A Precise Calculation Method for Hydroelastic Behaviors of Very Large Floating Structures

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1. Introduction

Very large floating structures have been recently considered as a possible airport. The size of the airport under consideration will be of order of 4km long and 1km wide, but the draft is very small compared to the dimensions of plan view. Therefore this type of structure will be very flexible and elastic deformations may be more important than the rigid body motions. To estimate hydroelastic behaviors of this type of structure, first-order wave loads must be accurately calculated in relatively very short wavelength (high frequency) region.

Due to the small draft compared to other two dimensions, the structure under consideration can be hydrodynamically represented as the pressure distribution on the free surface. Following Yamashita's pressure-distribution method[1)] calculating hydrodynamic loads on a box-shaped floating structure, Ikoma *et al.*[2)] showed numerical results of hydroelastic motions of a shallow-draft mat-like floating airport. However the accuracy seems not enough, because the zero-th order panel approximation is adopted in the discretization of integral equation for the unknown pressure.

For the airport under consideration, computations must be performed, with good accuracy, for wavelengths of the order of 1/10 or further down to 1/100 of the length of the structure. In this paper, emphasis is placed on the development of an accurate calculation method with fewer unknowns, using bi-cubic B-spline functions.

2. Formulation and integral equation

The coordinate system for the analysis is taken as in Fig.1. The water depth is assumed constant and denoted by h. The plan view of the structure is rectangle with length L and width B, and the draft is assumed very small. The incident angle of incoming wave is denoted by β.

Time-harmonic motions of small amplitude are considered, with the complex factor $e^{i\omega t}$ applied to all first-order oscillatory quantities. The boundary conditions on the body and free surface are linearized, and the potential flow is assumed. Then we write the velocity potential ϕ, pressure p, and vertical displacement of the structure ζ in the following normalized form:

$$\phi = i\omega a\Big\{\phi_I + \phi_S\Big\} + \sum_j i\omega X_j \phi_j \tag{1}$$

$$p = \rho g\Big[a\Big\{p_I + p_S\Big\} + \sum_j X_j p_j\Big] \tag{2}$$

$$\zeta = a\Big\{\zeta_I + \zeta_S\Big\} + \sum_j X_j \zeta_j \tag{3}$$

where a is the amplitude of incident wave, ω the circular frequency, ρ the fluid density, and g the gravitational acceleration.

Suffix I represents quantities related to the incident wave, S the scattering component, and j the radiation component of j-th mode of motion with amplitude X_j. Not only six conventional rigid-body motions, but also extended elastic deformations are included in the definition of j-th mode.

The dynamic and kinematic boundary conditions on the free surface are written as

$$p_j = K\phi_j + \zeta_j \, , \quad \frac{\partial \phi_j}{\partial z} = \zeta_j \quad \text{on } z = 0 \tag{4}$$

where $K = \omega^2/g$. Eliminating ζ_j from (4) gives the boundary condition:

$$\frac{\partial \phi_j}{\partial z} + K\phi_j = p_j \quad (j = S, 1, 2, \cdots) \tag{5}$$

This implies that the disturbance of the structure can be represented by the pressure distribution. It is known that the velocity potential satisfying (5) and the radiation condition at infinity is given by

$$\phi_j(x, y, z) = \iint_{S_H} p_j(\xi, \eta)\, G(x-\xi, y-\eta, z)\, d\xi d\eta \tag{6}$$

where $G(x, y, z)$ denotes the Green function satisfying the homogeneous free-surface condition with $p_j = 0$ in (5) and is written in the form

$$\begin{aligned} G(x, y, z) =& \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{k_n^2}{K - h(k_n^2 + K^2)} \frac{\cos k_n(z-h)}{\cos k_n h} K_0(k_n R) \\ &+ \frac{i}{2} \frac{k_0^2}{K + h(k_0^2 - K^2)} \frac{\cosh k_0(z-h)}{\cosh k_0 h} H_0^{(2)}(k_0 R) \end{aligned} \tag{7}$$

$$k_0 \tanh k_0 h = K, \;\; k_n \tan k_n h = -K \tag{8}$$

K_0 and $H_0^{(2)}$ in (7) are Bessel functions and $R = \sqrt{x^2 + y^2}$.

