A 3D Numerical Solution for a Surface-Piercing Plate Oscillating at Forward Speed

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Introduction

Calculation of the unsteady lifting potential flows about surface-piercing bodies is essential to prediction of ship manoeuvrability in waves. A simple but important case of such flows is the one about a vertical surface-piercing plate oscillating at forward speed. Chapman (1975), Yamasaki and Fujino (1985) have used slender body theory to analyse this case. Recently Wu (1994) obtained a numerical solution by a three-dimensional panel method using a Kelvin normal dipole distribution. In the present paper, we shall use a three-dimensional Rankine panel method to deal with this problem.

Formulation

We consider a surface-piercing plate in steady manoeuvring motion on the undisturbed free surface and at the same time undergoing small-amplitude rigid-body oscillations about its mean position in an otherwise calm water. We adopt two right-handed Cartesian coordinate systems: the system o-x, y, z which moves with the steady motion velocity of the plate, and the system o'-x', y', z' which is body-fixed.

The steady manoeuvring motion of the plate is described by the velocity $\vec{V}_s = (u - ry, v + rx, 0)$, where u, v are the velocity components in the longitudinal and lateral direction respectively, and r is the yaw rate about the z-axis. The small-amplitude unsteady motion is described by $\xi_j(t)$, where $j = 1, 2, 3$ denote the translation in x, y, z-direction (surge, sway, heave), and $j = 4, 5, 6$ denote the rotation about the x, y, z-axes (roll, pitch, yaw), respectively.

It is assumed that the fluid is inviscid and incompressible, and the flow is irrotational except in the wake which is to be modeled by a free vortex sheet, so that the flow can be represented by a disturbance velocity potential ϕ which is governed by Laplace's equation in the fluid domain.

Furthermore, we assume that the disturbance velocity $|\nabla\phi|$ is small relative to the steady velocity $|\vec{V}_s|$, and the free-surface elevation ζ is also small in some sense. Thus the velocity potential ϕ satisfies the following linearized free surface boundary conditions in the coordinate system o-x, y, z:

$$\zeta = \frac{1}{g}(\phi_t - \vec{V}_s \cdot \nabla\phi) \qquad \text{on } z = 0,$$

$$\phi_{tt} - 2\vec{V}_s \cdot \nabla\phi_t + \vec{V}_s \cdot \nabla(\vec{V}_s \cdot \nabla\phi) - g\phi_z = 0 \qquad \text{on } z = 0,$$

where g is the gravitational acceleration.

On the plate which is described by $F(x, y, z, t) = y - (\xi_2 + x\xi_6 - z\xi_4) = 0$, the linearized boundary condition is given by, assuming the yaw rate r is small,

$$\phi_y = v + rx + (\dot{\xi}_2 + x\dot{\xi}_6 - z\dot{\xi}_4) - u\xi_6 \qquad \text{on } \bar{S}_B,$$

where $\bar{S}_B$ denotes the wetted body surface in the mean position.

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We decompose the velocity potential ϕ into a steady part $\bar{\phi}(x,y,z)$ and an unsteady part $\varphi(x,y,z,t)$, and determine them separately. In this paper we shall confine ourselves to determination of the unsteady flow. We consider small-amplitude harmonic oscillations with frequency ω:

$$\xi_j(t) = Re\left\{\hat{\xi}_j e^{i\omega t}\right\},$$

where ˆ denotes the complex amplitude.

Correspondingly, the unsteady velocity potential may be expressed by

$$\varphi(x,y,z,t) = \sum_{j=1}^{6} \varphi_j(x,y,z,t) = \sum_{j=1}^{6} Re\left\{\hat{\xi}_j\,\hat{\varphi}_j(x,y,z)e^{i\omega t}\right\}.$$

The linearized boundary conditions on the free surface and body surface for $\hat{\varphi}_j(x,y,z)$ are

$$\hat{\zeta}_j = \frac{1}{g}(i\omega\hat{\varphi}_j - \vec{V}_s \cdot \nabla\hat{\varphi}_j) \qquad \text{on } z=0, \tag{1}$$

$$-\omega^2\hat{\varphi}_j - 2i\omega\vec{V}_s \cdot \nabla\hat{\varphi}_j + \vec{V}_s \cdot \nabla(\vec{V}_s \cdot \nabla\hat{\varphi}_j) - g\hat{\varphi}_{jz} = 0 \qquad \text{on } z=0, \tag{2}$$

$$\hat{\varphi}_{jy} = i\omega n_j + m_j \qquad \text{on } \bar{S}_B, \tag{3}$$

where $(n_1,n_2,n_3) = (0,1,0), \quad (n_4,n_5,n_6) = (-z,0,x),$
$(m_1,m_2,m_3) = (0,0,0), \quad (m_4,m_5,m_6) = (0,0,-u).$

Besides these boundary conditions, the velocity potential must satisfy a radiation condition at infinity, a Kutta condition along the trailing edge of the plate, and a dynamic boundary condition on the free vortex sheet.

