A 2-D Numerical Wave Flume Based on a Third Order Boundary Element Model

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Boundary Element Models (BEM's) based on potential theory are today widely used for numerical simulation of surface waves and wave-current-structure interaction. Since the full non-linear models are quite expensive in computer time, low order models based on Stokes' theory have been developed. The low order models are significantly faster than the full non-linear models, and thus great efforts are made to make these models feasible for as many problems as possible. For instance a second order model for wave-current interaction is still to be made.

The aim of the present study is to develop a second or third order 3-D wave tank for numerical simulation of wave-current-structure interaction. As intermediate results we have developed 2-D second order and a 2-D third order BEM for simulating wave-structure interaction. Comparing results for these two models to results by a full non-linear BEM by Skourup (1989, 1995) we can see how well each of the two models is performing.

The third order Boundary Element Model

The development of the third order BEM will be described briefly in the following. The second order BEM is basically the same, but with no third order terms and using a slightly different method for updating the free surface boundary conditions.

The 2-D transient flow problem considered here is restricted to a closed fluid domain bounded by a curve Γ. Γ includes a moving free surface Γ_f, a moving wave maker, Γ_{r1}, as well as the non-moving boundaries Γ_b and Γ_{r2}. Definitions of the fluid domain boundaries are shown as the fully drawn curve in Figure 1. Definitions of the wave height, H, and the wave length, L, as well as the length, l, of the computational domain and the local water depth, h, are also shown in Figure 1.

Following the normal procedure for low order BEM's we will transform the (nonlinear) boundary conditions on the moving boundaries to a set of linear conditions on a stationary boundary.

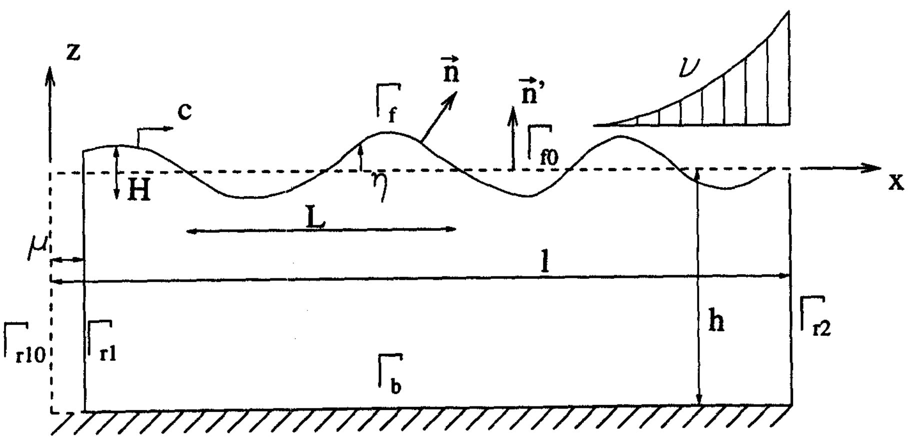

Figure 1: Definition sketch

Thus we define a computational domain bounded by the stationary boundary Γ_0, shown as the dashed curve in Figure 1 ($\Gamma_0 = \Gamma_{f0} \cup \Gamma_{r20} \cup \Gamma_b \cup \Gamma_{r1}$).

If ϕ denotes the velocity potential then the free surface boundary conditions, modified by dissipative terms for use in an absorbing layer, can be brought to the form

$$\frac{\partial \phi}{\partial t} + g\eta + \frac{1}{2}|\nabla\phi|^2 + \nu(x)\phi = 0 \ , \quad \text{at } z = \eta(x) \tag{1}$$

$$\frac{\partial \eta}{\partial t} - \frac{\partial \phi}{\partial z} + \frac{\partial \phi}{\partial x}\frac{\partial \eta}{\partial x} + \nu(x)\eta = 0 \ , \quad \text{at } z = \eta(x) \tag{2}$$

The free surface boundary conditions (1) and (2) are Taylor expanded from the still water level $z = 0$ to the actual surface elevation $z = \eta$. Then a perturbation expansion parameter ε is introduced such that

$$\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + O(\varepsilon^4) \tag{3}$$

$$\eta = \varepsilon\eta_1 + \varepsilon^2\eta_2 + \varepsilon^3\eta_3 + O(\varepsilon^4) \tag{4}$$

