Computational Method for Reaction Forces on a Heaving Sphere

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Abstract

We present a numerical three-dimensional time-domain method which is used to study the nonlinear force on a heaving sphere. The evolution of the fluid flow is computed using a fourth-order Runge-Kutta scheme. Laplace's equation for the velocity potential is solved by a higher order boundary element method. The fully nonlinear free-surface conditions are applied on the actual water surface. The waterline on the body is determined from an extrapolation of the free surface. The shape of the waterline determines how the sphere is regridded with quadrilateral panels. The pressure on the sphere is integrated over its *actual* wetted surface, which yields the hydrodynamic force on the sphere. We compare it with previous results.

1 Introduction

In the tradition of the Boundary Integral Equation Methods (BIEMs) our group has developed a time-domain panel method for solving general problems concerning the interaction of fixed or (freely) floating bodies with fully nonlinear waves in arbitrarily shaped domains. Broeze et al. (1992) and Broeze et al. (1993) used this method to model a wave passing over a shoal up to the point of breaking. The method has also been successful in the calculation of the force signal on oscillating bodies with small and large motion amplitudes in two dimensions, see van Daalen (1993) and Berkvens and Zandbergen (1996) (submitted for publication). A recent application was the computation of hydrodynamic loads on a bottom-mounted surface-piercing cylinder, see de Haas et al. (1995). We have extended the method to include the interaction of general three-dimensional floating bodies with waves.

We give a short outline of the mathematical model and the numerical method in Sections 2 and 3. In Section 4 we describe how a parameterization of the geometry of a sphere is used in determining the waterline and in regridding. The calculation of forces is outlined in Section 5. In Section 6 we apply the method to the calculation of the hydrodynamic force signal on a half-immersed sphere which is oscillated in the vertical direction.

2 Mathematical model

The present method can handle generally shaped bodies. Here we restrict ourselves to the interaction between water and a sphere. Consider a half submerged sphere in a fluid with a free surface. The fluid is contained within a three-dimensional region Ω bounded by a free surface F, the wetted part S of the sphere's surface and a bottom B which we choose to be flat, but which can be given an arbitrary shape. The fluid domain is closed an artificial lateral boundary A. See Figure 1 for an illustration of the geometry of the problem.

Under the assumptions that the fluid be inviscid and incompressible, and the flow irrotational, the flow can be described with a velocity potential ϕ. Once the initial conditions are provided, the flow is governed by Laplace's equation for the potential in Ω, together with the kinetic and the dynamic free-surface evolution equation on F, and boundary conditions on B and S. We use the impermeability condition on B, whereas the normal velocity on S is inferred from the prescribed motion of the sphere. Inherent to truncating the domain at the artificial boundary parts A is that a radiation condition has to be used.

3 Numerical method

The above mathematical problem can be numerically treated as follows. At any level in time a spatial problem governed by Laplace's equation is solved. The spatial problem is solved using a discretized form of a Boundary Integral Equation (BIE). To this end the domain boundary is divided into boundary parts, each with the topology of a rectangle. Each boundary part is then divided into quadrilateral panels, each of which contains a collocation point near its centre. An illustration of networks and panels is shown in Figure 2. After solving this BIE, ϕ and its normal derivative ϕ_n are known along the boundary of Ω.

When the spatial problem is solved, a timestep can be carried out. To obtain the shape of the fluid domain at the next time-level, first the kinetic boundary condition is integrated such that the new collocation points positions are obtained. The waterline on the sphere is then determined from extrapolation of the free surface to the sphere in its new (prescribed) position. Then its wetted surface S is regridded, where the waterline shape is used. The last two aspects are described in Section 4. By integrating from the previous timestep the new values of ϕ and ϕ_n along the boundary are calculated (using the dynamic free-surface evolution equation on F) or updated on Dirichlet and Neumann boundaries, respectively. Time integrations are carried out with the classical fourth-order Runge-Kutta method.

This panel method is set up in a way such that, globally, its accuracy is of second-order in the dimension of the panels. More detailed descriptions are given by Romate (1989), Broeze (1993) and van Daalen (1993). Broeze et al. (1993) give a concise description of the method and some recent developments.

4 Body and waterline

In the present work we use an sphere in heave. Any point $\vec{Y}$ on its surface can be described as a function of a polar angle φ in the x,y-plane and an azimuthal angle ϑ, thus $\vec{Y} = \vec{Y}(\varphi, \vartheta)$. This body description is used in determining the waterline and in regridding the wetted body surface S. For more general bodies we intend to implement a geometry description based on B-splines.

After the execution of a timestep, the positions of the collocation points in the free surface are known, as well as the new position (and orientation) of the sphere. In order to determine the waterline, we create a quadratic extrapolating curve as a function of a parameter s through three neighbouring collocation points of which the first lies in a panel bordering the waterline. A point on this curve is denoted $\vec{X}(s)$. For the (closest) intersection point with the sphere it is required that $\vec{G}(s, \varphi, \vartheta) \equiv \vec{X}(s) - \vec{Y}(\varphi, \vartheta) = 0$. This set of three nonlinear equations (one for each vector component) is solved for s, φ, ϑ using an iterative method. This procedure is executed for all collocation points in panels bordering the waterline, as well as for the network boundaries in the free surface that intersect the sphere. The latter yield the network corners in the waterline. The waterline is determined by these intersection points. In the present method we do not evaluate the motions of the waterline directly and therefore we have to rely on extrapolations of the free surface.

