Absorbing boundary condition for floating objects in current and waves

L.M. Sierevogel and A.J. Hermans
Department of Technical Mathematics, Delft University of Technology, The Netherlands

Introduction

To be able to take the non-linear effects into account for the wave-body interaction, Prins and Hermans [5, 6] recently developed a two- and three-dimensional time domain algorithm to compute the behaviour of floating objects in current and waves. The results for the linear problem and the second-order forces are very satisfying.
The physical fluid domain is an infinite (or large) fluid domain. The computational domain cannot be infinite, so we have to introduce artificial boundaries and proper boundary conditions. The disadvantage of the Sommerfeld conditions (which Prins uses) is the dependence of the wave frequency, so they cannot handle non harmonic waves. Keller and Givoli [1, 2] introduce a DtN-method to use an artificial boundary, which divides the original domain into a computational and a residual domain (the interior and exterior).

We derive a absorbing boundary condition independent of the wave frequency, using the idea of the method of Givoli with the algorithm of Prins. For both the two- and three-dimensional problem, we develop a special Green's function in the exterior. The results of the two-dimensional (2-D) case are satisfying; The condition absorbs the outgoing waves and the method also decreases computer time, when computing the behaviour of an object in harmonic waves. First the boundary will be closer to the object and secondly it is not necessarily to implement the conditions dependent of every frequency. For the three-dimensional (3-D) case we only have some preliminary results.

1 The interior problem

We divide the infinite fluid domain into an interior and exterior. In the interior S we use the mathematical model Prins and Hermans [5, 6] use. The object is floating in water of depth h and is free to oscillate in the x- and z-direction, and is free to roll. A uniform current with velocity U and regular incoming waves are travelling in the positive x-direction. The interior is enclosed by an artificial boundary $\mathcal{B}$. We assume the following restrictions: there is no viscosity, the fluid is incompressible and homogeneous, and the flow is irrotational, so we are able to introduce the velocity potential Φ, which has to satisfy the Laplace equation. By using the dynamic and kinematic conditions and splitting the potential into a steady and an unsteady part, we get the well-known linearized free-surface condition. We also use a linearized condition of the body boundary condition. The bottom is a rigid wall, so $\frac{\partial \phi_b}{\partial n} = 0$. In the interior, we can only say about the potential on the artificial boundary that it satisfies the Laplace equation and that it remain finite if we take $\mathcal{B}$ at infinity. We do not implement a Sommerfeld radiation condition at $\mathcal{B}$.

To solve the interior problem numerically, we introduce a Green's function satisfying the preceding conditions and use Green's second theorem for the potential in the interior fluid domain. This Green's theorem and discretizations makes it possible to rewrite the interior problem like

$$D_1 \psi_{i+1} = D_2 \psi_i + D_3 \psi_{i-1} + f_{i+1}, \tag{1}$$

with subscripts denoting the time level, f a time dependent vector and ψ a vector containing the resp. the potential on the free-surface, on the hull of the object and the artificial boundary, $(\phi_f | \phi_{\mathcal{H}} | \phi_{\mathcal{B}})$.

In our approach we make use of the same algorithm as developed by Prins except for the boundary $\mathcal{B}$. Experience learned that the Sommerfeld condition applied that the outer boundary $\mathcal{B}$ is efficient if $\mathcal{B}$ is taken at a distance of about three wavelength, while the coefficients for the two families of waves are dependent of the frequency. Hence the matrix has to be updated of each frequency. Our goal is to obtain a genuine time method where the matrix is independent of the frequency. In our case ϕ_n is not expressed in ϕ at the boundaries $\mathcal{B}$. Hence, our vector of unknowns becomes $\psi = (\phi_f | \phi_{\mathcal{H}} | \phi_{\mathcal{B}} | \phi_{\mathcal{B}n})$. To render the matrix equation uniquely solvable we have to add a matrix relation between $\phi_{\mathcal{B}}$ and $\phi_{\mathcal{B}n}$.

2 The exterior problem

A current with velocity U in the positive x-direction is the same as an object with a speed U in the negative x-direction. We suppose the interior S moving together with the object, while the exterior $\mathcal{D}$ is fixed to the earth. (See figure 1) In the exterior the velocity potential also has to satisfy Laplace equation and the bottom condition.

