Hydroelastic Behavior of a Very Large Floating Platform in Waves

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1. Introduction

Very large floating platforms such as a floating airport recently designed has a thin mat-like configuration of very large horizontal size: a proposed design, for example, is several kilometers length and about 10 meters thickness. Naturally bending rigidity is relatively very small with this thin configuration and elastic deflection due to wave action will be more crucial than the rigid body motions.

Numerical approaches used for analysis of wave-ship or wave-structures interaction will be applicable in principle but not appropriate for analyzing elastic responses of the platform of such particular configuration and large dimension. Wave length is very small relative to the body length and the platform hardly moves as a rigid body. In most cases little energy of waves penetrates deep underneath the platform from the edges and large deflection will occur only at the edge parts. When waves happen to penetrate into the beneath of the platform, the elastic deflection caused by the penetrated waves will be of different wave length from the incident waves. Therefore reflection and refraction will occur at the edge of the platform. Different analytical approach based on correct understanding of those physics must be developed for accurate prediction of the hydroelastic behaviors of the platform.

In this report we present a linear analysis of the behaviors of a thin elastic plate floating in waves. The approach is based on the idea that a thin plate is a part of water surface but with different characteristics from usual free surface of the water. This approach was used to analyze a similar problem of 2D by *Meylan and Squire (1994)*. We extend this approach to 3D case with the help of the wave diffraction theory of a slender ship which is familiar to ship hydrodynamicists.

2. The free surface condition

We consider long crested regular waves

$$\exp[-ikx\cos\theta - iky\sin\theta + i\omega t] \tag{1}$$

incident on a flat floating platform of length L, breadth B and draft d located in the $x-y$ plane which coincides with the still water surface (see Figure). It is assumed that $d << L, B$ and the platform deflects to the passing waves following a linear elastic model.

Assumption of $kL >> 1$ is rather natural considering several kilometers length of the platform. Another assumption is $kd << 1$. Hereafter we consider deep water case where $k = \omega^2/g$. Extension to the case of the finite water depth is straightforward.

Velocity potential ϕ has to satisfy Laplace equation in the fluid and a linear free surface condition on $z=0$ on the water surface outside of the platform. Underneath the platform we impose other condition. This condition is imposed at $z=0$ instead of at the actual platform surface in virtue of the assumption of very small d. Thin elastic plate theory will give an equation of the platform deflection w

$$m\frac{\partial^2 w}{\partial t^2} = -EI\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 w - \rho g w - \rho \frac{\partial \phi}{\partial t}_{z=0} \tag{2}$$

where m is mass of unit area of the platform, EI the equivalent flexural rigidity. The third term on the right hand side is the effect of buoyancy and the fourth is the dynamic pressure. Differentiating equation (1) with the time t, using the body boundary condition $\partial w/\partial t = \partial\phi/\partial z$ and factoring out the time component $e^{i\omega t}$, we obtain

$$\left[\frac{EI}{\rho g}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2-\frac{m\omega^2}{\rho g}+1\right]\frac{\partial\phi}{\partial z}-\frac{\omega^2}{g}\phi=0 \quad \text{at} \quad z=0 \tag{3}$$

This might be interpreted as a type of free surface condition with different mass and bending rigidity. This is to be satisfied on $0 < x < L, 0 < y < B$. *Stoker (1957)* used this idea for analysis of a flexible beam on the waves.

Our problem is a boundary value problem for the velocity potential ϕ with the usual free surface condition at $(z = 0, 0 < x < L - \infty < y < 0, B < y < \infty)$ and $(z = 0, -\infty < x < 0 \quad \text{or} \quad L < x < \infty)$, and another free surface condition (3) is imposed at $(z = 0, 0 < x < L, 0 < y < B)$

3. The method of solution

A. Oblique seas($\theta = 0$)

We assume the platform is long enough for L to be considered ∞. The solution ϕ must be periodical into the x direction and given is in the form of

$$\phi(x,y,z) = \psi(y,z)e^{-ikx\cos\theta+i\omega t} \tag{4}$$

It is well known that the governing equation of $\psi(y,z)$ is a 2D Helmholtz equation. Boundary conditions for ψ are:

$$k\psi - \frac{\partial\psi}{\partial z} = 0 \quad \text{at} \quad z = 0, 0 < y < B \tag{5}$$

$$\left[\frac{EI}{\rho g}\left(-k^2\cos^2\theta+\frac{\partial^2}{\partial y^2}\right)^2-\frac{m\omega^2}{\rho g}+1\right]\frac{\partial\psi}{\partial z}=-\frac{\omega^2}{g}\psi \quad \text{at} \quad z=0, y<0, \text{or} \quad y>B \tag{6}$$