Next the four network corners that lie in the waterline are used in the regridding of the wetted body boundary S. We have chosen to describe S using five networks, four of which are bounded by the fifth network and the waterline, see Figure 2. In this way we prevent the use of triangular panels. It also gives us a relatively large freedom in determining the panel-size *distribution* on the sphere: this is simply a matter of choosing the positions of the corners of the fifth network in the right way. Finally the collocation points on S are determined using the position of the waterline.

5 Hydrodynamic forces

We need to know the pressure p in order to obtain the reaction force exerted by the fluid onto the moving sphere. The pressure p is given by Bernoulli's equation:

$$p = -\rho\left(gz + \phi_t + \frac{1}{2}\nabla\phi \cdot \nabla\phi\right) \ . \tag{1}$$

$\nabla\phi$ is computed from ϕ_n, which follows from the prescribed body motion and its shape and orientation, and from the tangential derivatives which are approximated using finite differences. ϕ_t satisfies a set of equations and boundary conditions similar to those for ϕ and is solved for in a manner, which is described by Cointe et al. (1991) and van Daalen (1993).

On every panel (on the sphere) a local second-order approximation is made of the boundary shape and of the pressure. The force on the sphere is then integrated using a nine-points Gaussian method for each panel, thus yielding the total force on the body.

6 Experiments

The method described above is now ready to be used for calculations. The two-dimensional version has already been validated for small-amplitude oscillations of a circle and a square and it has been used successfully for large-amplitude oscillations of a square, see Berkvens and Zandbergen (1996). In the months ahead we plan to carry out similar calculations using the three-dimensional method for validation purposes.

In the experiments we will use a half-immersed sphere in a circular domain of sufficient cross section and depth. As explained before there will be five networks on S, four on F, four on A and five on B. We expect to use up to 2000 panels on the domain boundary, but we have the resources to use more than 4000 panels. The sphere will be forcedly oscillated harmonically in the vertical direction at amplitude small compared to the sphere's radius and at frequencies near the resonance frequency for the sphere when it is half immersed. This oscillation will be sustained during a few periods. During this time the force signal on the sphere will be calculated. Since the amplitude is much smaller than the radius and since the frequency is of the order of the natural frequency in heave of the half-immersed sphere, we expect almost linear behaviour of the fluid-body system. Therefore we expect that after a start-up interval, the force signal will be harmonic as well. This signal will then be used to find the added-mass and damping coefficients of the sphere in heave at this particular frequency. The results will be compared with previous results by Kudou (1977), Pinkster (1980) and Prins (1995) in order to verify the quality of the results. Later we will use the method for more generally shaped bodies and larger motions.

Acknowledgements

This work was sponsored by the National Computing Facilities Foundation (NCF) for the use of supercomputer facilities, with financial support from the Technology Foundation (STW).

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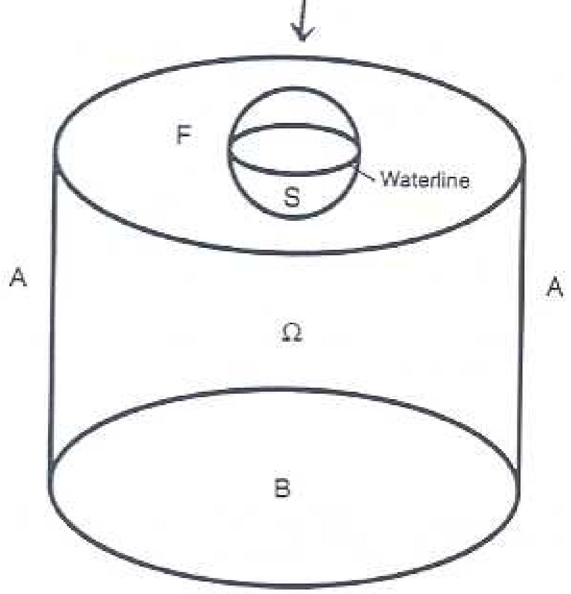
Figure 1: Fluid domain and bounding surfaces.



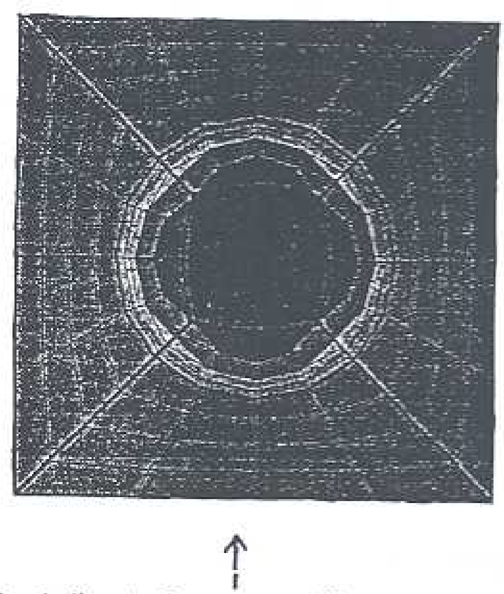


Figure 2: Top view of a sphere at rest in a domain similar to the one we will carry out the calculations with. In the middle is the wetted part of the sphere divided into five networks containing 16 panels each. The surrounding free surface is divided into four networks containing 16 panels each. The heavy lines indicate the network boundaries. The grey shades give an indication of the pressure distribution along the boundary when the sphere and the fluid are at rest.