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Uniformly valid solution of the wave-current-body interaction problem

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Wave diffraction-radiation by offshore structures or ships in the low forward speed regime has been analyzed in a number of studies, including *Huijsmans and Hermans* (1985), *Wu and Eatock Taylor* (1990), and *Nossen et al.* (1991). In these studies, a perturbation expansion of the Green function and/or the velocity potential with respect to the Strouhal number $\tau = U\omega/g$, where U, ω and g respectively stand for the forward speed, the encounter wave frequency, and the acceleration of gravity, is used. Interesting results, notably for wave drift dampings, have been obtained. Although this perturbation analysis yields useful $O(\tau)$ corrections for forward-speed (or current) effects, it suffers from an aesthetic blemish associated with the property that solutions (velocity potential and Green function) contain secular terms unbounded in the far field. Furthermore, this physically-unacceptable far-field behavior can seriously limit practical applications to bodies of large size.

In this paper, we present a uniformly valid solution of the wave-current-body problem which is based on a decomposition of the unsteady time-harmonic potential into linear and nonlinear parts. The linear unsteady potential satisfies the classical linearized condition on the free surface within a $O(\tau)$ approximation. Interactions between the local steady potential represented by the double-body solution and the linear unsteady potential are taken into account by the nonlinear unsteady potential which satisfies a non-homogeneous condition on the free surface. The integral equations corresponding to the source method for both the linear and nonlinear time-harmonic potentials are solved by making use of the Green function G for small τ given in *Noblesse and Chen* (1995), which is uniformly valid in space.

Another new feature of the present solution is the direct evaluation of m_j-terms on the right hand side of the body boundary condition for the nonlinear unsteady potential. A method inspired from *Wu* (1991) is developed to evaluate all the terms of the double gradient of the steady potential, without computing the double gradient of the Green function. Numerical results using lower-order panel methods show this method is effective and the double derivatives are evaluated with a very good precision.

Numerical results for the double gradients of the local steady potential on an ellipsoid hull, wave drift dampings and free-surface elevations in the vicinity of a group of vertical cylinders are presented. Some comparisons with results obtained by the perturbation method are also included.

Wave-current-body interaction problem

We define a moving system of coordinates (x,y,z) in steady translation with the mean forward velocity of the ship: the x axis points in the direction of forward speed, the z axis points upward and $z=0$ is the mean free-surface plane. Based on the assumptions of perfect fluid, irrotational flow and small wave steepness, the flow potential can be written as a superposition of a steady part $\bar{\phi}$ and a time-harmonic unsteady part ϕ by :

$$\Phi = U(\bar{\phi} - x) + \mathrm{Re}[\phi \exp(-i\omega t)] \tag{1}$$

Both the steady and unsteady potentials satisfy the Laplace equation in the fluid domain and an appropriate radiation condition at infinity. In particular, assuming small forward speed, the steady potential $\bar{\phi}$ satisfies :

$$\bar{\phi}_z|_{z=0} = 0 \qquad \text{and} \qquad \bar{\phi}_n|_{S_B} = n_1 \tag{2a,b}$$

at the free surface $S_F(z=0)$ and the body surface S_B, respectively. In $(2a,b)$, $\bar{\phi}_{z,n}$ mean the derivatives of $\bar{\phi}$ with respect to z and n where the normal vector $\vec{n} = (n_1, n_2, n_3)$ points toward the fluid.

Neglecting terms of order $O(\tau^2)$ and higher, the time-harmonic potential satisfies boundary conditions at the free surface $S_F(z=0)$ and the body surface which are written (e.g. as given in *Malenica* 1994) :

$$(-k + \partial_z + 2i\tau\,\partial_x)\,\phi - 2i\tau\,\nabla\bar{\phi}\,\nabla\phi + i\tau\,\bar{\phi}_{zz}\,\phi = 0 \quad \text{on } S_F \quad \text{and} \quad \phi_n = V_n \quad \text{on } S_B \tag{3a,b}$$

where $k = \omega^2/g$ is the wavenumber. On the rignt hand side of $(3b)$ we have $V_n = 0$ for the diffraction problem and $V_n = n_j + i\tau m_j/k$ with $j=1,2,...,6$ for the radiation problem where $(n_4, n_5, n_6) = \vec{r} \times \vec{n}$ are rotational components of the generalized normal vector. The well-known m_j terms are defined by double derivatives of the local steady flow.