From the linearized boundary conditions it can be seen that the modes of motion $j = 1,3,5$ do not induce disturbance flow, so that we only need to determine the velocity potential due to the transverse modes of motion.

Numerical Solution

We solve the problem by a three-dimensional panel method using Rankine type singularities. The disturbance velocity potential is represented by a vortex distribution on the plate and its wake, and by a source distribution on or above the undisturbed free surface. Consistently with the linearization, the free vortex sheet in the wake may be put on the plane $y = 0$.

The semi-infinite vortex sheet is divided horizontally into strips. The vortex strength vector on the strips is decomposed into a bound component in the vertical direction and a free component in the longitudinal direction. The bound component is given by, see e.g. Newman (1977),

$$\hat{\gamma}_j(x,z) = -\frac{\partial}{\partial x}(\hat{\varphi}_j^+ - \hat{\varphi}_j^-) \qquad -\infty < x < X_f, \tag{4}$$

where $\hat{\varphi}_j^+ = \hat{\varphi}_j(x,+0,z)$, $\hat{\varphi}_j^- = \hat{\varphi}_j(x,-0,z)$, and X_f denotes the x-coordinate of the leading edge.

On the other hand, from the dynamic boundary condition on the free vortex sheet, the pressure-equality Kutta condition at the trailing edge and Kelvin's theorem of the conservation of circulation, it can be derived:

$$\hat{\gamma}_j(x,z) = -\frac{i\omega}{u}e^{-\frac{i\omega}{u}(X_a-x)}\,\hat{\Gamma}_j(z) \qquad -\infty < x \le X_a, \tag{5}$$

where X_a denotes the x-coordinate of the trailing edge, and

$$\hat{\Gamma}_j(z) = \int_{X_a}^{X_f} \hat{\gamma}_j(x,z)\,dx = (\hat{\varphi}_j^+ - \hat{\varphi}_j^-)|_{x=X_a}. \qquad (6)$$

Hence the free vortex sheet does not contain unknown vortex strengths.

In order to construct the numerical solution, the plate and the free surface are discretized into panels. Correspondingly, the vortex distribution is discretized and replaced by a vortex filament of strength $\hat{\Gamma}_{jm} = \hat{\gamma}_j(x_m, z)\Delta x_m$, resulting in a system of horseshoe vortices, while the source distribution is discretized into quadrilateral source panels, with a constant source strength on each of these panels. In this paper, the longitudinal and vertical locations of the collocation points and the vortex filaments on the plate are chosen according to the well-known quasi-continuous method. On the other hand, the vortex filaments on the free vortex sheet and the panels on the free surface are equidistant in the longitudinal direction. In order to satisfy the radiation condition numerically, the source panels or source points on or above the undisturbed free surface are shifted backwards one panel length relative to the corresponding collocation points on the free surface. For the harmonic oscillatory motion with forward speed, it is required that the parameter $\tau = \frac{\omega u}{g} > \frac{1}{4}$ to ensure no waves are radiated forward.

The velocity potential and the disturbance velocity induced by a source panel element can be calculated by using a conventional panel method. The velocity induced by a horseshoe vortex can be obtained by the Biot-Savart law. Since a horseshoe vortex is equivalent to a semi-infinite normal dipole sheet bounded by the vortex filament, the velocity potential due to a horseshoe vortex can also be easily evaluated. Then, by satisfying the boundary conditions (2) and (3), we obtain a linear equation system for the unknown source and vortex strengths.

Once the flow is determined, we can calculate the free-surface elevation and the hydrodynamic forces acting on the plate. The complex amplitude of the free-surface elevation is given by (1), while the complex amplitudes of the lateral force, roll moment and yaw moment can be obtained by integrating the dynamic pressure over the plate. The jump of the linearized pressure on the plate is given by

$$\hat{p}_j^+ - \hat{p}_j^- = -\rho\left[i\omega(\hat{\varphi}_j^+ - \hat{\varphi}_j^-) - u\frac{\partial}{\partial x}(\hat{\varphi}_j^+ - \hat{\varphi}_j^-)\right]. \qquad (7)$$

