Entry Problem for Body with Attached Cavity

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When a blunt body approaches a liquid free surface, the air which is pushed ahead the body cannot escape completely from the gap between the bottom and the water surface and, as a result, the cavity filled with the entrapped air may be formed. This phenomenon can be observed for bodies bottom of which has a very small deadrise angle. To study the air-cushion effect, the fall of a flat-bottomed construction onto the water surface is usually considered in experiments. On the other hand, it is well-known that the process of the cavity formation is unstable and depends very much on small variations of parameters (attack angle, fine geometry of the bottom near the edge and so on). This instability is the main obstacle to analyse the problem in detail.

The main idea of the present paper is to consider first such shapes of the falling bodies for which a cavity formation can be guaranteed in advance, and to investigate the influence of the cavity on the flow parameters and the pressure distribution.

The falling of the blunt body with a shallow depression at its top onto the free surface of an ideal incompressible liquid (plane problem) is considered. As an example of the shapes under consideration, a catamaran cross-section can be taken. The theory of water impact given by Wagner (1932) is generalized with regards to the presence of the cavity filled with a compressible gas. It is assumed that the pressure inside the cavity is a time function only. We shall determine the pressure in the cavity, the cavity volume and shape, the dimension of the wetted part of the bottom, and the body motion. The main part of calculations is carried out analytically. The system of four ordinary differential equations of the first order with respect to the unknown functions is solved numerically by the Runge-Kutta method. The results and their discussion are presented for the body shape of which is described by the function $y = h(x^2/l^2 - 1)^2$ where $2l$ is the initial horizontal dimension of the cavity and h is its initial vertical dimension.

1 Formulation of the problem

The entry of a blunt contour with a shallow depression at its top is considered. The body is symmetrical with respect to the vertical axis Oy'. The line $y' = 0$ corresponds to the initial undisturbed position of the free surface. Dimensional variables are denoted by a prime. At the initial instant of time ($t' = 0$), the body touches the liquid surface at two points $x' = \pm l$. The cavity filled with a compressible gas is bounded by the free surface, $|x'| < l$, $y' = 0$, from below and by the body surface from above. The maximum thickness of the cavity is at its centre and is equal to h. We assume that $h/l \ll 1$. Then the body starts to penetrate the liquid vertically, the initial impact velocity being V_0. We assume that (1) the pressure P' inside the cavity is a function of time only, $P'(0) = 0$; (2) the equation of state of the gas is taken in the Tate form $P' = P_*[(\rho'/\rho_0)^n - 1]$, where P' is the deviation of the pressure from its initial value, ρ' is the gas density, ρ_0 is the initial gas density, n and P_* are constants which depend on the gas properties, $P_* = \rho_0 c_0^2/n$, c_0 is the sound speed in the gas at rest;

(3) the gas and the liquid can not be mixed and the mass conservation law $\rho'\mathcal{V}' = \rho_0\mathcal{V}_0$ is hold, where $\mathcal{V}'(t')$ is the cavity volume, $\mathcal{V}_0 = \mathcal{V}'(0)$. The position of the entering body is given by the equation $y' = f'(x') - s'(t')$ where $s'(t')$ is the depth of penetration, $s'(0) = 0$, $(ds'/dt')(0) = V_0$, $f'(0) = h$, $| df'/dx' | \ll 1$ where $| x' | < l$. We take l as the length scale, the ratio h/V_0 as the time scale, V_0 as the scale of the liquid velocity, h as the scale of the penetration depth, P_* as the scale of the pressure in the cavity, $\mathcal{V}_0$ as the scale of the cavity volume.

In the dimensionless variables, which are designated by the above-mentioned terms without a prime, the boundary conditions can be put on the undisturbed initial level of the liquid and linearized together with the equations of liquid motion. The linearization leads to the well-known Wagner approach where the velocity potential $\varphi(x, y, t)$ satisfies the Laplace equation in the lower half-plane ($y < 0$) and the mixed boundary conditions on the line $y = 0$. The main feature of the problem is the fact that the division of the liquid boundary into a cavity surface, $| x | < a(t)$, a contact region between the body and the liquid, $a(t) < | x | < c(t)$, and an outer free surface, $| x | > c(t)$, is unknown and must be determined with the help of an additional conditions. Those conditions are :
(a) The velocity potential on the liquid boundary $\varphi(x, 0, t)$ is a continuous function of x;
(b) Displacements of the liquid particles are bounded;
(c) The liquid particles of the free surface can move only vertically.