Decomposition of the unsteady time-harmonic potential

The free-surface condition $(3a)$ suggests a decomposition of the time-harmonic potential ϕ into a linear part $\phi_I + \phi_L$ which is independent of the steady flow, and a nonlinear part ϕ_N which takes into account the interaction with the steady flow. Furthermore, low forward speed is assumed (for small Strouhal number τ), so that the nonlinear part ϕ_N can be supposed to be of order τ. We write then :

$$\phi = \phi_I + \phi_L + \tau\,\phi_N \tag{4}$$

where ϕ_I is the incident wave potential. Introducing the decomposition (4) in the free-surface condition $(3a)$, the linear time-harmonic potential ϕ_L satisfies a linear condition :

$$E\phi_L = 0 \qquad \text{with} \qquad E = -k + \partial_z + i2\tau\,\partial_x \tag{5a}$$

and the nonlinear potential ϕ_N satisfies a non-homogeneous condition :

$$E\phi_N = Q \qquad \text{with} \qquad Q = 2i\,\nabla\bar{\phi}\,\nabla(\phi_I + \phi_L) - i\,\bar{\phi}_{zz}\,(\phi_I + \phi_L) \tag{5b,c}$$

In the same way, $(3b)$ and (4) give the conditions on the body surface for the linear potential :

$$\partial_n \phi_L = -\partial_n \phi_I \quad \text{and} \quad \partial_n \phi_L = -n_j \tag{6a,b}$$

for the diffraction and radiation problems, respectively. The condition on the body surface for ϕ_N is written by :

$$\partial_n \phi_N = im_j/k \tag{6c}$$

since m_j includes the interaction effects with the local steady flow.

The new decomposition (4) of the time-harmonic potential leads to a decomposition of the wave-current-body interaction problem for the unsteady potential (3*a,b*) into two distinct components represented by equations (5*a*,6*a,b*) for the linear unsteady potential ϕ_L and (5*b*,6*c*) for the nonlinear unsteady potential ϕ_N, respectively. The linear unsteady potential ϕ_L (the incident potential ϕ_I is known analytically) satisfies the classical linearized condition on the free surface (5*a*) in which terms of order $O(\tau^2)$ and higher are neglected. The nonlinear unsteady potential ϕ_N satisfies a non-homogeneous condition on the free surface (5*b*) where the term on the right hand side depends on the local steady flow $\bar{\phi}$ and the linear unsteady potential ϕ_L. The solution of these two coupled problems depends on an elementary solution - the Green function - satisfying the same linearized free-surface condition (5*a*) as the linear unsteady potentials ϕ_L, since we apply the Green theorem in the fluid domain.

By applying the Green theorem to the linear potential and the Green function which satisfies the same free-surface condition (5*a*) as the linear unsteady potential, and introducing the boundary condition on the ship hull S_B, we obtain the expression of ϕ_L and the corresponding integral equation :

$$\phi_L = \iint_{S_B} \sigma_L \, G \, dS \quad \text{with} \quad \frac{1}{2}\sigma_L + \iint_{S_B} \sigma_L \, \partial G/\partial n \, dS = V_n \tag{7a,b}$$

where $V_n = -\partial \phi_I / \partial n$ for the diffraction problem and $V_n = n_j$ with j=1,2,...,6 for the radiation problem, following (6*a,b*). In the same way, the nonlinear potential ϕ_N can also be expressed by :

$$\phi_N = \iint_{S_B} \sigma_N \, G \, dS - \iint_{S_F} Q \, G \, dS \quad \text{with} \quad \frac{1}{2}\sigma_N + \iint_{S_B} \sigma_N \, \partial G/\partial n \, dS = \iint_{S_F} Q \, \partial G/\partial n \, dS + im_j/k \tag{8a,b}$$

with $Q(x,y)$ defined by (5*c*). The integral equations (7*b* and 8*b*) which determine the unknown source densities σ_L and σ_N, are established via an analysis based on applying the Green theorem in both the fluid domain exterior to the body surface and the fictitious interior domain enclosed by the body surface and the interior waterplane. No line integral along the waterline is present in the source method, and the integrands of the integrals on the free surface in (8*a,b*) decay very rapidly with increasing distance from the body, so that only a relatively small region of the free surface around the body must be discretized.

