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A NOTE ON THE YAW-MOMENT ACTING ON BODIES ADVANCING IN WAVES

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1 Introduction

We consider the mean yaw-moment acting on floating bodies advancing in incoming monochromatic waves. The fluid is assumed incompressible and inviscid, and the flow irrotational. The amplitude of the waves (A) and the speed of the body (U) are both assumed small. The mean yaw-moment is obtained taking into account terms up to $O(A^2, U)$. In Grue and Palm (1993), hereafter referred to as GP, formulae were derived for evaluating this moment. One part of the moment stems from second order coupling between the linear motion and itself. Another part is due to interaction between time-averaged second-order velocities in the fluid and the steady flow due to the moving body. The moment was obtained by two methods, viz.

a) the near field method (pressure integration),

b) the far field method (conservation of angular momentum).

It is convenient to denote the moments obtained by the near and far field methods by M_z^{near} and M_z^{far}, respectively. These moments are composed by

$$M_z^{near} = M_z^{near(1)} + M_z^{near(2)}, \qquad M_z^{far} = M_z^{far(1)} + M_z^{far(2)}, \tag{1}$$

where $M_z^{near(1)}$, $M_z^{far(1)}$ stem from second order couplings between the linear part of the flow and itself, and $M_z^{near(2)}$, $M_z^{far(2)}$ stem from coupling between time-averaged second-order velocities and the steady flow due to the moving body. It turns out that the two different procedures lead to different analytical expressions and numerical values for $M_z^{near(1)}$ and $M_z^{far(1)}$, $M_z^{near(2)}$ and $M_z^{far(2)}$. More precisely: While $M_z^{near} = M_z^{far}$, we have in general that $M_z^{near(1)} \neq M_z^{far(1)}$, and $M_z^{near(2)} \neq M_z^{far(2)}$.

Questions concerning these results have been raised: Are they inconsistent or wrong? And, in practical computations: Can the contribution due to the second-order velocities be avoided? These questions have provoked this study. We find that the answers to these questions are: No. In the present note we will by some examples try to make the issue clearer.

We note that a corresponding result also applies to the drift force.

2 The potentials

A coordinate system $Oxyz$ is introduced in the frame of reference following the forward speed of the body, with the x and y axes in the mean free surface and the z axis vertical upwards. Unit vectors $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ are introduced accordingly. The forward speed direction is along the x axis. According to the assumptions above, the fluid motion is governed by a Laplacian velocity potential Φ. This may be decomposed by $\Phi = U\chi_s + \phi^{(1)} + \psi^{(2)}$, where $U\chi_s = U(-x+\chi)$ denotes the velocity potential for the steady flow around the body, $\phi^{(1)}$ the linear wave potential proportional to A, and $\psi^{(2)}$ a steady second order potential proportional to A^2. The potential Φ may also have other components, but these do not contribute to the present analysis.

Assuming time-harmonic oscillations with frequency of encounter σ, the linear potential $\phi^{(1)}$ can be written

$$\phi^{(1)}(\mathbf{x},t) = Re\{\phi(\mathbf{x})e^{i\sigma t}\} = Re\{A(\phi_I + \phi_B)e^{i\sigma t}\}. \tag{2}$$

The incoming wave potential reads $\phi_I = (ig/\omega)(\cosh K(z+h)/\cosh Kh)e^{-iK(x\cos\beta + y\sin\beta)}$, where g denotes the acceleration of gravity, ω the wave frequency, K the wave number ($\omega^2 = gK\tanh Kh$), h the fluid depth, and β the angle of incidence. σ is related to ω and K by $\sigma = \omega - UK\cos\beta$. The potential ϕ_B represents the sum of the scattering and radiation potentials. Here we consider only the far field form of this potential which is given by

$$\phi_B = R^{-1/2}H(\theta)\frac{ig}{\omega}\frac{\cosh k_1(\theta)(z+h)}{\cosh k_1(\theta)h}e^{-ik_1(\theta)R(1+O(\tau^2))} + O(1/R) \quad \text{as } R \to \infty, \tag{3}$$

where $H(\theta)$ denotes amplitude distribution, $x = R\cos\theta$, $y = R\sin\theta$, $k_1(\theta) = k_0(1 + 2\tau^h\cos\theta)$, $k_0(\beta) = K(1 - 2\tau^h\cos\beta)$, $\tau^h = \tau/C_g(Kh)$, $\tau = U\sigma/g$, and $C_g(Kh) = \tanh Kh + Kh/\cosh^2 Kh$.

