which must be treated in the linear, second- and third-order problems permits the selection of a free-surface mesh which allows the computation of the respective solutions with desired accuracy.

Figure 2 compares the linear sway added-mass and damping coefficients of a truncated circular cylinder computed by WAMIT and SWAN over a broad range of frequencies. The very good agreement, particularly for the damping coefficient, confirms the performance of the method in the linear problem.

The effectiveness of the beach is not expected to deteriorate in the second- and third-order problems over the wavelength regime considered in the linear problem. Computations will be presented for the the second- and third-order wave disturbance and exciting forces on a truncated circular cylinder, illustrating the performance of the method.

6. References

[1] Nakos, D. E., Kring, D. and Sclavounos, P. D. (1993) Rankine Panel Methods for Time-Domain Free Surface Flows. Proc. 5th Conf. Numer. Ship Hydrodynamics. Iowa City, Iowa.

[2] Nestegard, A. and Stokka, T. (1994). Comparison of Second- and Third-Order Wave Models. Det Norske Veritas Research Technical Report.

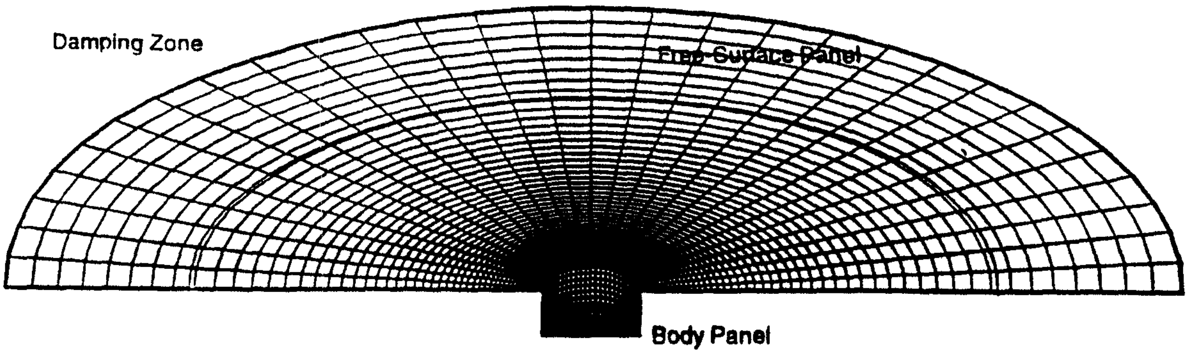
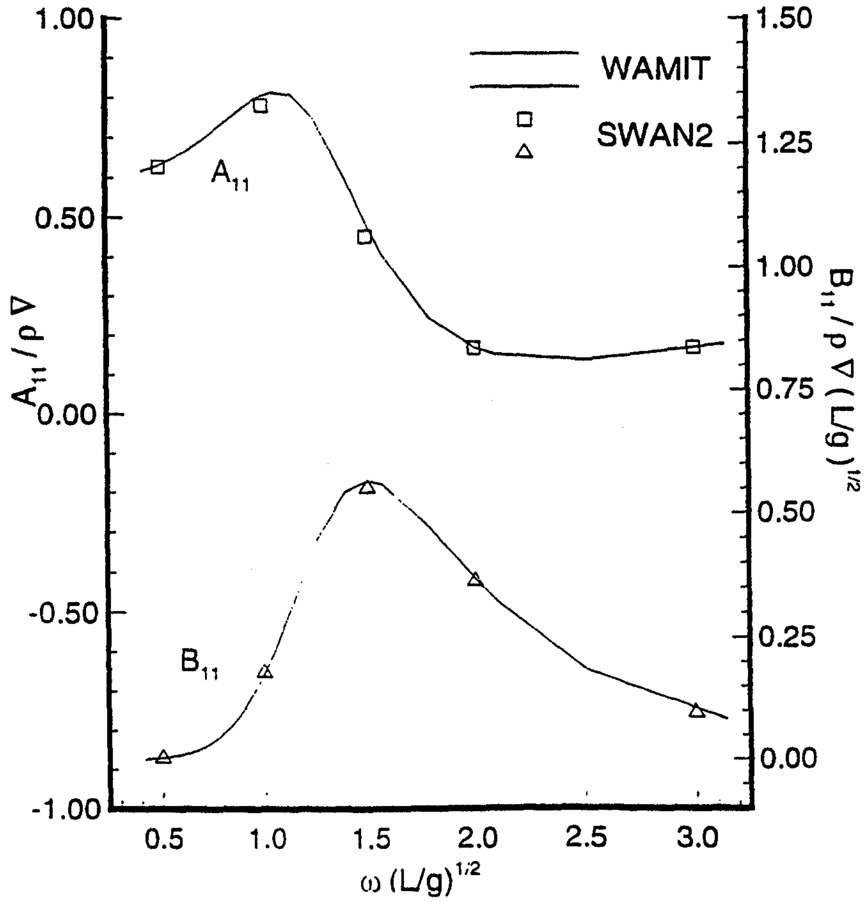