Double gradients of the local steady flow

The well-known m_j terms are present on the right hand side of the integral equations (8*b*) to determine the source density corresponding to the linear radiation potential. As the m_j terms contain the second-order derivatives of the steady potential $\bar{\phi}$, it has been shown that we cannot get good accuracy by usual direct computations using double gradients of the Green function. The integral relation given in *Ogilvie and Tuck* (1969) is generally used to reduce the derivatives to the first-order. Unfortunately, this tranformation cannot be applied to the integral equation (8*b*). In addition, the second-order derivative $\bar{\phi}_{zz}$ must be evaluated over the free surface, as it appears in the integrand $Q(x,y)$ of the free-surface integral (8*a,b*).

A new method inspired from *Wu* (1991) is developed to evaluate all the terms of the double gradients of $\bar{\phi}$. The method consists in treating the second-order derivatives of $\bar{\phi}$ as solutions of a problem of Dirichlet type, using the first-order derivatives of $\bar{\phi}$ as the right-hand side term of the condition on the body boundary surface. By applying the Rankine source method, we can evaluate the source densities $\sigma_{1,2}$ by the integral equation :

$$\iint_{S_B} \sigma_{1,2} \, (1/r + 1/r') \, dS = \bar{\phi}_{x,y} \tag{9}$$

Thus, five second-order derivatives on S_B or at any point in fluid can be computed by :

$$\bar{\phi}_{xx,xy,xz} = \iint_{S_B} \sigma_1 \, \partial_{x,y,z} \, (1/r + 1/r') \, dS \quad \text{and} \quad \bar{\phi}_{yy,yz} = \iint_{S_B} \sigma_2 \, \partial_{y,z} \, (1/r + 1/r') \, dS \tag{10a,b}$$

and the sixth derivative $\bar{\phi}_{zz}$ on both S_B and S_F is obtained via the Laplace equation :

$$\bar{\phi}_{zz} = -\bar{\phi}_{xx} - \bar{\phi}_{yy} = -\iint_{S_B} (\sigma_1 \, \partial_x + \sigma_2 \, \partial_y) \, (1/r + 1/r') \, dS \tag{10c}$$

The three m_j (j=1,2,3) terms on S_B are defined by the three components of $-\partial_n[\nabla(\bar{\phi}-x)]$ and the three other m_j (j=4,5,6) terms, corresponding to the rotational modes, by $-\partial_n[\vec{r} \times \nabla(\bar{\phi}-x)]$.

The procedure defined by (9) and (10) makes it possible to obtain double gradients of the potential without using double gradients of the Green function. This source method to evaluate the six double derivatives and then the m_j (j=1,2,3) is quite similar to that of Wu who used directly the Green theorem to establish the integral equations (corresponding to so-called mixed source-dipole method). This mixed method is unlikely to be able to evaluate separately the six components of the second-order derivatives. Only one integral equation (9) is to be solved to obtain afterwards the six derivatives by (10), since the other three double derivatives are not independent.

Numerical results and comparisons

The double gradients of the local steady flow are evaluated by using the source method (9-10) for a half-immersed ellipsoid with three principal axes (a=3,b=1.5 and c=1.2). Numerical results obtained from a mesh composed of 1956 flat panels are presented by markers on Fig.1 and compared with analytical solutions described by continuous or discontinous lines. The computed points are located at the centroids of the first line of panels (total of 35) close to the free surface for $0 < x < 3$ and $-1.5 < y < 0$. The abscisse on Fig.1 is the x-coordinates. The comparison with analytical solutions shows that the second-order derivatives, and hence the m_j-terms, are evaluated with a very good precision.

Using the source method (7-8), the linear and nonlinear unsteady potentials are evaluated numerically. The validation has been carried out by comparing with the non-secular analytic solution for a vertical cylinder, given in *Malenica* (1995). The global loadings (first and second-order) and the radiation coefficients evaluated for single cylinders and half-immersed spheres agree well with the solutions obtained by using the perturbation method (e.g. *Nossen et al.* 1991). However, this is not the case for bodies of large size such as an oil platform composed of multi-columns.