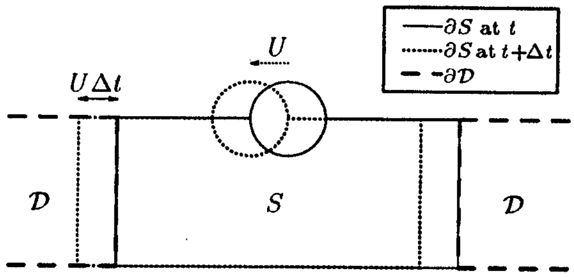

FIGURE 1: *The 2-D geometry on t and $t+\Delta t$*

By making ϕ_z explicit and discretizing ϕ_{tt} by a first-order difference and because the exterior is not moving in a time step, the linearized free-surface condition can be written as

$$\phi_{z,i+1} + \mu\phi_{i+1} = \mu\,(2\phi_i - \phi_{i-1}) \qquad \text{at } z = 0, \tag{2}$$

with $\mu = \frac{1}{g(\Delta t)^2}$. To solve the exterior problem numerical, we introduce a Green's function, which satisfies

$$\begin{array}{ll} G_z + \mu G = 0 & \text{at } z = 0 \\ \nabla^2 G = 0 & x, \xi \in \mathcal{D} \\ G_z = 0 & \text{at } z = -h \\ G = 0;\ \ G_n = 0 & \lim r \to \infty, \end{array} \tag{3}$$

with x, ξ the coordinates of resp. the source- and field point, and r the distance between the two points. Now we can also use Green's theorem in the exterior. In analogy with Wehausen et al.[7] we derive a 2-D Green's function, which satisfies (3)

$$2\pi G = \ln r + \ln r_2 - \frac{2}{\pi}\int_0^\infty \frac{e^{-kh}}{k}\left(\frac{(k-\mu)\cosh k(z+h)\cosh k(\zeta+h)\cos kX}{k\sinh kh + \mu\cosh kh} + 1\right)dk \tag{4a}$$

$$= -\sum_{k=1}^{\infty} \frac{2\pi}{m_k}\frac{m_k^2+\mu^2}{hm_k^2 + h\mu^2 + \mu}\cos m_k(z+h)\cos m_k(\zeta+h)e^{-m_k|X|}, \tag{4b}$$

with X the horizontal distance, $r_2 = |x - \xi'|$, with ξ' the image of ξ with respect to the bottom and m_k imaginary parts of the purely imaginary poles. The integral equation(4a) is used for small X, because the sum equation(4b) does not converge.
The same way we derive the 3-D Green's function for infinite deep water

$$4\pi G = -\frac{1}{r} - \int_0^\infty \frac{k-\mu}{k+\mu} e^{kZ} J_0(kX)dk,$$

end for water with a finite depth

$$4\pi G = -\frac{1}{r} - \frac{1}{r_2} - \int_0^\infty \frac{2(k-\mu)e^{-kh}\cosh k(z+h)\cosh k(\zeta+h)}{k\sinh kh + \mu\cosh kh} J_0(kX)dk.$$

These Green's functions can be rewritten to functions more friendly to computed numerically, the same way Noblesse [4] and Newman [3] rewrite their functions.
By using Green's theorem we are able to write for the potential in the exterior

$$\boldsymbol{D}_{\mathcal{D}}\boldsymbol{\psi}_{\mathcal{B}} = \boldsymbol{E}_{\mathcal{D}}\tilde{\boldsymbol{\phi}},$$

with $\boldsymbol{\psi}_{\mathcal{B}}$ a vector $(\phi_{\mathcal{B}}|\phi_{\mathcal{B}n})$ and $\boldsymbol{E}_{\mathcal{D}}\tilde{\boldsymbol{\phi}}$ is the right-hand side of (2).

If there is no current, at time $t+\Delta t$ the artificial boundary $\mathcal{B}$ of the exterior is the same as the artificial boundary of the interior. When we have a uniform current speed we apply Green's theorem on the domain between the boundaries. Now we are able to to write the interior problem as a overall matrix equation, like equation(1).

$$\boldsymbol{D}_1\boldsymbol{\psi}_{i+1} = \boldsymbol{D}_2\boldsymbol{\psi}_i + \boldsymbol{D}_3\boldsymbol{\psi}_{i-1} + \boldsymbol{f}_{i+1} + \boldsymbol{E}_{\mathcal{D}}\tilde{\boldsymbol{\psi}},$$

with $\boldsymbol{\psi}$ a vector containing $(\phi_f|\phi_{\mathcal{H}}|\phi_{\mathcal{B}}|\phi_{\mathcal{B}_n})$.

