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Nonlinear Water Waves in a 2-D Damaged Compartment

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Capsize may occur when the ship hull is damaged and water enters the compartment. If the bottom of the compartment is close to the center of gravity of the ship, water sloshing inside the compartment is hazardous to the ship's safety. The trapped water cannot escape from the compartment and may all move to one side of the ship due to the roll motion. Then, a large heeling moment is generated. The resonant frequency of sloshing is different from that in an intact compartment. Near the resonant frequency, large wave motion inside the compartment can be observed, even under the excitation of small ship motion. To understand the flow properties, we conducted a model test of a two-dimensional, damaged rectangular compartment in a wave flume. The opening is located at the center of the bottom. The damaged compartment is fixed in the flume so that the diffracted wave effect is studied. The water wave motion inside the compartment was measured under regular incident waves. A typical time history of wave motion inside the compartment is shown in Fig.1. The ratio of the wave height inside the compartment to the incident wave height, A_w, is shown in Fig. 2. Various beam(B) to draft(T) ratios and the sizes of opening were selected in the test. Numerical study of the nonlinear wave motion is under investigation. Our purpose is to develop a numerical scheme which is able to simulate the wave motion for a long time interval, and finally to apply it to the ship capsize study.

First, the nonlinear radiated wave problem is considered. We assume that the fluid is inviscid and incompressible, and the flow is irrotational. Vortex shedding is not considered. The fluid domain R is bounded by the free surface S_f, the body surface S_b and the bottom S_{bt}, as shown in Fig.3. The potential function satisfies the Laplace equation, the nonlinear free surface condition, the solid surface boundary condition, and the radiation condition at the far field. Initial conditions are also provided for the potential function and free surface elevation.

Finite Element Approach: The finite element method is applied to solve the initial boundary value problem. The horizontal, infinite domain is truncated at $x = \pm L$ (Fig.3) and appropriate boundary conditions are posed at these two boundaries. This will be discussed later. At each time step, the Laplace equation is solved, which is subjected to the essential boundary condition on S_f: $\phi(x,y) = f_1(x,y)$; and the natural boundary condition on other portion of boundary, $S_2 = S_b + S_{bt} + S_l + S_r$, of the computational region: $\frac{\partial \phi}{\partial n} = f_2(x,y)$. The appoximated potential function on an element is represented by:

$$\phi^e(x,y) = \sum_{j=1}^{m} \phi_j^e N_j^e(x,y) \tag{1}$$

where m is the number of nodes of one element and N_j^e are the shape functions. The Galerkin method is used in the finite element analysis which gives

$$\sum_{j=1}^{m} \alpha_{ij}^e \phi_j^e = \beta_i^e \tag{2}$$

where $\alpha_{ij}^e = \int_{\Delta R^e} \nabla N_i \cdot \nabla N_j dR$ and $\beta_i^e = \int_{\Delta S_2^e} f_2 N_i ds$. As in the paper by Wu and Eatock-Taylor (1995), the linear shape function on triangular elements is used for the nonlinear water wave problem. The potential functions at nodal points are solved from a linear algebraic system which is assembled from equation (2).

Free Surface Advection: Instead of using the kinematic free surface boundary condition, the Height-Flux method, which is used by Mashayek and Ashgriz (1993) in their jet stability study, is adopted for the free surface advection. The fluid domain is divided into n_x subdomains in the horizontal direction. At the time step t_n, when the velocity and wave surface are solved, the volume of each subdomain $v_i(t_{n+1})$ at the time step t_{n+1} will be determined from:

$$v_i(t_{n+1}) = v_i(t_n) + dv_i^l - dv_i^r + dv_i^b \tag{3}$$

where $v_i(t_n)$ is the volume of the ith subdomain at the time step t_n, dv_i^l is the volume flux into the ith subdomain through its left boundary, dv_i^r is the volume flux out of the ith subdomain through its right boundary and dv_i^b is the volume flux into the ith subdomain through its bottom boundary:

$$dv_i^l = \Delta t \int_{-d_{i-1}}^{\zeta_{i-1}} u(x_{i-1}, y)dy, \quad dv_i^r = \Delta t \int_{-d_i}^{\zeta_i} u(x_i, y)dy \ \text{ and } \ dv_i^b = -\Delta t \int_{s_{i-1}}^{s_i} v_n(s)ds \tag{4}$$

where $-d_i$ is the y-coordinate of the bottom, s is the curvilinear coordinate of the bottom, n is the outward normal and v_n is the normal velocity.