The Laplacian potential $\psi^{(2)}$ has the boundary conditions

$$\partial\psi^{(2)}/\partial z = -(\sigma/2g)Im(\phi\phi^*_{zz}) \quad \text{on } z = 0 \tag{4}$$

where a star denotes complex conjungate,

$$\partial\psi^{(2)}/\partial n = -\mathbf{n}\cdot\overline{[(\boldsymbol{\xi}^{(1)} + \boldsymbol{\alpha}^{(1)}\times\mathbf{x})\cdot\nabla]\nabla\phi^{(1)}} + \overline{(\boldsymbol{\alpha}^{(1)}\times\mathbf{n})\cdot[(d/dt)(\boldsymbol{\xi}^{(1)} + \boldsymbol{\alpha}^{(1)}\times\mathbf{x}) - \nabla\phi^{(1)}]} \text{ on } S_B, \tag{5}$$

where $\boldsymbol{\xi}^{(1)} = Re\{(\xi_1,\xi_2,\xi_3)e^{i\sigma t}\}$ and $\boldsymbol{\alpha}^{(1)} = Re\{(\xi_4,\xi_5,\xi_6)e^{i\sigma t}\}$ denote the first order translations and rotations of the body, respectively, and a bar time-average. S_B denotes the wetted part of the body in the mean position. In addition, $\partial\psi^{(2)}/\partial z = 0$ at $z = -h$, $\nabla\psi^{(2)} \to 0$ for $(x^2+y^2)^{1/2} \to \infty$.

3 The yaw-moment

3.1 The near field method

The mean yaw-moment is first obtained by pressure integration over the wetted body surface (M_z^{near}). The moment has two components as defined in (1). $M_z^{near(1)}$ is found by integrating the parts of the fluid pressure determined by $\phi^{(1)}$ and χ_s over the wetted body surface. (The expression is not given here.) $M_z^{near(2)}$ is determined by

$$M_z^{near(2)} = -\rho U\int_{S_B}\nabla\chi_s\cdot\nabla\psi^{(2)}n_6 dS = -\rho U\int_{S_F+S_B}\left(\Psi + x\frac{\partial\chi}{\partial y} - y\frac{\partial\chi}{\partial x}\right)\frac{\partial\psi^{(2)}}{\partial n}dS, \tag{6}$$

where $n_6 = \mathbf{k}\cdot(\mathbf{x}\times\mathbf{n})$, $\mathbf{n}$ the normal vector positive out of the fluid, $\mathbf{x} = (x,y,z)$, S_F denotes integration over the mean free surface and Ψ is determined by $\Psi = y + \Psi_s$. The latter is a Laplacian velocity potential satisfying $\partial\Psi_s/\partial z = 0$ on $z = 0, -h$, $\partial\Psi_s/\partial n = 0$ on S_B, $\Psi_s \to -y$ for $(x^2+y^2)^{1/2} \to \infty$. We note that Ψ_s is the steady potential due to a current with unit speed along the negative y axis, corresponding to χ_s, the potential due to the current along the negative x axis.