Substituting (6) into (4) gives the integral equation for the unknown pressure:

$$p_j(x, y) - K\iint_{S_H} p_j(\xi, \eta)\, G(x-\xi, y-\eta, 0)\, d\xi d\eta = \zeta_j(x, y) \tag{9}$$

Here the right-hand side is the normalized shape function of j-th mode and given specifically as

$$\left.\begin{array}{l} \text{Scattering: } \zeta_S(x, y) = -\zeta_I(x, y) = -\exp\{-ik_0(x\cos\beta + y\sin\beta)\} \\ \text{Heave: } \zeta_3(x, y) = 1, \quad \text{Roll: } \zeta_4(x, y) = y, \quad \text{Pitch: } \zeta_5(x, y) = -x \end{array}\right\} \tag{10}$$

and for elastic deformations, extending heave and pitch modes, the appropriate mode shape functions[3] are

$$\zeta_{4j+3}(q) = \frac{1}{2}\left\{\frac{\cos \kappa_{2j} q}{\cos \kappa_{2j}} + \frac{\cosh \kappa_{2j} q}{\cosh \kappa_{2j}}\right\} \quad (j = 1, 2, \cdots) \tag{11}$$

$$\zeta_{4j+5}(q) = \frac{1}{2}\left\{\frac{\sin \kappa_{2j+1} q}{\sin \kappa_{2j+1}} + \frac{\sinh \kappa_{2j+1} q}{\sinh \kappa_{2j+1}}\right\} \quad (j = 1, 2, \cdots) \tag{12}$$

where $q = 2x/L$ and factors κ_j are the positive real roots of the equation:

$$(-1)^j \tan \kappa_j + \tanh \kappa_j = 0 \tag{13}$$

Similar elastic mode shape functions in the y-direction must be considered for the general hydroelastic analysis.

3. Generalized first-order wave forces

With an appropriate numerical solution method, the pressure on the bottom of the structure can be directly obtained from integral equation (9).

The j-th component of normal vector is equal to the shape function of the j-th mode, because in the present case $n_1 = n_2 = 0$ and $n_3 = 1$. Namely $n_j = \zeta_j(x, y)$ $j = 1, 2, \cdots$.

In the radiation problem including elastic modes, the forces in the i-th direction due to the j-th mode of motion can be summarized as

$$F_i = \sum_j T_{ij} X_j = \sum_j \left[\omega^2 A_{ij} - i\omega B_{ij} - C_{ij} \right] X_j \tag{14}$$

where

$$\left. \begin{aligned} \omega^2 A_{ij} - i\omega B_{ij} &= -\rho g \iint_{S_H} (p_j - \zeta_j) \zeta_i \, dxdy \\ C_{ij} &= -\rho g \iint_{S_H} \zeta_j \zeta_i \, dxdy \end{aligned} \right\} \tag{15}$$

Here A_{ij} and B_{ij} are generalized added-mass and damping coefficients respectively, and C_{ij} is hydrostatic restoring force coefficient. We notice from (15) that the symmetry relation $C_{ij} = C_{ji}$ is satisfied in the present case.

In the diffraction problem, the generalized wave-exciting force in the i-th direction can be computed from

$$E_i = -\rho g a \iint_{S_H} p_S \zeta_i \, dxdy \tag{16}$$

4. Haskind's relation and energy conservation

In order to check the numerical accuracy, let us consider various hydrodynamic relations, which may be obtained from the reciprocity theorem. The analysis is somewhat lengthy but the final results are already known and the same in form as those for general shaped bodies.

Haskind's relation can be written as

$$E_j = \rho g a H_j(k_0, \beta + \pi) \tag{17}$$

where $H_j(k_0, \theta)$ is the Kochin function defined by

$$H_j(k_0, \theta) = \iint_{S_H} p_j(\xi, \eta) \, e^{ik_0(\xi \cos\theta + \eta \sin\theta)} d\xi d\eta \tag{18}$$

Eq. (17) must be equal to (16) for $i = j$; this relation is used in this paper as a check of numerical accuracy.

The energy relation concerning the damping coefficient is also written by use of the Kochin function, in the form

$$B_{ij} = \frac{\rho\omega}{4\pi} \frac{k_0^2}{K + h(k_0^2 - K^2)} \int_0^{2\pi} \overline{H_i(k_0, \theta)} \, H_j(k_0, \theta) \, d\theta \tag{19}$$

where the overbar means the complex conjugate.

Eq. (19) must be equal to the same coefficient to be obtained from (15); this relation is also used as a check of numerical accuracy, although (19) requires numerical integration with respect to θ, which may be another source of inaccuracy.

5. Numerical calculation method

The problem here is how accurately we can solve the integral equation (9) for very short wavelengths. In order to reduce the number of unknowns, it may be indispensable to adopt a higher-order panel method. In this paper, the unknown pressure is represented by use of the cubic B-spline function, in the form

$$p(x, y) = \sum_{m=0}^{NX+2} \sum_{n=0}^{NY+2} \alpha_{mn} B_m(x) B_n(y) \tag{20}$$

Here $B_m(x)$ and $B_n(y)$ are normalized cubic B-spline functions, which can be obtained by Boor-Cox's recursion formula. NX and NY are number of panel division in the x- and y-directions; therefore the number of total unknowns α_{mn} in (20) is $(NX + 3) * (NY + 3)$. To determine these unknowns, the collocation method is used, at equally-spaced positions of the same number of unknowns. The property of symmetry and anti-symmetry of the fluid domain is taken into account, reducing unknowns to 1/4 of the total number.