Figure 1

Ring-Type Panel, Ring-Type Damping Zone

Hydrodynamic Coefficient : Surge

Figure 2



DISCUSSION

Clément, A. : The physical meaning of the extra term you add proportional to ζ is clearly a mass flux across the surface, removing potential energy. Is there a physical interpretation of the second term and how can you ensure that this term is always dissipative?

Sclavounos, P. D. & Kim, Y-W.: Indeed the term proportional to ζ is a mass removal term which should be dissipative. The second term is often referred to in the literature as a Newtonian cooling which I find difficult to interpret in classical mechanical terms. Yet, it is possible to show that the combination of the two terms in the beach free surface condition is dissipative, by a frequency domain analysis. The substitution of a harmonic wave component in the free surface condition yields a dispersion relation which can immediately be seen to be dissipative, i.e. the frequency corresponding to a real wavenumber has a uniformly negative imaginary part for all wavenumbers. Our numerical experiments confirm this property.

Kim, M-H. : If you want to solve the 2nd order difference frequency problem, you have to deal with the 2nd order free waves whose wavelengths are usually very long. How can you apply your numerical beach technique in such a case?

Sclavounos, P. D. & Kim, Y-W.: We are not interested in solving the 2nd order difference frequency problem. Our objective is to model accurately the sum-frequency and third order problems as they are known to affect the high-frequency end of the force spectrum on floating structures. In principle we expect our numerical beach to be able to handle long-wavelength disturbances if its distance from the body and its size are properly chosen. Yet, it is important to recall that such long wavelengths carry a small amount of energy and their accurate modelling may not be as important as that of shorter wavelengths in typical wave spectra.

Faltinsen, O. a) It is important to be sure about the accuracy in the linear problem for small frequencies when in the future you apply your method to ringing. Errors at low frequencies may have consequences for higher harmonic loads of importance to ringing.

b) I do not understand the meaning of Rayleigh viscosity for nonlinear wave radiation and scattering problems.

Sclavounos, P. D. & Kim, Y-W. a) We agree that in principle we need to be careful about the accuracy of our numerical technique at low frequencies in connection with the solution of the third order problem and the prediction of ringing loads. We have tested the method for non dimensional wavenumbers ka (a is the cylinder radius) of less than 0.5 with very good accuracy. So we expect the method to be reliable in the prediction of the higher harmonics of importance to ringing. Also, the amount of energy carried by low-frequency primary harmonics corresponding to non-dimensional wavenumbers less than 0.5 will be tempered by the typical shape of sea spectra which carry very little energy below a certain low cut-off frequency.