The perturbation expanded potential and surface elevation are inserted in the Taylor expanded boundary conditions, and terms of same order in ε are collected. The first and second order boundary conditions have been used before (Isaacson and Cheung, 1991), so only the third order terms will be given here:

$$\begin{aligned}\frac{\partial \phi_3}{\partial t} + g\eta_3 + \nu(x)\phi_3 =& -\frac{\partial \phi_1}{\partial x}\frac{\partial \phi_2}{\partial x} - \frac{\partial \phi_1}{\partial z}\frac{\partial \phi_2}{\partial z} - \eta_1\frac{\partial^2 \phi_2}{\partial t\partial z} - \eta_2\frac{\partial^2 \phi_1}{\partial t\partial z} - \frac{1}{2}\eta_1\frac{\partial}{\partial z}\left(|\nabla\phi_1|^2\right) \\ & - \nu(x)\eta_1\frac{\partial \phi_2}{\partial z} - \nu(x)\eta_2\frac{\partial \phi_1}{\partial z} - \frac{1}{2}\eta_1^2\frac{\partial^3 \phi_1}{\partial t\partial z^2} - \frac{1}{2}\nu(x)\eta_1^2\frac{\partial^2 \phi_1}{\partial z^2}\end{aligned} \tag{5}$$

$$\frac{\partial \eta_3}{\partial t} - \frac{\partial \phi_3}{\partial z} + \nu(x)\eta_3 = -\frac{\partial \phi_1}{\partial x}\frac{\partial \eta_2}{\partial x} - \frac{\partial \phi_2}{\partial x}\frac{\partial \eta_1}{\partial x} + \eta_1\frac{\partial^2 \phi_2}{\partial z^2} + \eta_2\frac{\partial^2 \phi_1}{\partial z^2} - \eta_1\frac{\partial^2 \phi_1}{\partial x\partial z}\frac{\partial \eta_1}{\partial x} + \frac{1}{2}\eta_1^2\frac{\partial^3 \phi_1}{\partial z^3} \tag{6}$$

Surprisingly no third order derivatives need to be approximated numerically in this third order model, since (5) and (6) can be rewritten to containing only first and second order derivatives in space. Both third and fourth order spacial derivatives will be introduced in the fourth order terms, and since it is difficult numerically to find accurate higher order derivatives, this *could* be troublesome when a fourth or higher order model is developed. Thus we restrict ourself to third order.

Similar to (5) and (6) boundary conditions correct to third order have been developed for the piston boundary (not shown here).

In discrete form the boundary integral equations obtained for ϕ_3 will have the same coefficients as the equations for ϕ_1 and ϕ_2. This means that the calculation time per time step will be increased by roughly 50% by introducing third order terms compared to a second order model. For weakly nonlinear waves it may not be worth the extra calculation time to obtain the third order potential, but for waves of intermediate nonlinearity the third order terms may make this model feasible, and this model will still be a lot faster than a full nonlinear BEM.

The numerical time stepping procedure of the third order BEM at hand is a three step procedure for each order:

1. Updating using a Adams-Bashforth-Moulton method
2. Solving the boundary integral equations

3. Finding new values for time derivatives using the free surface boundary conditions

Correction methods have been implemented to avoid wiggles at the intersections between the free surface and the lateral boundary. Both a Boundary Condition correction method (Otta 1992) and extrapolation from the interior of the free surface to the corners are used. Apart from this no artificial smoothing have been used in the model.