3.2 The far field method

Next we consider the mean yaw-moment obtained by the far field method. The formula for $M_z^{far(1)}$ reads (see GP, Grue and Biberg 1993)

$$\frac{M_z^{far(1)}}{\rho g A^2} = -\frac{g}{4\omega^2}\int_0^{2\pi}(C_g(k_1 h) - 2\tau\cos\theta) Im\left[H\frac{\partial H^*}{\partial\theta}\right]d\theta \tag{7}$$

$$-\frac{g}{2\omega^2}\left\{\left(1-\frac{K}{C_g(Kh)}\frac{dC_g(Kh)}{dK}\right)\tau\sin\beta Im[S] + (C_g(Kh) - 2\tau\cos\beta)Im[S']\right\}, \tag{8}$$

where $S = \sqrt{2\pi/k_0}e^{i\pi/4}[H(\beta + 2\tau^h\sin\beta)]^*$, $S' = \sqrt{2\pi/k_0}e^{i\pi/4}[H'(\beta + 2\tau^h\sin\beta)]^*$, a prime denotes differentiation with respect to the argument, and ρ the density of the fluid.

The formula for $M_z^{far(2)}$ derived by GP reads

$$M_z^{far(2)} = -\rho U\int_{S_F+S_B}\Psi_s\frac{\partial\psi^{(2)}}{\partial n}dS = -\rho U\int_{S_F+S_B}(\Psi - y)\frac{\partial\psi^{(2)}}{\partial n}dS. \tag{9}$$

The integral over the body surface in (9) involves evaluation of a spacial second derivative of $\phi^{(1)}$, and precise evaluation may not be trivial when applying a low-order panel method. By applying a variety of Stokes' theorem we may, however, obtain integrals which require evaluation of spacial first derivatives of $\phi^{(1)}$ only, being suitable for efficient and accurate computations. In addition to integrating over S_B we must also integrate along the water line of the body, C_B. The final result for $M_z^{far(2)}$ reads

$$\frac{M_z^{far(2)}}{\rho g A^2} = \frac{\tau}{2K}\iint_{S_F}\Psi_s Im(\varphi\varphi_{zz}^*)dS + \frac{\tau}{2}\int_{C_B}\Psi_s Im[(B_3+\varphi)\mathbf{B}^*\cdot\mathbf{n}]dl$$

$$+\frac{\tau}{2K}\iint_{S_B} Im[(\mathbf{B}\cdot\mathbf{n})(\nabla\varphi^* + K\mathbf{B}^*)\cdot\nabla\Psi_s - K\Psi_s\mathbf{C}\cdot\mathbf{B}^*]dS, \tag{10}$$

where $\varphi = (\omega/ig)\phi$, $\mathbf{B} = [(\xi_1,\xi_2,\xi_3) + (\xi_4,\xi_5,\xi_6)\times\mathbf{x}]/A$, $\mathbf{C} = [(\xi_4,\xi_5,\xi_6)\times\mathbf{n}]/A$.

4 Discussion

In the near field method the formulae for the moment are obtained by integrating the presssure over the instantaneous wetted body surface. When the far field method is used, the moment is found from conservation of angular momentum. This is equivalent to integrating the vector product of the space coordinate and the equation of motion of the fluid. The moment is in both procedures obtained consistently to second order in the wave amplitude and to first order in U.

It turns out that the contributions to the moment from the steady second order potential are different. Thus, the mathematical forms of $M_z^{near(2)}$ and $M_z^{far(2)}$ are different, see eqs. (6) and (9). The mathematical deductions thus show that in general $M_z^{near(1)} \neq M_z^{far(1)}$ and that $M_z^{near(2)} \neq M_z^{far(2)}$ (but $M_z^{near} = M_z^{far}$).

We have performed computations of the components of the moment using the near and far field methods. Several geometries are considered: Offshore platforms, ships, and also idealized bodies. In general we find that the contribution due to the second-order velocities is not small and cannot be neglected in the computations. It turns out that this effect is most pronounced when the wave direction is orthogonal to the steady motion of the body.