Within the framework of the Wagner approach the boundary-value problem for the velocity potential $\varphi(x, y, t)$ has the form

$$\Delta\varphi = 0 \quad (y < 0), \tag{1}$$

$$\varphi_y = -V(t) \quad (y = 0, a(t) < | x | < c(t)), \tag{2}$$

$$\varphi_t = -\beta P(t) \quad (y = 0, | x | < a(t)), \tag{3}$$

$$\varphi_t = 0 \quad (y = 0, | x | > c(t)), \tag{4}$$

where $\beta = (1/n)(\rho_0/\rho_L)(c_0/V_0)^2(h/l)$, ρ_L is the liquid density. Equations (3), (4) can be integrated with respect to time at the initial stage when $\dot{a}(t) < 0$ and $\dot{c}(t) > 0$. Dot stands for the derivative in time.

2 General theory

The function $\varphi(x, 0, t)$ will be continuous if and only if

$$-\beta \int_0^t P(\tau)\, d\tau = \int_{a(t)}^{c(t)} \varphi_x(\xi, 0, t)\, d\xi \tag{5}$$

where

$$\varphi_x(x, 0, t) = \frac{V(t) \cdot (x^2 - B) - G(t)}{\sqrt{(c^2 - x^2)(x^2 - a^2)}}, \tag{6}$$

$B = (c^2 + a^2)/2$. The function $G(t)$ is unknown. Asymptotic behaviour of the velocity at the infinity is given as $\varphi_y \sim -G(t)x^{-2}$ as $| x | \to \infty$. Equations (5), (6) provide

$$G(t) = V(t)\left[c^2\frac{E(m)}{K(m)} - B\right] - c\frac{W(t)}{K(m)} \tag{7}$$

where

$$\dot{W} = \beta P(t), \tag{8}$$

$K(m), E(m)$ are the complete elliptic integrals of the first and second kind, respectively.

The variation of the cavity volume $\mathcal{V}(t)$ is governed by the equation

$$\dot{\mathcal{V}} = -2\kappa[aV(t) + \int_0^{a(t)} \varphi_y(\xi, 0, t)\, d\xi]$$

where

$$\varphi_y(x, 0, t) = \frac{G(t) - V(t)(x^2 - B)}{\sqrt{(c^2 - x^2)(a^2 - x^2)}} - V(t),$$

$\kappa = hl/\mathcal{V}_0$. By algebra we find

$$\dot{\mathcal{V}} = \kappa \left[2\frac{K(1-m)}{K(m)} W - \frac{\pi c}{K(m)} V(t) \right], \quad m = 1 - \frac{a^2}{c^2}. \tag{9}$$

In order to derive the boundary-value problem for the displacements of the liquid particles, we need to integrate the relations (1)-(4) with respect to time. The elevation of the outer free surface, $x > c$, is

$$Y(x, 0, t) = \left[\frac{2}{\pi} \int_a^c y_b(\xi, t) \frac{\xi\sqrt{(c^2 - \xi^2)(\xi^2 - a^2)}\, d\xi}{\xi^2 - x^2} - D \right] \cdot \left[(x^2 - c^2)(x^2 - a^2) \right]^{-1/2}, \tag{10}$$

and of the inner free surface, $|x| < a$, is

$$Y(x, 0, t) = \left[-\frac{2}{\pi} \int_a^c y_b(\xi, t) \frac{\xi\sqrt{(c^2 - \xi^2)(\xi^2 - a^2)}\, d\xi}{\xi^2 - x^2} + D \right] \cdot \left[(c^2 - x^2)(a^2 - x^2) \right]^{-1/2}. \tag{11}$$

Here $y_b(x, t) = f(x) - s(t)$. The vertical displacements of both the outer free surface (10) and the inner one (11) will be bounded if

$$D = -\frac{2}{\pi} \int_a^c y_b(\xi, t) \xi \sqrt{\frac{\xi^2 - a^2}{c^2 - \xi^2}}\, d\xi, \qquad \int_a^c \frac{\xi y_b(\xi, t)\, d\xi}{\sqrt{(c^2 - \xi^2)(\xi^2 - a^2)}} = 0.$$

These two equalities can be simplified

$$D(t) = -\frac{1}{\pi} A \int_0^\pi f[\sqrt{A\cos\theta + B}] \cos\theta\, d\theta, \tag{12}$$

$$s(t) = \frac{1}{\pi} \int_0^\pi f[\sqrt{A\cos\theta + B}]\, d\theta, \quad A = (c^2 - a^2)/2. \tag{13}$$

Equation (10) predicts that $Y(x, 0, t) \sim -D(t)x^{-2}$ as $x \to +\infty$. Therefore

$$\dot{D} = G(t) \tag{14}$$

The body motion is governed by the equation

$$-\dot{V} = \frac{\mu}{\pi} \int_{-c}^{c} p(x, 0, t)\, dx - \lambda, \quad \mu = \pi\rho_L l^2/M, \quad \lambda = gh/V_0^2,$$

where M is the body mass, and g is the acceleration due to gravity. The last equation can be integrated in time. This yields

$$\dot{s} = V(t), \quad V(t) = 1 + \mu G(t) + \lambda t. \tag{15}$$

The differential equations (8), (9), (14), (15) with the initial conditions $W(0) = 0$, $V(0) = 1$, $D(0) = 0$, $s(0) = 0$ and the formulae (7), (12), (13) allow us to determine all characteristics of the process. It is worth noting that in the present approach the liquid particles of the free surface can move only vertically, therefore the approach cannot be valid for all times.