3 Results

3.1 Reflections (2-D)

First we are looking at the reflections of the artificial boundary using the 2-D algorithm mentioned in the previous sections. We studied the case for both infinite depth (i.e. choosing the bottom relatively far away) and the ratio depth/draught is 3 (i.e. $h/R = 6$). To be able to compare our results with the results of Prins et al.[5] we look for both $U = 0$, so the Froude number $Fn = U/\sqrt{gR}$, and for $Fn = .14$ and we forced the cylinder to oscillate harmonic. The time integration was carried out over a time interval of four period according to the frequency of encounter. On this interval 200 time steps were taken.
The length of the interior turned out to be one wavelength, to get about the same reflection Prins get using three wavelength.
The length of the exterior depends on the length of the time integration. Givoli [1] chooses the length of the exterior such that it is always just ahead of the wave fronts. This length is not constant during time-stepping. We choose the length such that a wave will not be back at the boundary during the time integration. To decrease the length of the exterior we are able to put some artificial damping ϵ into the exterior. By using this damping we are able to use the nearly same Green's function and theorem.
Another advantage of our method is we do not have to update the matrices $\boldsymbol{D}_1$, $\boldsymbol{D}_2$, etc. for every wave frequency. This will decrease the computer time a lot. We divide the frequency domain in such a way that for one group of frequencies the interior is one wavelength for the smallest ω_0 and two wavelength for the largest ω_0. Figure 2 shows the reflection of a wave, made by a forced oscillation in sway of the cylinder during one period, after three periods.

3.2 The added mass, damping, movement and second-order forces (2D)

Knowing the potential due to the diffracted waves, we are able to compute the hydrodynamic coefficients, the added mass and damping matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, by fitting the first-order forces to the acceleration and the velocity. The movement is computed by using an incoming potential and solving a differential equation. Knowing the total unsteady potential we are able to compute the average second-order forces.
Computing the added mass and damping coefficients and the second-order forces, (figure 4 & 3), is done in groups of frequencies in the same way as mentioned in the previous subsection. The time to compute the coefficients is much less than the way Prins computes them. For instance, to compute the added mass and damping coefficients in sway at $\omega_0\sqrt{R/g} = 1.$ and $Fn = 0$ we need about 20 sec [1] to fill the matrices and 40 sec to time iterate and compute the coefficients. This is respectively $\frac{2}{3}$ and $\frac{1}{3}$ of the time Prins needed. This is only for one frequency. By using the groups of frequencies we do not have to update the matrices. So by using groups of about five frequencies we need 30% of the time Prins needs to compute them.

3.3 Reflections (3-D)

The artificial boundary for the 3-D problem with infinite deep water, using this method, seems to absorb the outgoing waves better than the Sommerfeld radiation condition. So we are able to place the artificial boundary closer to the object and the computer time will decrease. The 3-D problem for water with a finite depth is theoretically worked out, but the implementation on the computer is not ready yet.

4 Conclusions

In this abstract we show a new absorbing boundary condition for floating 2-D (and 3-D) objects in current and waves, using time integration. By dividing the physical fluid domain in a computational and a residual part, we derive a frequency independent numerical algorithm to compute the hydrodynamic coefficients. Computing the special 2-D Green's function in the residual part take relatively a lot of time, a faster algorithm will be developed in the future. Still this method uses about one third of the computer time if we implement a Sommerfeld radiation condition at the artificial boundaries. To be able to compare our 2-D results with those of Prins [5] we only compute the interesting coefficients for several frequencies.

[1] Computations are done on a HP 9000/720 workstation

In the mean time Prins booked satisfying results computing some 2-D retardation function with this method. We will be still working on the implementation of the 3-D problem for infinite and finite water.

Acknowledgements

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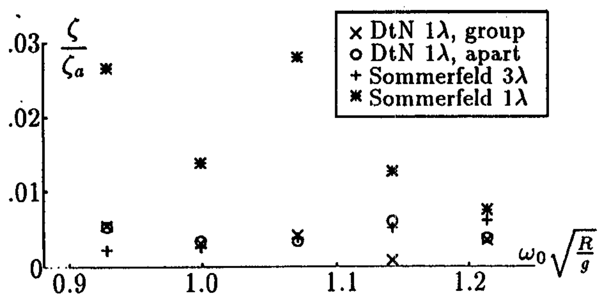