Results

The results presented here represents the generation and propagation of a transient wave train over a horizontal bottom. The simulations are compared with results by Skourup (1995) using a full nonlinear BEM on the same problem. The chosen test uses first order (sinusoidal) piston generation, which is widely used in physical wave flumes. The following parameters are used for the simulation: wave period $T = 3.0$s, wave height $H = 0.14$m and water depth $h = 0.70$m. This gives a wave length of $L = 7.4$m and a wave steepness of $H/L = 1.9\%$. The spatial and temporal discretizations are respectively $\Delta x = 0.10$m and $\Delta t = 0.025$s. The generated waves

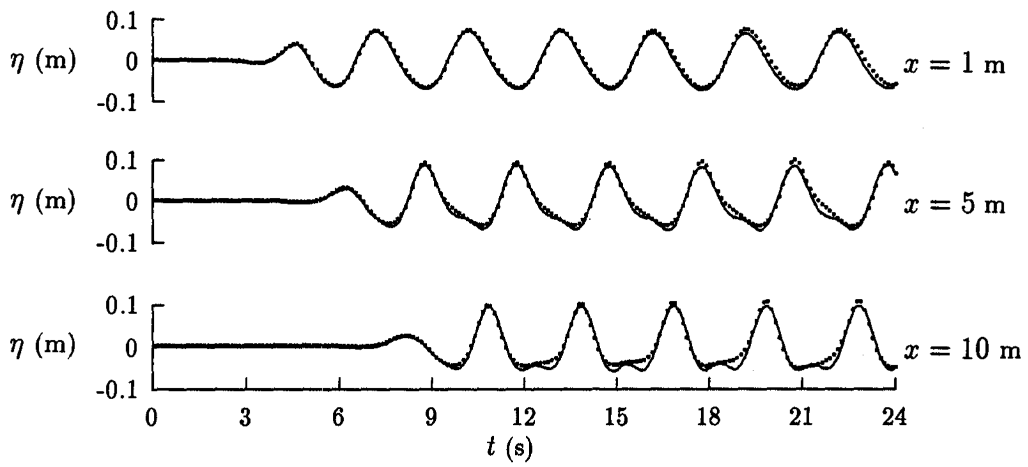

Figure 2: Time series for surface elevations at different locations for the present *second* order BEM (—) and the full nonlinear BEM by Skourup (···) $T = 3.00$s, $h = 0.70$m.

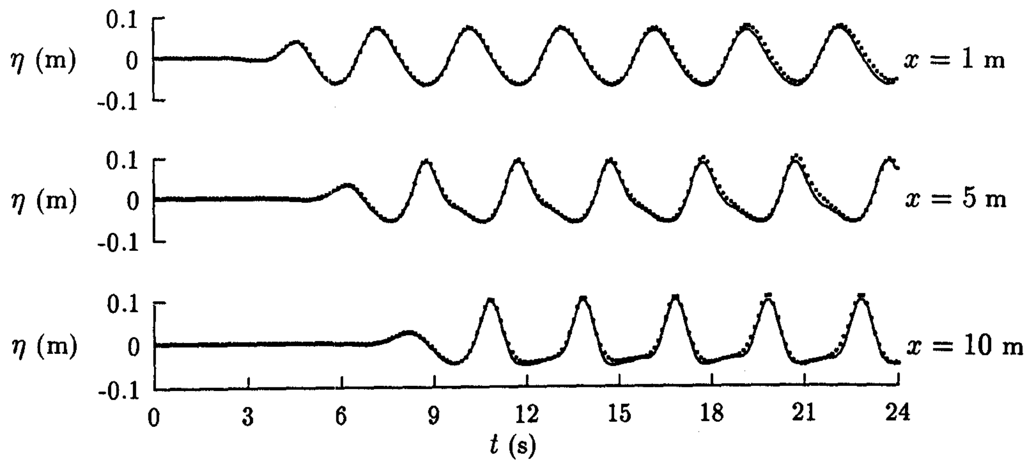

Figure 3: Time series for surface elevations at different locations for the present *third* order BEM (—) and the full nonlinear BEM by Skourup (···), $T = 3.00$s, $h = 0.70$m.

are quite long in shallow water, and thus this simulation represents a challenging test for a low order BEM.