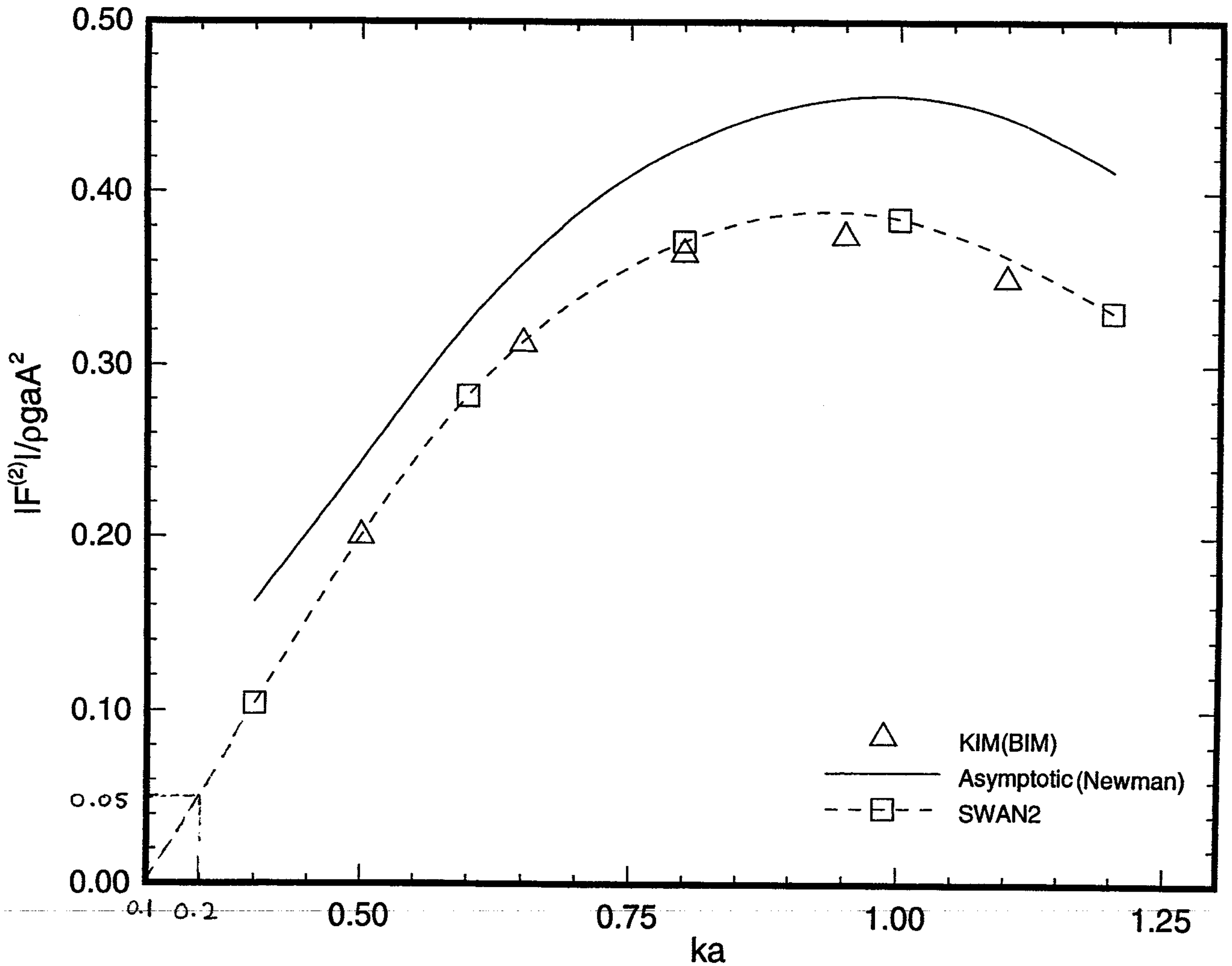
b) we are not solving a fully nonlinear problem but rather a sequence of linear ones. Moreover, far from the body the free surface flow asymptotes to a linear one. The dissipative mechanism implemented on our beach is simply designed to remove energy for all wavelengths. The use of the Rayleigh viscosity concept is merely a way of explaining why energy is removed rather than added into the fluid domain. By virtue of the linearity of the linear, 2nd and 3rd order free surface conditions the mechanism for removing energy is the same in all three problems. In response to the question by Rod Rainey which follows, we have computed the sum-frequency vertical second-order force on a truncated cylinder and have found that the numerical beach works as well in the second-order problem as it does in the linear case.