FIGURE 2: *The relative surface elevation.*

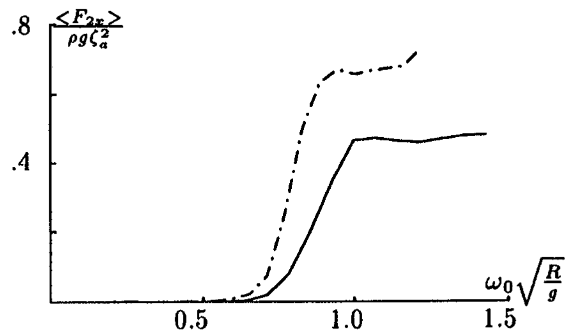


FIGURE 3: *Horizontal drift force for inf. depth.*

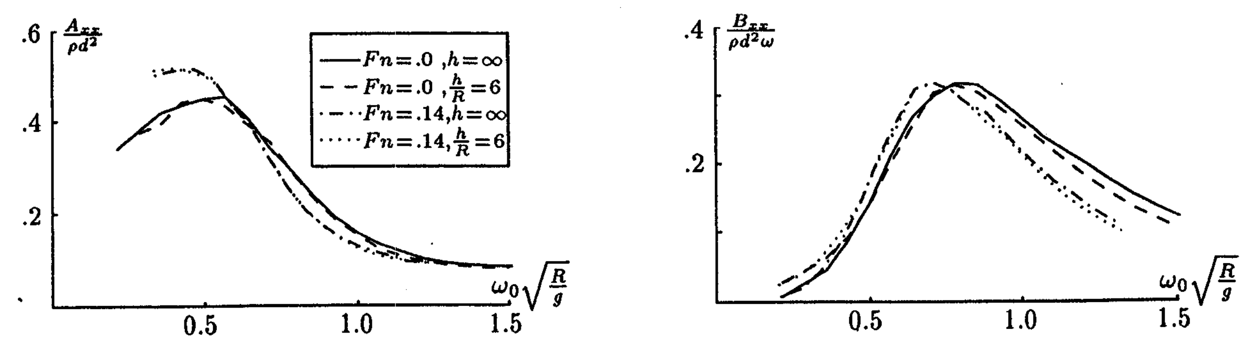


FIGURE 4: *Added mass and damping in sway.*

DISCUSSION

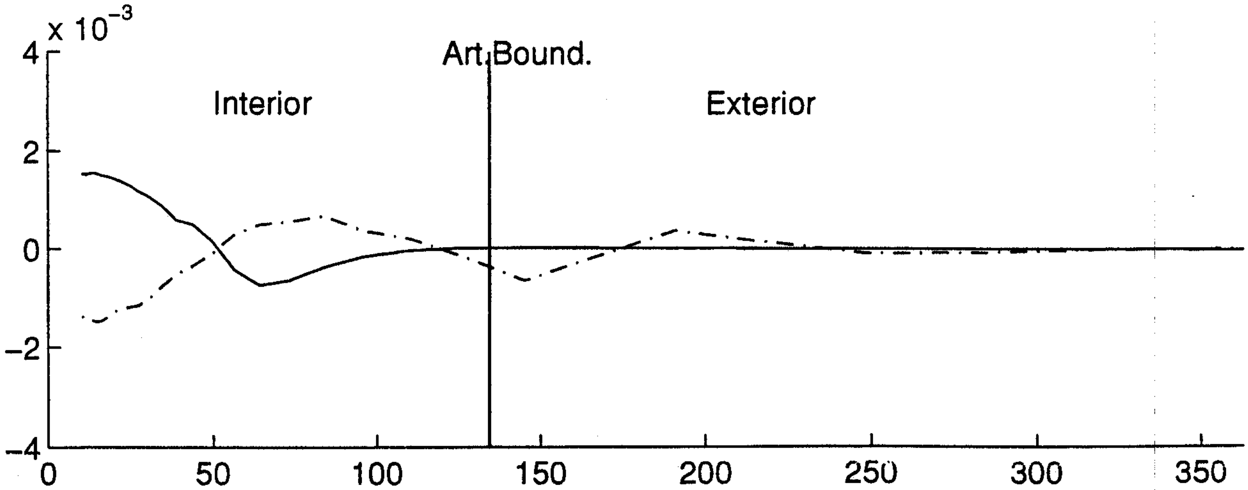
Newman, J. N.: What is the advantage of the DtN method compared to using the conventional time-domain Green function in the exterior solution?

Sierevogel, L. M. & Hermans, A. J.: Without forward speed, both methods will give good results. The advantage of our method is that we have to compute the Green's function just once, on the other hand the conventional time-domain Green's function doesn't need to cover the free surface of the exterior with panels.

Till now, we didn't see any results, using the conventional time domain Green's function including forward speed. We think that it is easier to implement the effects of the double body potential by using the DtN method instead of the conventional time domain Green's function. By taking into account the double body potential the conventional time domain Green's function also needs panels at the free surface of the exterior.

Yeung, R. W.: You have an interesting way of treating the exterior problem, based primarily on a discrete time treatment. I wonder if you take $\Delta t \rightarrow 0$, the exterior solution actually represents the effects due to a wavemaker, which we know is represented by a convolution integral in time. Some years ago, I proposed a generalized form of absorption/matching boundary condition of type DtN in an IUTAM paper (1985, Lisbon, Portugal), which was recently implemented in the 8th IWWWFB (Yeung & Cermelli) in a very efficient way. The extension to include current effects is presumably possible.

Sierevogel, L. M. & Hermans, A. J.: See the discussion by Newman. For finite values of Δt the discretized free surface conditions can be analyzed by means of the z transform. The numerical results show that the scheme is stable (see figure). Hence for large values of t or $\Delta t \rightarrow 0$ we expect that the discrete convolution converges to the same Green's function as in the time domain.



surface elevation due to forced oscillation

———— after 1.5 periods

-·-·-·- after 4 periods