3 Numerical results

In the simplest case, $f(x) = (x^2 - 1)^2$, the integrals in (12), (13) can be evaluated analytically

$$D(t) = -A^2(B-1), \quad A^2 = 2s(t) - 2(B-1)^2$$

and the system (8), (9), (14), (15) can be essentially simplified. The cavity is filled with air, $\rho_0 = 1kg/m^3, c_0 = 330m/sec, n = 1.4$, the body enters water, $\rho_L = 1000kg/m^3$. We take $l = 1m, h = 0.1m, V_0 = 3m/sec, g = 9.81m/sec^2, M = 500kg$. This means that $\beta = 0.8643$, $\kappa = 0.9375$, $\mu = 2\pi$, $\lambda = 0.109$. The following four cases are distinguished:

1. There is no gas inside the cavity, $\beta = 0$, $\lambda \neq 0$, $\mu \neq 0$;
2. There is no gas inside the cavity, and the body velocity is constant, $\beta = 0$, $\lambda = 0$, $\mu = 0$;
3. The cavity is filled with the air, and the body velocity is constant, $\beta \neq 0$, $\lambda = 0$, $\mu = 0$;
4. The cavity is filled with the air, $\beta \neq 0$, $\lambda \neq 0$, $\mu \neq 0$.

In the first case the cavity disappears at the moment $t \approx 1.4$, the body velocity being five times less than the initial one. In the second case the cavity disappears at $t \approx 0.63$. In both cases the dimension of the wetted part of the bottom increases monotonically with time.

In the third case the Wagner scheme of the flow fails at $t \approx 0.86$. The pressure in the cavity is maximum at $t \approx 0.62$ and is equals to 7.7 at this moment. After this instant of time the cavity volume starts to grow and the pressure drops down to 2.89 at the end of the stage. It is worth noting that at the beginning of the penetration the inner free surface is convex down, but close to the end of the stage under consideration it is convex up, being approximately tangential to the body surface near the inner contact points. This means that after the moment $t = 0.86$ the liquid particles can leave the contact region and escape onto the inner free surface. The horizontal velocity of the flow close to the contact points, $x = \pm a(t)$, is continuous and differs from zero. This stage is not considered here.

In the forth case the liquid particles from the contact region start to escape onto the inner free surface at the moment $t \approx 0.67$. Due to reduction of the entry velocity, the pressure in the cavity is quit low. The Wagner stage of the entry process is over at $t \approx 0.51$, after that the body starts to exit the liquid moving up. The exit velocity is equal to 0.37 at the end of the time interval under consideration. The cavity volume drops at the initial stage, $0 < t < 0.42$, and drows arter that. The pressure in the cavity peaks at $t = 0.42$ and is equal to 0.931.

The analysis demonstrates that the presence of the cavity essentially changes the flow structure and the body motion under its penetration into water.

DISCUSSION

Schultz: Your results show interesting contact angle behaviour. Since length scales could be small, surface tension could be included. Then a contact angle condition could be imposed. Have you considered this? How would it change your results.

Korobkin: Surface tension effects were not included in the approach. Indeed, they can be of importance near the contact points. The further analysis of those effects are desirable. I think the effects can change the flow only locally near the contact points but not globally. This reasoning is based on analysis of similar problems on water impact without the presence of a cavity.

Tanizawa: (1) You showed general theory for bottom impact with given amount of attached cavity. But in general, the initial amount of cavity or trapped air is unknown and so many researchers tried to estimate it (Verhagen, Miyamoto, Iwanowski, etc). Is there any idea to estimate it? (2) Many numerical or experimental data are available on this problem. Please show agreement of your theory to those data.

Korobkin: (1) The approach presented cannot help us to estimate the amount of trapped air in cavity. It describes the evolution of pressure in the cavity and motion of the body after the moment when the cavity has been closed already. (2) Main part of experimental results are for the impact by a box-like structures, when the air-cushion phenomenon is of major importance, on initially horizontal free liquid surface. This is not exactly the case the present approach was developed for, but it looks possible to apply this approach for the flat-bottomed body impact as well. I hope to modify the approach to carry out the numerical analysis and to compare my results with experimental results available in the near future.