We consider a structure composed of 4 vertical columns (1m of radius, 3m of draft and 7m of axis-to-axis distance) which is represented by a mesh including 1120 panels on the hull, together with 880 panels over the free surface. The wave frequency and the current speed considered are 2.5rad/s and 0.2m/s, respectively. Free-surface elevations due to the diffraction linear and nonlinear potentials, are calculated by the present method and the method using the secular Green function. The variation of free-surface elevations (moduli) along a straight line from $-2\lambda < x < 8\lambda$ (y =0.35m and z=0) where λ is the wavelength, is presented in Fig.2 in which the solid and dotted lines correspond to the results obtained by the present method and the secular method, respectively. The figure shows the free-surface elevation obtained by the secular method, increases at a moderate distance away from the body while the results from the present method decay.

Even in the vicinity of the structure, noticable differences (more than 20%) are found between the two approaches as shown in Fig.3 where the computed points are located along two centre lines on the free surface : between the two downstream cylinders (-2.5m < y < 2.5m with x = -2.5m) and between the two upstream cylinders (-2.5m < y < 2.5m with x = 2.5m). The square symbols and the circle symbols show results (moduli) along the first centre line and the second centre line, respectively. The solid and dotted lines correspond respectively to the results obtained by the present method and the secular method.

Finally, the wave drift dampings (represented by circle symbols) on the fixed 4-columns in the current direction are presented in Fig.4 and compared with analytical results (solid line) using the exact formulae given in *Clark et al.* (1993). The good agreement shows the present method gives the same results as previous perturbation methods. This can be explained by the fact that the wave drift dampings are obtained by numerical differenciation between the drift loadings due to two different forward speeds which are assumed to be very small (a current speed of ±0.02m/s was used) in which case the secular solution is known to be good.

Conclusions

Unlike the analysis based on a perturbation expansion which yields secular terms unbounded in the far field, the decomposition (4) gives a consistent expression of the time-harmonic potential as the sum of the linear component and the term representing the nonlinear effects of the local steady flow. The uniformly valid solution is obtained by using the Green function for small τ given in *Noblesse and Chen* (1995), and by using the source method to evaluate accurately the m_j-terms of the local steady flow. Numerical results show the significance of the present method which removes the limitation of applications to bodies of large size and gives correctly fluid kinematics around the body, in addition to the global first and second-order loadings.

Furthermore, the present method can be extended, without theoretical difficulties, to solve the second-order diffraction-radiation problem with small forward speed (up to terms $O(\varepsilon^2\tau)$), since the potentials are well behaved in space and the integrals over the free surface involved in the second-order problem are convergent. This is not the case if we use the classical perturbation method which, as already mentioned, leads to secular terms for the potentials.

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Fig.1 Double gradients of the local steady flow : (left) for $\bar{\phi}_{xx,yy,zz}$ and (right) for $\bar{\phi}_{xy,xz,yz}$