6. Numerical results and status

Fundamental parts of the programming has been completed and the accuracy check is now being performed using Haskind's relation (17) and the energy relation (19).

One example is shown in Fig. 2, which is the numerical error in Haskind's relation for the heave exciting force acting on a square plate ($L = B$) in head waves ($\beta = 0°$) of infinite depth. It is found that the error in the energy relation is almost the same as that in Haskind's relation.

For wavelengths longer than $\lambda/L = 0.2$, the error is very small even with small number of panels. However for very small wavelength, say $\lambda/L = 0.05$, we still need relatively large number of panels to keep the error small, for which the computation time may be long from the viewpoint of practical calculation method. It seems necessary to develop an approximate method which is effective in the region of high frequencies.

Further improvement and computations of hydroelastic responses (modal analysis) of very large floating structures are now in progress.

References

1) Yamashita, S. (1979): Motions and Hydrodynamic Pressures of a Box-Shaped Floating Structure of Shallow Draft in Regular Waves, J. Soc. of Nav. Arch. of Japan, Vol.146

2) Ikoma, T. *et al.* (1995): Hydroelastic Responses of Very Large and Shallow Draft Floating Structures, Proc. of 13th Ocean Engineering Symposium, Soc. of Nav. Arch. of Japan

3) Newman, J. N. (1994): Wave Effects on Deformable Bodies, Appl. Ocean Res., Vol.16

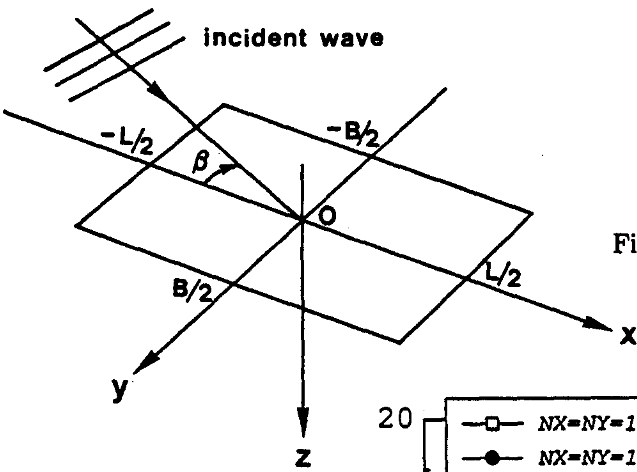

Fig.1 Coordinate system and notations

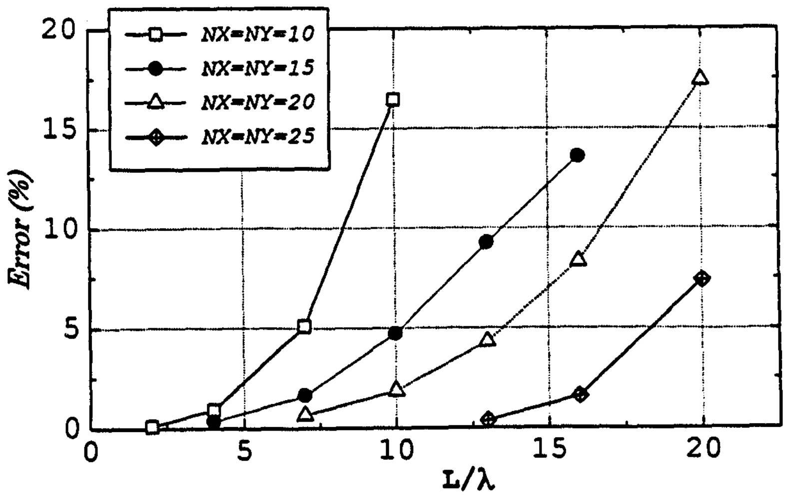

Fig.2 Numerical error in Haskind's relation for the heave exciting force acting on a square plate ($L = B$) in head wave of infinite depth

DISCUSSION

Korsmeyer: I think this is an ideal case for a brute force computation. I estimate it could be done with a bit less than 10^5 higher-order panels.

Kashiwagi: I suppose you are thinking to use your method (Precorrected-FFT Algorithm with Multiple Acceleration) for the present case. I think that is another good choice, but the best choice may be a combination of the use of higher-order panel method (such as B-spline function) with the multiple acceleration to solve dense linear systems.

Eatock Taylor: The dynamic responses will be mainly influenced by the coincidence of the "wave-structure matching" condition (kL), and a condition of structural resonance (ω/ω_n): ie the relation between k and k_n. If you consider realistic values of plate flexibility, how high are the mode numbers corresponding to the most relevant generated forces?

Kashiwagi: The present paper is just concerned with the computations of first-order wave loads responding to higher-order natural mode shapes; which are to be used in the modal analysis later on, i.e. in the computations of hydroelastic responses. Therefore, at present, I can't say any definitive answer to your question, but I'm thinking to compute to rather higher mode numbers, and that's why the emphasis in this paper is placed on the accuracy in the very short wavelength region.