Wave diffraction by a long array of circular cylinders

by

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To examine the feasibility of using a three-dimensional spline–Galerkin panel method for analyzing wave diffraction due to large structures, we have considered the diffraction problem for a single row of equally-spaced identical bottom mounted circular cylinders due to an obliquely incident plane wave. The total number of cylinders (N) in the array is large but finite. The panel method is not restrictive in the choice of the body shape or the configuration of the array, but this particular example allows a quantitative comparison with the results obtained by the semi–analytic approach of Linton and Evans [3], and leads to some unique results not reported before.

We employ a higher-order panel method which approximates the potential as a B–spline expansion and utilizes a semi–discrete Galerkin procedure to solve for the diffraction potential on each cylinder (for details see Maniar [5]). The cylinders are of radius a and placed with their centers $2d$ apart in a fluid of depth $H = 1$. The incident wave has a wavelength $\lambda = 2\pi/k$ and a heading angle β, measured relative to the long axis of the array. We present results for $N = 100$ with the spacing parameter $a/d = 0.5$. One plane of symmetry is used, with each half–cylinder discretized by 6 panels. Cubic B–splines are used to represent the potential.

Figure 1 shows the force on each cylinder for head seas, for the four wavenumbers $kd = \pi/3, 1.3907, \pi/2, \pi$. The pairs (left and right) of plots show the distribution of the magnitude and phase respectively. The force on each cylinder is evaluated by direct pressure integration, and the magnitude is normalised by the McCamy–Fuchs force on a single cylinder. Note that the scales of the plots of magnitude differ. The phase is relative to the incident-wave elevation at the array center. The cylinder with index $I = 1$ is the head of the array. These results have been validated by the semi–analytic approach of Linton and Evans [3], which we have programmed independently of the panel method. The maximum relative discrepancies in the computed local force (sampled at a few cylinders in the array) for the four wavenumbers are $0.003\%, 1\%, 0.001\%, 0.02\%$ respectively. The singular aspect of the second wavenumber will be discussed below; for this wavenumber the summation over Fourier modes does not appear to converge in the approach of Linton and Evans [3], but there is satisfactory numerical evidence of convergence for the two modes, $n = \pm 1$, which contribute to the force.

For relatively long waves in head seas, Figure 1(a) indicates that the magnitude of the force increases slowly along the array, and the phase is essentially the same as the incident wave. Conversely for the short waves in Figures 1(c-d), the force is a maximum at or near the head of the array with substantial reduction along the array due to sheltering. This behaviour is qualitatively similar to the results of Faltinsen [2], who used matched asymptotic expansions to show that for head-sea diffraction by a long slender body the potential decays like $1/\sqrt{x}$, where x is the distance measured from the head of the structure.

As in the long-wavelength regime of Figure 1(a), the phase distributions in the shorter waves are essentially the same as the local incident-wave elevation. Thus in Figure 1(c), where the wavelength is equal to twice the cylinder spacing, the forces on consecutive cylinders are nearly 180 degrees out of phase whereas in Figure 1(d), where the wavelength is equal to the cylinder spacing, the phase is nearly constant along the array.

Seeking to explain the transition from the results of Figure 1(a) to Figure 1(c), we discovered the result shown in Figure 1(b) at $kd = 1.3907$. In this case the magnitude of the force is much larger than for the other wavenumbers, and the phase differs by 180 degrees between adjacent cylinders. This wavenumber is virtually identical to the value $kd = 1.39131$ reported by Callan *et. al.* [1], for the existence of trapped waves in the diffraction problem for a cylinder of radius a in the center a channel of width $2d$ when $a/d = 0.5$.

The connection between these two diffraction problems can be explained in the following manner. Trapped waves are known to exist in a channel, corresponding to the case of beam seas incident upon an infinite array of equally-spaced cylinders. In the latter case the trapped wave is associated with a homogeneous solution, of indeterminate magnitude. Since it is anti-symmetric about the centerplane of the channel, the corresponding force on the cylinder is transverse (perpendicular to the channel walls).