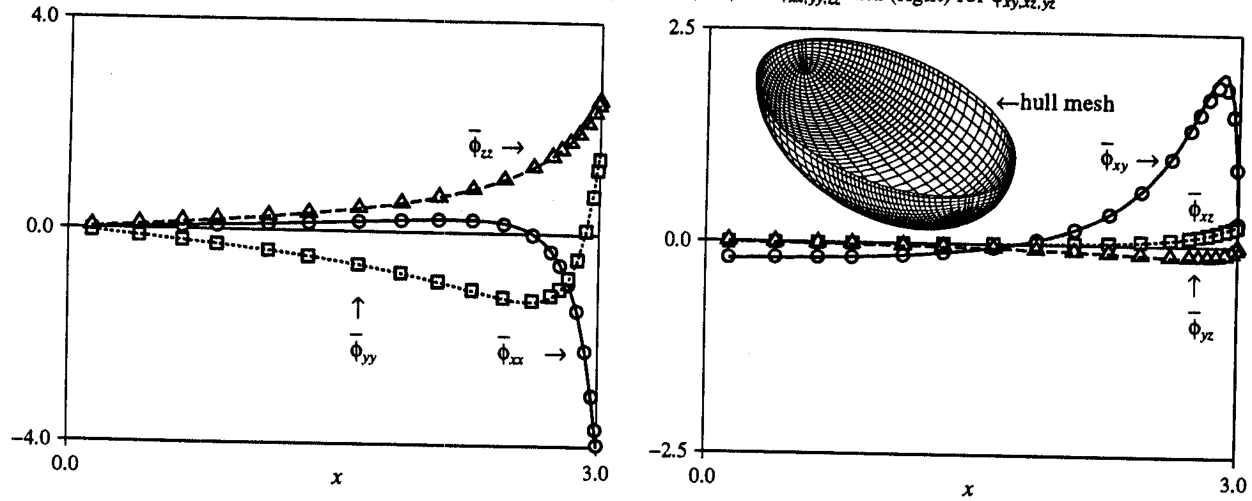

Fig.2 Diffraction wave elevation along a longitudinal line over the free surface

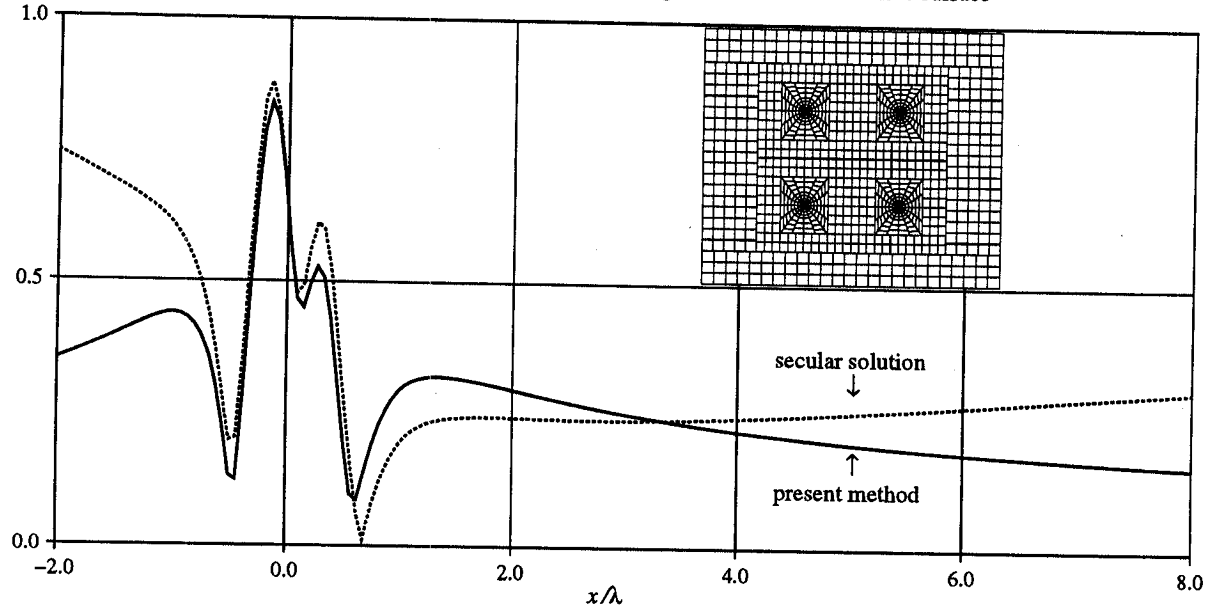

Fig.3 Diffraction wave elevation around the body

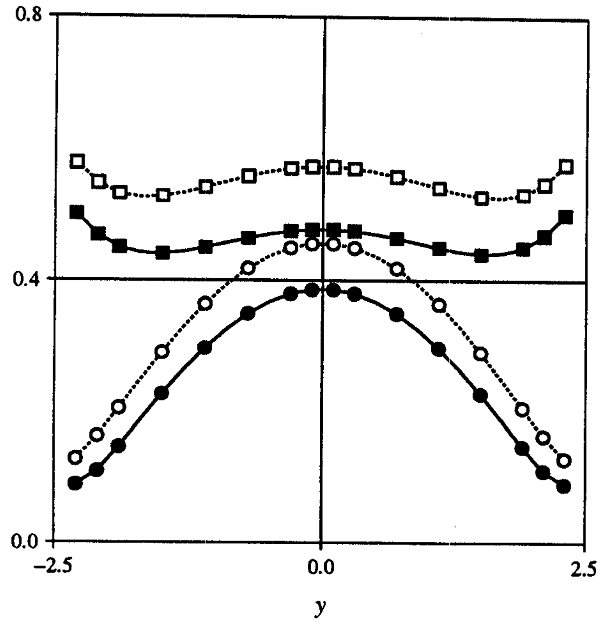

Fig.4 Wave drift dampings on 4 cylinders