The height between two subdomains is determined as follows. It is assumed that the volume in each subdomain is known and the free surface height can be approximated by a straight line:

$$\zeta(\xi) = a\xi + b \tag{5}$$

where $\xi = x - x_{i-1}$. The subvolumes are:

$$v_i = \frac{1}{2}[b + (a\xi_i + b)]\xi_i \quad \text{and} \quad v_{i+1} = \frac{1}{2}[(a\xi_i + b) + a(\xi_i + \xi_{i+1}) + b]\xi_{i+1} \tag{6}$$

Then, we have

$$a = \frac{4(v_i\xi_{i+1} - v_{i+1}\xi_i)}{2\xi_i\xi_{i+1}(\xi_{i+1} - \xi_i)} \quad \text{and} \quad b = \frac{2[v_{i+1}\xi_i^2 - v_i(2\xi_i\xi_{i+1} + \xi_{i+1}^2)]}{2\xi_i\xi_{i+1}(\xi_{i+1} - \xi_i)} \tag{7}$$

The Height-Flux method is tested for a sine function and a step function. We used the the above formulas to compute the height function. The results are given in Fig.4 and Fig.5. When the free surface is predicted at the time t_{n+1}, the potential value on the free surface can be updated from the dynamic boundary condition. The boundary value problem can be solved at t_{n+1}.

Absorbing Boundary Condition: A boundary condition (absorbing boundary condition) simulating the radiation condition at the truncated boundary $x = \pm L$ should be devised. It is to allow the wave propagating out of the domain without reflecting back into the domain from the boundary. The absorbing boundary condition can be devised under certain ideal conditions. However, in a water wave problem (especially in the nonlinear cases), any differences from the ideal conditions will cause reflection from the boundary. In addition, we hope the truncated domain to be as small as possible to reduce the computational effort. In this work, a sponge layer is adopted in $x_0 < |x| < L$. The wave motion in the sponge layer is damped and the wave amplitude is decreasing as it approaches to the open boundary. If a reflection exists, it will be damped out by the artificial damping in the sponge layer. The fluid domain of interest is: $-x_0 < x < x_0$, and $-d(x) < y < \zeta$. The free surface condition is modified to include the damping term:

$$\frac{d\phi}{dt} + g\zeta + \frac{1}{2}[(\frac{\partial\phi}{\partial x})^2 - (\frac{\partial\phi}{\partial y})^2] + \mu\phi - \frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial y}\frac{\partial\zeta}{\partial x} = 0, \quad \text{on} \quad y = \zeta \tag{8}$$

where μ is an artificial 'frictional' damping coefficient. Furthermore, a simple relation for a dispersive outgoing wave propagation is adopted as the absorbing boundary condition:

$$\frac{\partial\phi}{\partial t} + c\frac{\partial\phi}{\partial x} = 0, \quad \text{at} \quad x = \pm L \tag{9}$$

where c is the local phase velocity. The damping coefficient given by Israeli and Orszag (1981) is as follows:

$$\mu(x) = \begin{cases} 0, & \text{for } |x| < x_0 \\ A(n+1)(n+2)(|x| - x_0)^n(L - |x|)(L - x_0)^{-(n+2)}, & \text{for } x_0 < |x| < L \end{cases} \tag{10}$$

We tested the absorbing boundary condition together with a sponge layer for the nonlinear Klein-Gordon equation.

$$\phi_{tt} = \alpha\phi_{xx} - \gamma\phi + \beta\phi^3, \quad \text{for } 0 < x < \infty \tag{11}$$

with $\phi(0,t) = a\cos\omega t$, for $t > 0$; $\phi(x,0) = \phi_t(x,0) = 0$, for $0 < x < \infty$; and $\phi(x,t) = 0$, at $x \to \infty$. The equation is the hyperbolic type, however its solution represents a dispersive wave (Whitham, 1974). In the numerical computation, we take $A = 4$, $n = 2$, $\alpha = 1$, $\beta = 0.1$, $\gamma = 1$, $a = 0.1$, $\omega = \sqrt{2}$, and $x_0 = 20$. The effects of absorbing boundary condition and the damping in the sponge layer have been studied. The numerical solutions with the sponge layer (L=30) and without sponge layer (L=20) are shown in Fig.6. The results are compared with the analytical solution. The time instant is $t = 60$ sec, if there is a reflection from $x = L$, it should have enough time to travel back to the computational domain. It shows clearly that errors exist for the reflection in the case of without the sponge layer. The sponge layer not only damps out the reflection but also reduces the outgoing wave motion, hence this provides an appropriate outgoing wave for the absorbing boundary condition at $x = L$. According to our numerical experiments, the sponge layer should be about 1.5λ (λ is wave length).

The water wave generated by a wavemaker located at the bottom of a tank is computed. The water depth is 0.5 m. The wave maker moves with a displacement: $\zeta_w(x,t) = \zeta_0[\frac{1}{2}(1 - \cos\frac{\pi t}{t_c})H(t_c - t) + H(t_c - t)]H(b^2 - x^2)$, where $b = 0.61$ m is the length of the wavemaker and H is the Heaviside step function. The numerical results are compared with the experimental data (Hammer, 1973). The wave elevations at $x = 0$ and $x = b$ are shown in Fig. 7, and Fig. 8, respectively. $t_c = 0.248$ sec is used in the computation.