Now consider the diffraction problem for a finite array. This can be analysed in general from the interaction theory of Linton and Evans [3], and it is generally assumed that the same theory applies in the limit $N \to \infty$ (see [4]). Thus, as $N \to \infty$, the linear system of equations which represent the interactions between the cylinders ([3], equation 2.15) admits a homogeneous solution at the wavenumber where trapped waves exist, and a nearly singular solution can be expected when N is finite but large. Since the left-hand-side of this linear system is independent of the wave heading angle, the same singular behaviour can occur for head seas, or more generally for any oblique angle of incidence. From this argument we conclude that the force acting on each cylinder in head waves may become large at the same wavenumber that trapped waves exist in a channel, with the understanding that the channel walls correspond to the midplanes between adjacent cylinders in the array.

In the absence of physical walls, the velocity potential and its normal derivative must be continuous across these midplanes. This condition is satisfied if the local 'trapped' solution, which is anti-symmetric with respect to the corresponding cylinder axis, is 180 degrees out of phase with the local trapped solutions at the adjacent cylinders of the array. Thus the force on adjacent cylinders will have the same phase difference, as observed in Figure 1(b).

If this explanation is valid, results similar to Figure 1(b) should be observed at other wave headings. This is confirmed in Figure 2, where the magnitude of the in-line force component is plotted at $\beta = 0$, 30, 60, and 90 degrees. Even in beam seas there is a substantial in-line force, although it is reduced in magnitude relative to the other headings. The phase, which is omitted in Figure 2, differes by 180 degrees between adjacent cylinders for all wave headings. An exception occurs in beam waves if N is odd instead of even. This follows from symmetry, since the transverse force on the cylinder at the center of the array must be zero. The results shown in the right column of Figure 2 confirm the expectation that the cases $N = 100$ and $N = 101$ are practically identical except in beam waves.

Our results suggest that care must be exercised in using simplified approximations to analyze very large structures composed of periodic arrays of sub-elements, as in the configurations which have been proposed for floating airports.

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References

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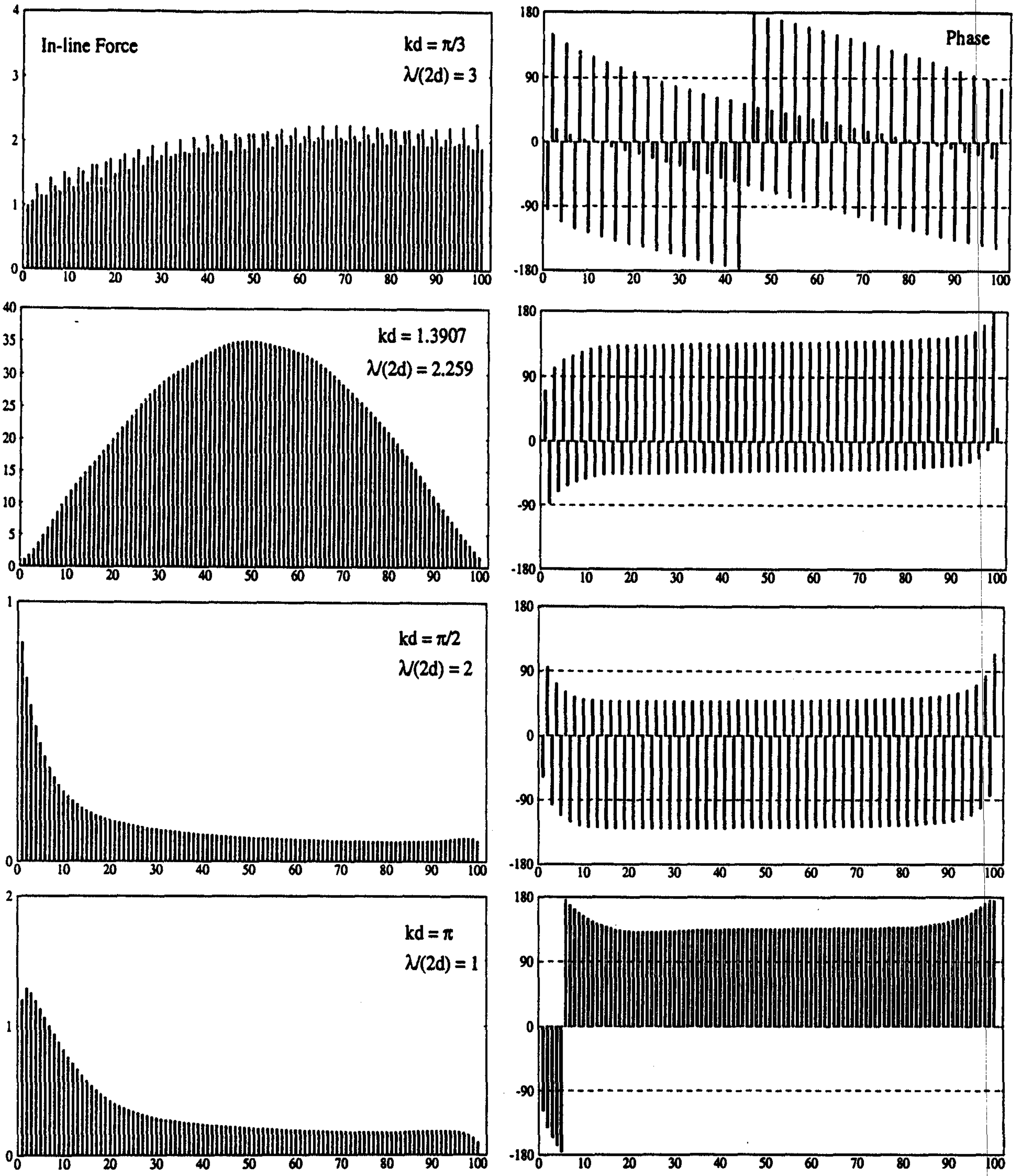