As can be seen from Figure 2 the second order model does describe the waves reasonable well, but secondary crests in the wave troughs do occur. This is not unusual given the rather high nonlinearity of the problem. The waves are clearly seen to by transient, since the shape of the waves changes with the distance x from the wavemaker. This is caused by the wave generation being only first order. We see from Figure 3 that the third order model is simulating the wave much better than the second order model. The secondary crests in the troughs of the waves have vanished, and the elevations generally fit the results from the full non-linear BEM by Skourup (1995) nicer. It must be said that the Stokes theory could not a priori be expected to be working at its best, even though we have included third order terms. The transient waves generated are quite high ($H/h = 20\%$) in relative shallow water ($L/h = 10.6$), and thus this is a quite demanding test for the model.

As expected the third order model gives solutions closer to the full nonlinear BEM than the second order model. Since the third order model is a lot faster than a full nonlinear BEM and only 50% more CPU time than the second order model, it can be used as an alternative to existing second order or full nonlinear BEM's when modelling waves of small to intermediate nonlinearity.

The third order model at hand can include wave-structure interaction, but it has not been shown here.

Acknowledgements

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DISCUSSION

Kashiwagi: Could you explain about the form of the sponge layer, $\nu(x)$, i.e. length, strength, etc.? Are the results not sensitive to phase values?

Büchmann: We are using

$$\nu(x) = \begin{cases} 0 & ,x \in [0; x_s[\\ C \cdot \frac{(x-x_s)^2}{(l-x_s)^2} & ,x \in [x_s; l] \end{cases}$$

i.e. a parabolic variation. After a few numerical experiments, we have chosen to follow Cointe (1989) and define:

$$C = \omega = \frac{2\pi}{T} \qquad ; \qquad l - x_s = L = \frac{gT^2}{2\pi}\tanh kh$$

i.e. angular frequency and wave length based on linear theory. Of course the absorption of a sponge layer is not perfect, but this choice of sponge layer seems to absorb the waves quite good. As you indicate, the length and strength of the sponge layer should be chosen with care.

Cointe, R. (1989), *Nonlinear simulation of transient free surface flows*, 5th Int. Conf. on Num. Ship Hydrodyn., Hiroshima, 168-179

Korsmeyer: Can you elaborate on the computation time comparison of your 3rd order and nonlinear codes?

Büchmann: The model presented here has roughly 200 nodes. On an IRIS-INDIGO2 (Silicon Graphics) the computation time per time step was approximately 10 seconds for a full nonlinear code. Only 0.7 seconds were used in the inversion, so most of the time was spent on building the matrix (9.2s). For the third-order model, the computation time per time step is roughly 0.09s or less than 1% of the CPU time for the full nonlinear code.

Molin: In Stokes third-order theory of regular waves, a correction must be done to the wave-length (or frequency) to get rid of secularity. Otherwise the wave amplitude would slowly vary with time t or distance x. In your model the wave-length is set by the first-order solution. So it seems to me that you should get a slowly varying wave amplitude at third-order.

Büchmann: We do not impose the wave length to be equal to the wave length at first order. It just turns out that the third order terms modify the wave height rather than the wave length. Unfortunately this does not fulfill the dispersion relation for waves of permanent form to 3rd order. We do in fact see a slowly varying amplitude of 3rd order wave component. We do not think that this is caused by secular terms in the third order differential equations, though. This is for two reasons: First, the amplitude tends to a constant as time grows, and secondly the second order potential shows a slowly oscillating behaviour with the same period as the variation in the third order amplitude. The slow oscillation of the potential also vanishes as time grows. Thus, we think that the observed variation of the third order amplitude is caused by initial conditions rather than secular terms. We do not see a problem in fulfilling the dispersion relation to third order since the first order solution should then depend on the wave amplitude, which it does not! We have not been aware of this problem before, and efforts should be made to deal with this. I should like to thank Prof. Molin for pointing out this problem.

Schultz: Since the KBC stays the same in viscous and potential formulations, it would seem better to put all the damping in the dynamic B.C. for the beach. Then mass is conserved locally in the beach, but energy is not. This seems to make more sense. Any comment?

Büchmann: Yes, you are absolutely right. Physically it would make more sense to keep the KBC without dissipative terms. The damping term in the KBC modifies the mean water level at the sponge layer. But since we are using a 2D model and only have one sponge layer, we can use damping terms in the KBC to improve damping without great risk. Care should be taken using multible sponge layers or in 3D, since net currents may be induced.