It follows from (4), (6) and (7) that

$$\hat{F}_{2j} = \rho\int_0^T\left[i\omega\int_{X_a}^{X_f}(x - X_a)\,\hat{\gamma}_j dx + u\int_{X_a}^{X_f}\hat{\gamma}_j dx\right]dz, \qquad (8)$$

$$\hat{F}_{4j} = -\rho\int_0^T\left[i\omega\int_{X_a}^{X_f}(x - X_a)\,\hat{\gamma}_j dx + u\int_{X_a}^{X_f}\hat{\gamma}_j dx\right]z\,dz, \qquad (9)$$

$$\hat{F}_{6j} = \rho\int_0^T\left[i\omega\int_{X_a}^{X_f}\frac{1}{2}(x^2 - X_a^2)\,\hat{\gamma}_j dx + u\int_{X_a}^{X_f}x\,\hat{\gamma}_j dx\right]dz, \qquad (10)$$

where ρ is the fluid density, T is the draft of the plate.

Numerical Results

The described method has been applied to flat plates oscillating at forward speed $\vec{V}_s = (U, 0, 0)$. Preliminary numerical results are obtained for two plates with length-draft ratio $L/T = 10$ and 5, respectively.

Fig.1 shows calculated wave contours (the real part of the complex amplitude of the free-surface elevation) for the plate of $L/T = 10$ in sway oscillation at $F_n = 0.40$ and $\tau = 0.32$, with 20×5

panels on the plate and 65 × 23 panels on half of the free surface. The free-surface elevation is antisymmetric about the plane $y = 0$.

The calculated hydrodynamic coefficients versus non-dimensional frequency for the plate of $L/T = 5$ at $F_n = 0.48$ are compared with experimental results by Van Den Brug et al. (1971) and numerical results of slender-body theory by Chapman (1975). The agreement between the present results and the measurements is quite well for the coefficients obtained from yawing oscillation. On the other hand, for coefficients obtained from swaying oscillation, the present results mostly show some deviations from the measurements. At some frequencies, the maximal errors reach 50% compared with the measurements. In general, the frequency-dependence for the present results is not so strong as for the measurements. It seems necessary to make further efforts to clarify this discrepancy.

References

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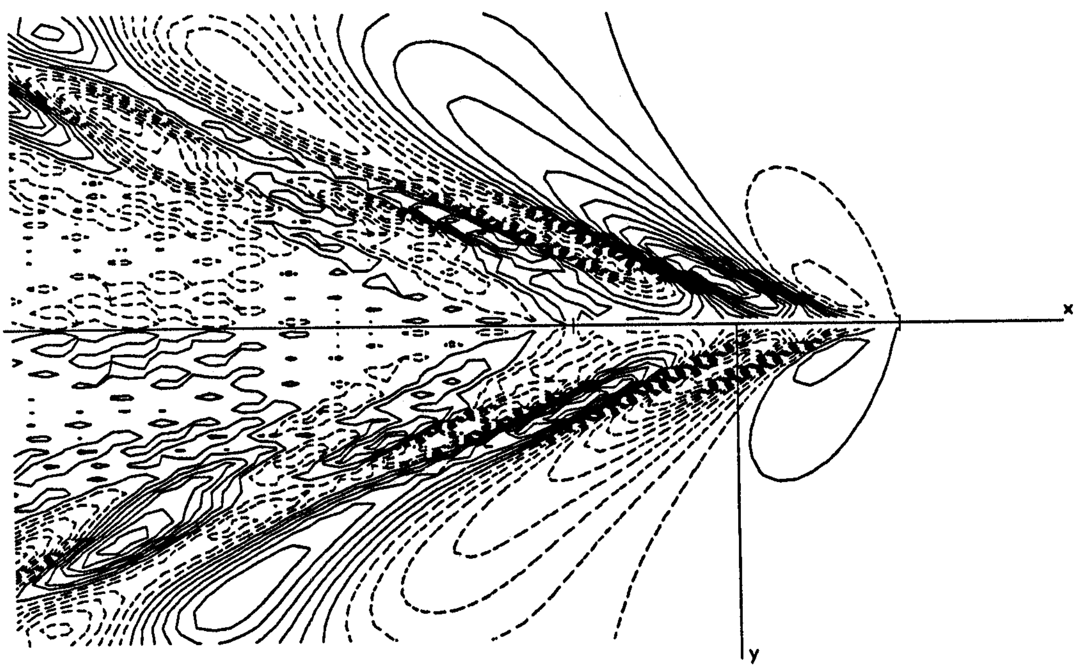

Fig.1 Calculated wave contours of the time-harmonic wave field for a plate of $L/T = 10$ during swaying oscillation at $F_n = 0.40$ and $\tau = 0.32$
The distance between the contour lines is $0.9 \times 10^{-3} L$.