Figure 1: The distribution of the force magnitude (left) and phase (right) along an array of 100 circular cylinders in head seas. Starting at an array end and numbering sequentially, the Ith. cylinder in the array is identified by the integer I on the absissa. The cylinder at the head of the array corresponds to $I = 1$.

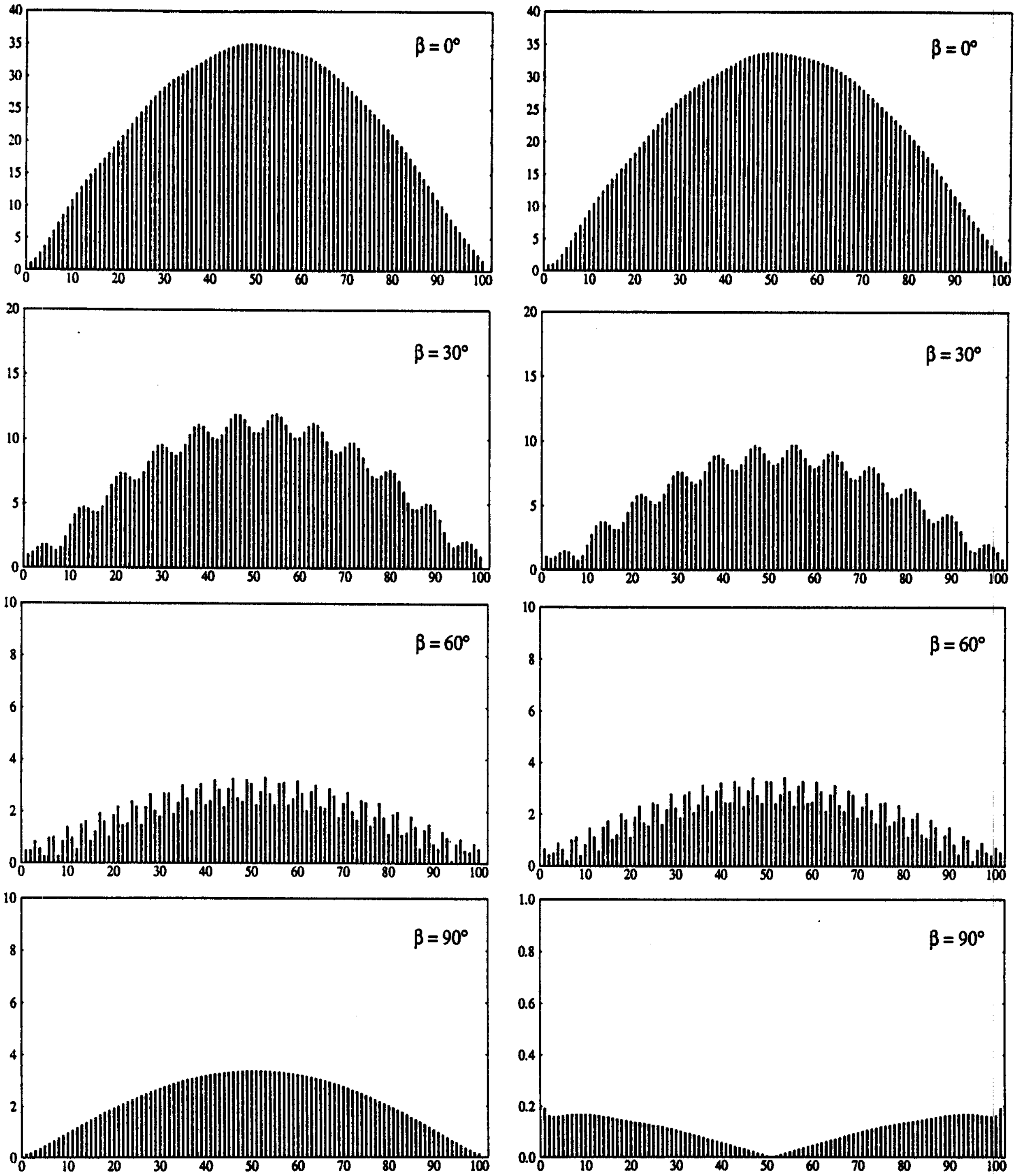

Figure 2: The distribution of the in–line force magnitude along an array of 100 (left) and 101 (right) circular cylinders. The in–line force is the component of the exciting force along the length of the array. From top to bottom, the four pairs of plots correspond to the heading angles, $\beta = 0, 30, 60, 90$ degrees respectively. The heading angle is measured relative to the long axis of the array.

Discussion of paper by Maniar & Newman

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The results described for the large in-line exciting forces on the centre cylinders of a finite but large number of identical cylinders in a row, at the frequency of the corresponding trapped mode for a single cylinder on the mid-line of a narrow wave tank are remarkable and the authors are to be complimented on their discovery. Prompted by their results, Dr. Richard Porter and I at Bristol University undertook a study of the total force on each of the cylinders in a regular *circular* array of cylinders. Although a pure trapped mode for such a configuration seemed unlikely, we speculated that 'near-trapping' might occur in some cases, resulting in an exceptionally large force on the cylinders at a particular incident wavelength and cylinder spacing. We began by assuming that a large number of cylinders would be needed but the results for sixteen cylinders in a circular array exhibited such a bewildering number of sharp spikes in the forces that we progressively reduced the number to four cylinders where some sort of clearer picture emerged.

The cylinders, each of radius a were placed at the corners of a square of side d and the incident wave of wavelength $\lambda = 2\pi/\kappa$, approached in a direction along a diagonal of the square. The figures show the magnitude of the total force on each of the cylinders as a function of κa as the size of the square is reduced, or as a/d increases. It can be seen that starting with a gap between cylinders equal to 1.5 cylinder diameters and reducing to 0.25 cylinder diameters a sharper and sharper peak is observed for which all cylinders experience an increasingly large force as the gap reduces until at $a/d = 0.4$ the peak is as high as 54 times the maximum force on a single cylinder in isolation in the direction of the incident wave. The frequency at which this occurs corresponds to a wavelength almost exactly equal to the gap between cylinders at the ends of the diagonals of the square array whilst the direction of the forces on these cylinders are opposite to each other in such a way that if the forces on one pair are acting towards the centre of the array, the forces on the other pair are acting away, and vice versa. This is consistent with a near-perfect trapped wave consisting of two exactly out of phase standing waves along the diagonals.