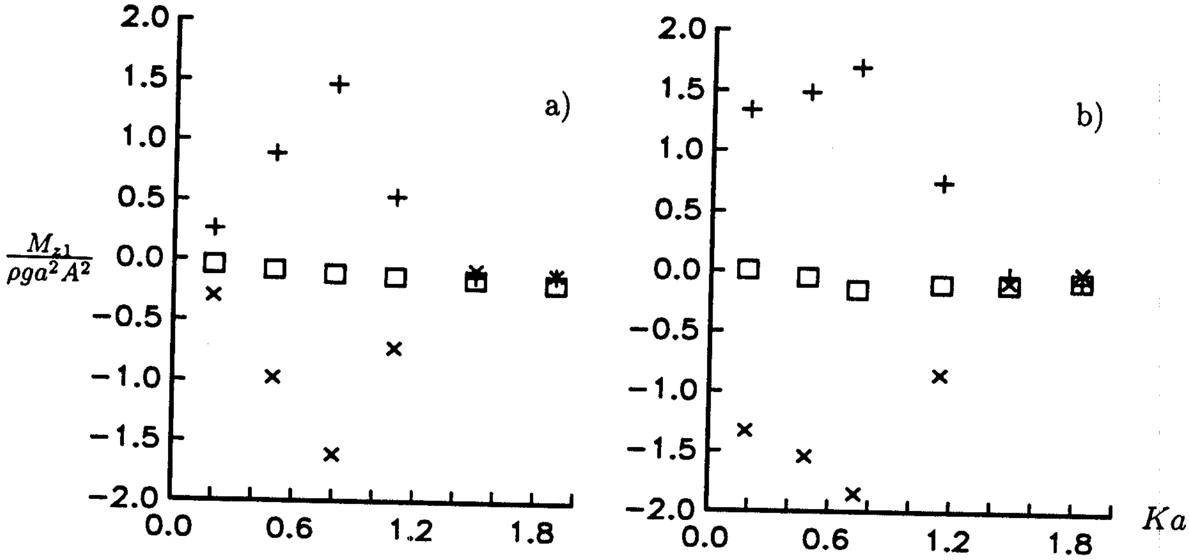

Figure 1: Wave drift damping moment vs. Ka due to slow surge motion of a freely floating half-immersed sphere with radius a. Wave angle: $\beta = 90°$. Squares: M_{z1}^{far} (total moment), $\times\ \times$: $M_{z1}^{far(1)}$, + + +: $M_{z1}^{far(2)}$. a) Water depth: $h = \infty$. b) Water depth: $h/a = 1.2$.

To illustrate this we consider the moment on a sphere with respect to the center-axis. This moment is zero, which must be predicted by both methods. Noting that $n_6 = 0$ for the sphere we find that $M_z^{near} = M_z^{near(1)} = M_z^{near(2)} = 0$. In the far field method we find that $M_z^{far(1)}$ and $M_z^{far(2)}$ are not zero in general. These individual terms are largest when the direction of the waves and the forward speed are orthogonal. In figure 1 are displayed results obtained by numerical methods to illustrate the contributions due to these terms. For convenience we introduce

$$M_z^{far} = M_{z0}^{far} + \frac{U}{\sqrt{ga}} M_{z1}^{far}, \qquad M_{z1}^{far} = M_{z1}^{far(1)} + M_{z1}^{far(2)}, \tag{11}$$

where a denotes the radius of the sphere. The figure shows that both $M_{z1}^{far(1)}$ and $M_{z1}^{far(2)}$ are large, that $M_{z1}^{far(1)} \simeq -M_{z1}^{far(2)}$ and that $M_{z1}^{far} = M_{z1}^{far(1)} + M_z^{far(2)} \simeq 0$. In these examples the contribution to $M_{z1}^{far(1)}$ is due to the stationary phase term (8). The right of (7) is always very small in these computations.

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References

Grue, J. and Biberg, D. (1993), Wave forces on marine structures with small speed in water of restricted depth. *Appl. Ocean Res.* **15**, 121-135.

Grue, J. and Palm, E. (1993), The mean drift force and yaw moment on marine structures in waves and current. *J. Fluid Mech.* **250**, 121-142.