Currently, the computed results of water wave in a damaged compartment are being validated against experimental data from our lab.

References

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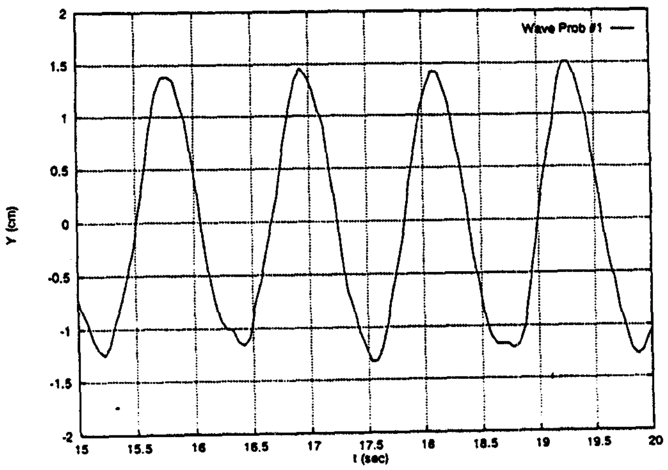

Fig. 1 Measured Wave Elevation inside a Damaged Compartment

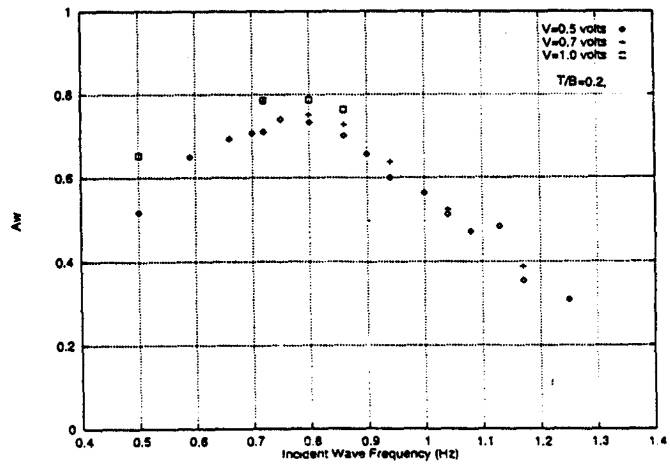

Fig.. 2 Wave Amplitude Ratio inside a Damaged Compartment

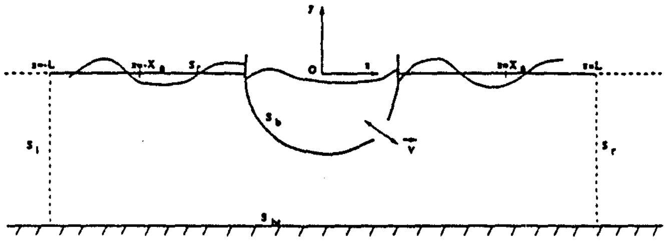

Fig. 3 Coordinate System

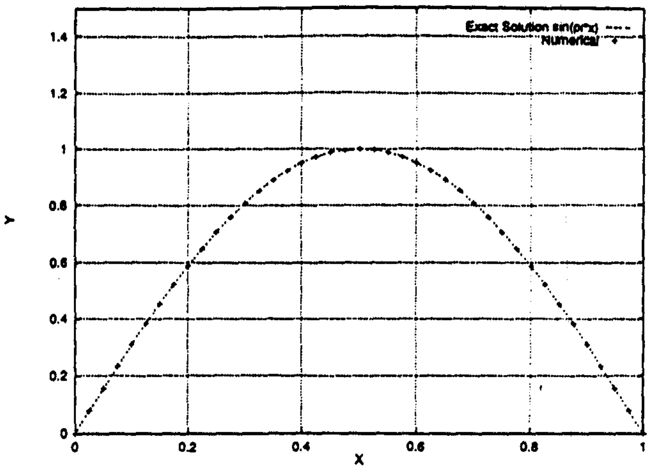

Fig. 4 Height Function of $\sin(\pi x)$

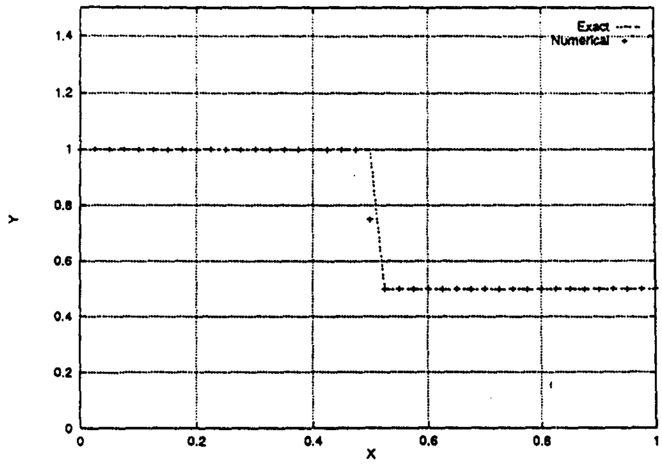

Fig. 5 Height Function of a Step Function

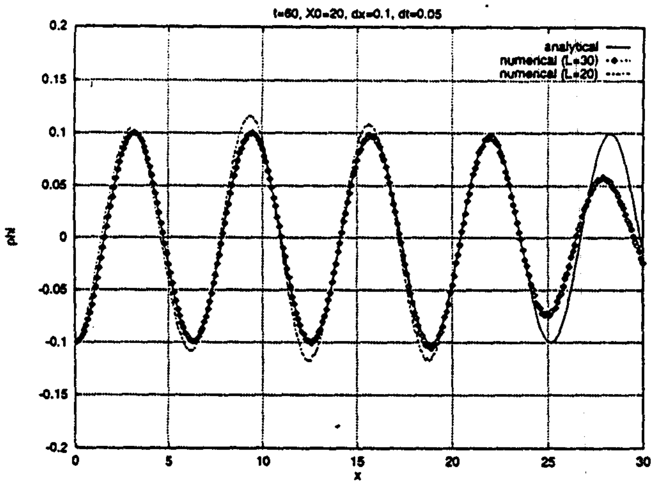

Fig. 6 Solution of Klein-Gordon Equation

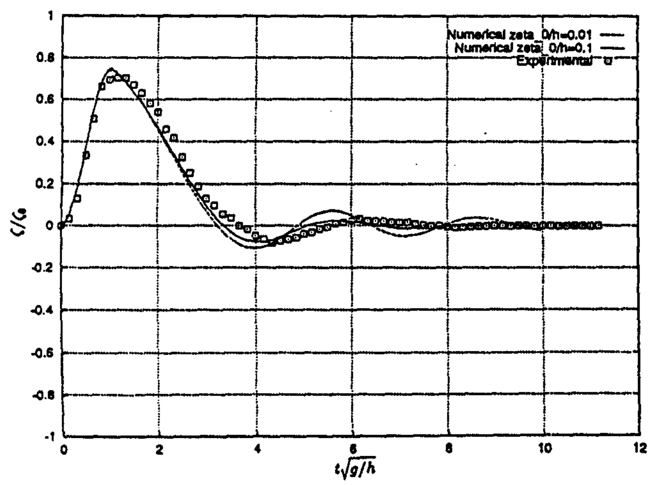

Fig. 7 Wave Elevation at x=0

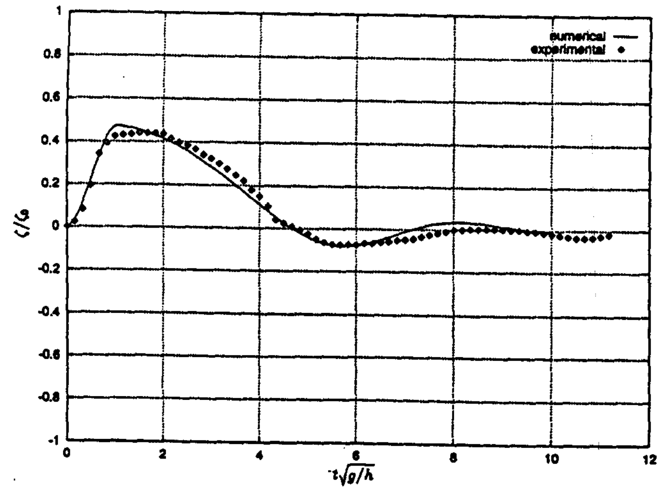

Fig. 8 Wave Elevation at x=b

DISCUSSION

Clement: You use an Orlanski's condition on the truncation boundary, and you show (good) results in monochromatic waves where the coefficient c is known and it is constant. But how should you compute $c(t)$ if the the wave were non-monochromatic (e.g. combination of two, or more, frequencies)?

Huang et al.: At each time step, the position of the wave crest close to the truncated boundary can be located from the numerical results of the wave profile. Then, the local phase velocity $c(t)$ is calculated based on the travelling distance of the crest within one time step. Therefore, our approach is suitable to both monochromatic and non-monochromatic waves.

Delhommeau: Why do you prefer FEM to other method?

Huang et al.: We need to know the velocity not only on the free surface but also in the whole fluid domain for the free surface advection. The FEM is efficient for this purpose and is adaptable to complicated forms of boundaries of the fluid domain.