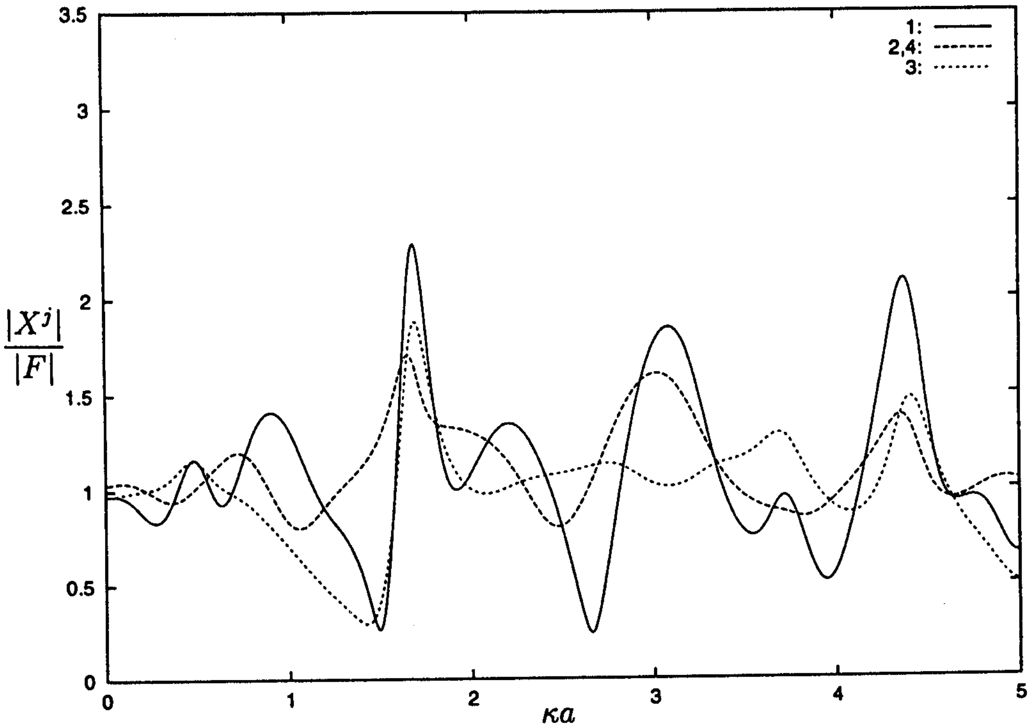

Figure 1: Resultant force on 4 cylinders, $\beta = 0°$, $\mathbf{a/d = 0.25}$

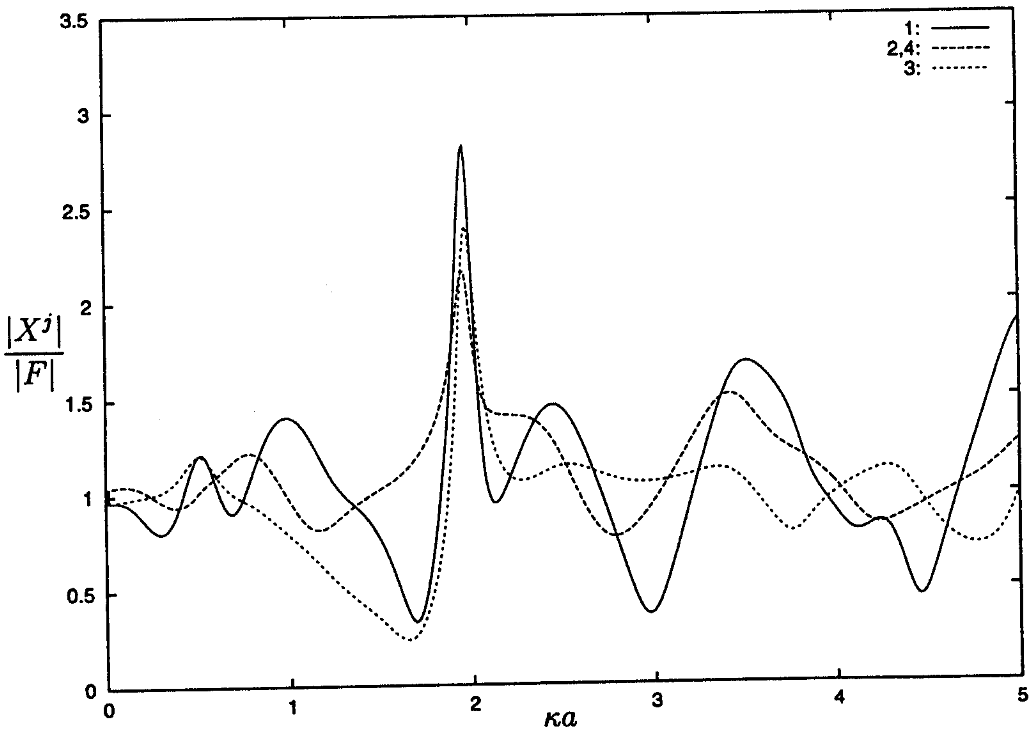

Figure 2: Resultant force on 4 