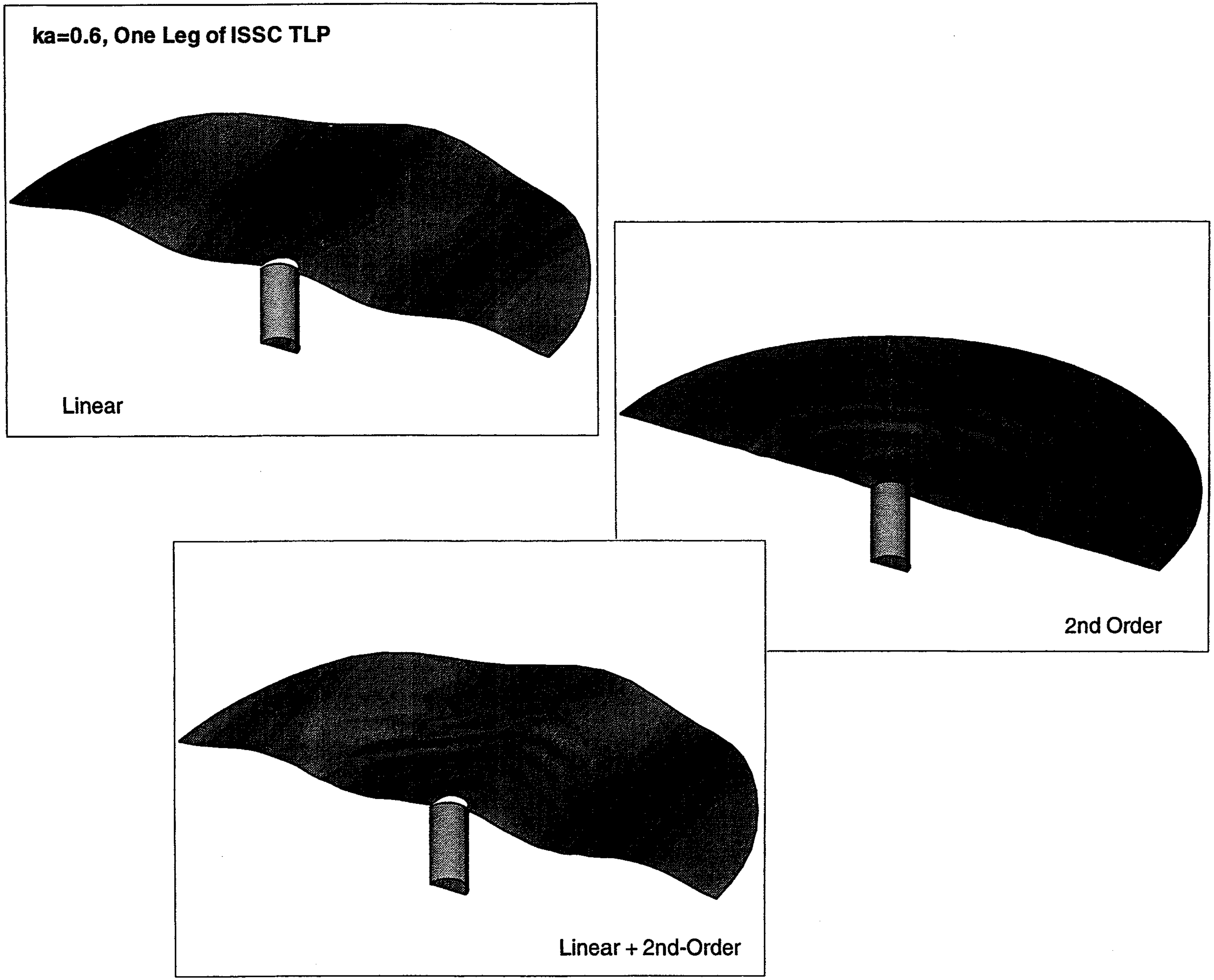
Rainey, R. C. T.: I believe that your finding that the 2nd order problem is unaffected by the far field is special to the geometry you have chosen, i.e. the bottom mounted cylinder. For cylinders of deeper draught like the TLP legs, the "far-field" driven microseism effect dominates the 2nd order vertical loads in all but the longest waves. Nick Newman (the following day) also made this point. The implication here is that the damping of the far field wave disturbance by the numerical beach will be detrimental to the predictions of the second-order vertical force which is dominated by far-field effects.

Sclavounos, P. D. & Kim, Y-W.: As a result of a fax communicated to us by Rod Rainey and the same point made by Nick Newman the last day of the Workshop, we went ahead and computed the second-order vertical force acting on a truncated cylinder for which benchmark computations have been carried out by M-H Kim. A page illustrating very good correlation of our results with M-H Kim's has been made available to the Workshop organizers for inclusion in the discussion. We have found a clear lack of sensitivity of the vertical second-order force on the distance and size of the beach which was selected as in the computation of the second-order sway force.

So it is likely that the dependence of the vertical second-order force on the far field wave disturbance stated by the discussers is the result of the mathematical technique used in its definition and interpretation, rather than a genuine physical effect. Our numerical algorithm suggests that it is not the latter.

Comparison of Second-Harmonic Heave Force ON TRUNCATED CYLINDER




ka=0.6, One Leg of ISSC TLP
Linear
2nd Order
Linear + 2nd-Order