cylinders, $\beta = 0°$, $\mathbf{a/d = 0.275}$

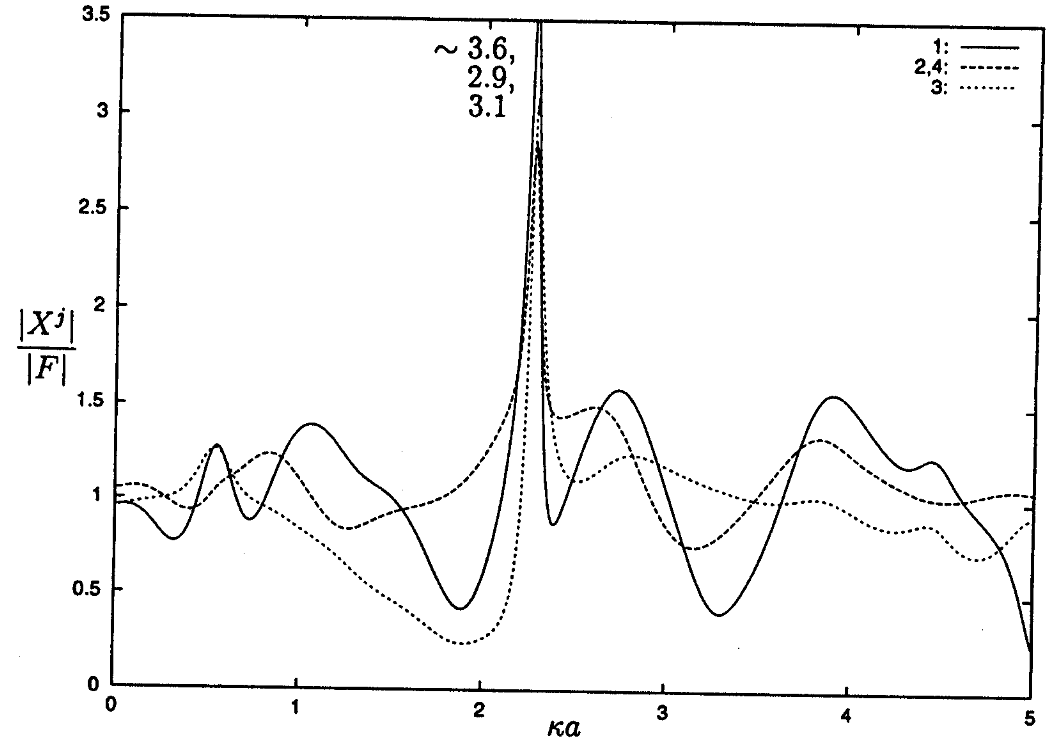

Figure 3: Resultant force on 4 cylinders, $\beta = 0°$, **a/d = 0.3**

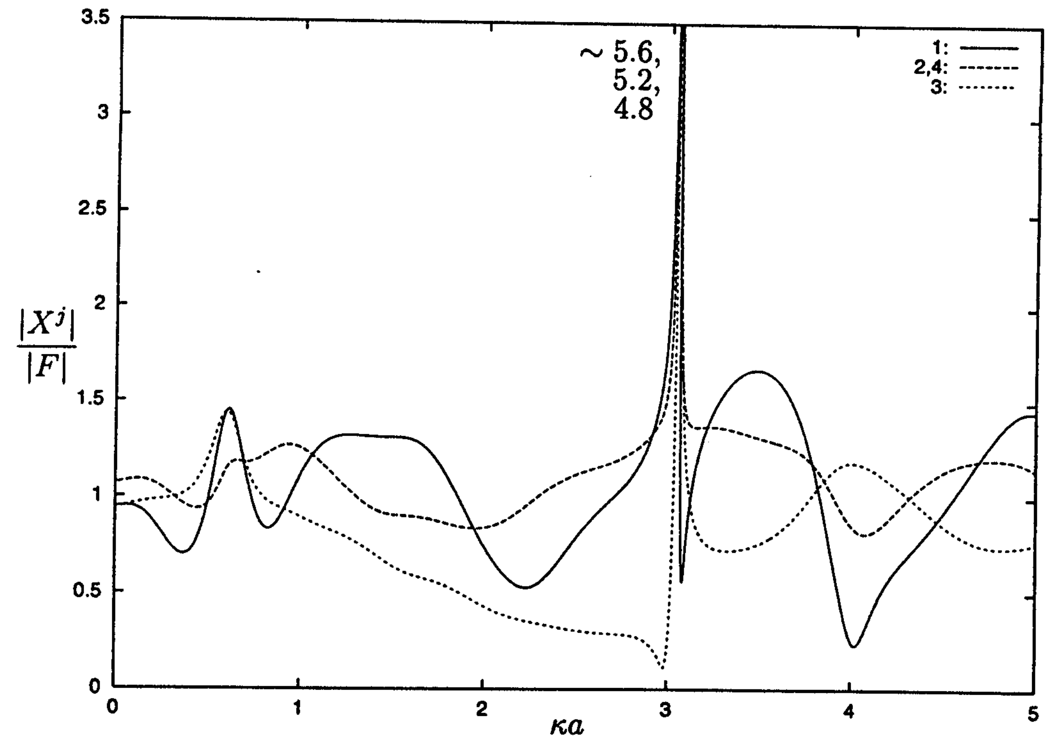

Figure 4: Resultant force on 4 cylinders, $\beta = 0°$, **a/d = 0.35**

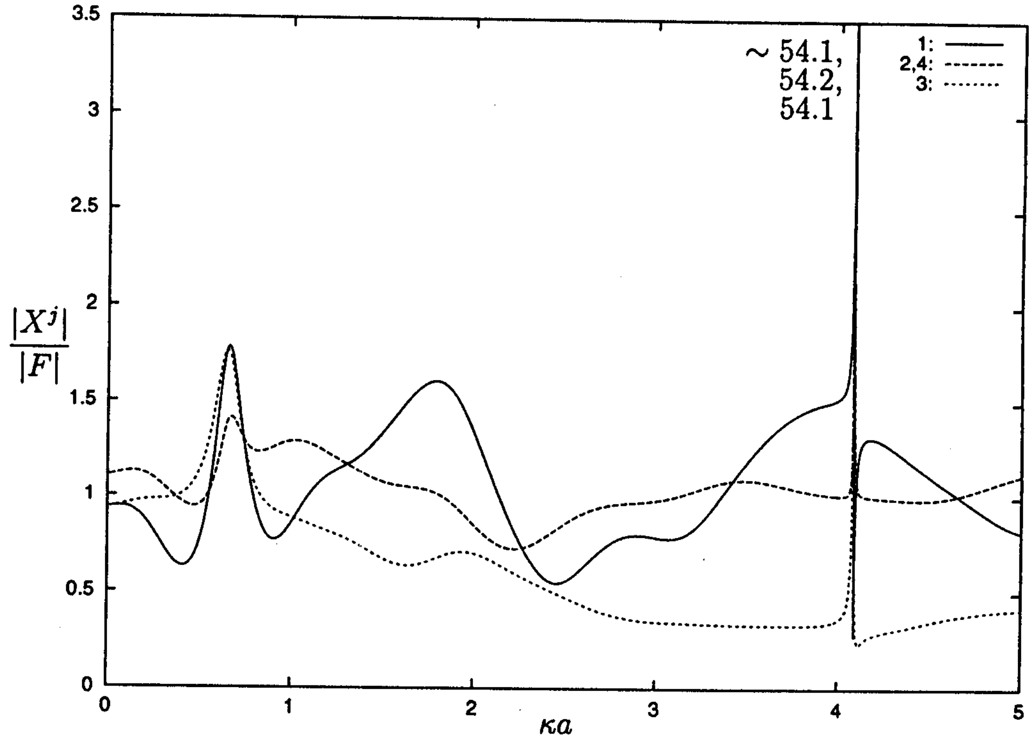

Figure 5: Resultant force on 4 cylinders, $\beta = 0^\circ$, $\mathbf{a/d = 0.4}$