Let us consider equation (6) as a differential equation for $\partial\psi/\partial z$. Then a Green function $f(y,y')$ of equation (6) with letting the right hand side zero and satisfying the zero moment and shear force condition $\partial^3/\partial y^3(\partial\psi/\partial z) = \partial^2/\partial y^2(\partial\psi/\partial z) = 0$ at the edge $y = 0, B$ is readily found. Then the linear condition (6) is rewritten as

$$\frac{\partial\psi}{\partial z} = -\frac{\omega^2}{g}\int_0^B f(y,y')\psi(y')dy' \tag{7}$$

where $f(y,y')$ is given by

$$f(y,y') = \sum_{j=1}^{4} A_j(y')e^{\beta_j y} \tag{8}$$

Naturally expression of A_j is different for $y > y'$ or $y < y'$ so that the derivatives of f have singularity at $y = y'$. β_j are the roots of

$$\beta^4 - 2k^2\beta^2\cos^2\theta + \frac{\rho g}{EI}(1 - \frac{m\omega^2}{\rho g} - \frac{EI}{\rho g}k^2\cos^2\theta) = 0 \tag{9}$$

Wave source function $S(y,z;y',z')$ of the Helmohotz equation satisfying (5) and an appropriate radiation condition has been well studied. The Green's second identity and the wave source function gives a linear Fredholm integral equation for ψ at $z = 0, 0 \le y \le B$ after some algebra.

$$\begin{aligned}\psi(y) =& e^{-iky\sin\theta} + k\int_0^B \left(\psi(y') - \int_0^B f(y',\eta)\psi(\eta)d\eta\right) \\ & \times \left[S(y,0;y',0) + \frac{i}{\sin\theta}\cos(k(y-y')\sin\theta)\right]dy'\end{aligned} \tag{10}$$

where

$$\begin{aligned}S(y,0;y'.0) =& -\frac{k\cos\theta}{\pi}\int_0^\infty d\mu \frac{\mu K_1(k\sqrt{(y-y')^2+\mu^2})}{\sqrt{(y-y')^2+\mu^2}} \\ & + \frac{i}{2\sin}\left[e^{-k|y-y'|\sin\theta} - e^{ik|y-y'|\sin\theta}\right]\end{aligned} \quad)(11)$$

Numerical implementation of this integral equation is not so difficult as it looks since the double integral part can be handled analytically in virtue of the simple form of f.

$\psi(y,0)$ numerically determined underneath the platform will give ψ_z through equation (7) and the deflection w of the platform is readily computed from ψ_z.

B.Head seas

Configuration of the airport will be very slender because B will be several hundreds meters. Although the slender body theory is not perfectly legitimate for this configuration since the wave length is not necessarily of the order of B, we apply it to our problem as a preliminary stage of investigation. Naturally we assume a slow variation of $B(x)$ even at the bow and the stern of the platform.

In the inner field close to the body we can factor out a rapidly varying part. The velocity potential is written in the form.

$$\phi(x,y,z) = \psi(y,z;x)e^{-ikx} \tag{12}$$

Theory of wave and slender ship interaction tells that the problem of the slow modulation part ψ is almost the same as in (B. Oblique seas). Difference is radiation condition to be determined by the analysis of the outer potential. The inner solution $\psi(y,z;x)$ will be written as

$$\psi(y,z:x) = F(x)[e^{-kz} + \psi_0(y,z:x)] \tag{13}$$

where ψ_0 is a solution of an integral equation similar to (10)

$$\psi(y,0:x) = \int_0^{B(x)} kg(y,,0;y',0)\left[(\psi_0(y,0:x)+1) - \int_0^{B(x)} f(y',\eta)(\psi_0(\eta,0:x)+1)d\eta\right]dy' \tag{14}$$

$g(y,z:y',z')$ is again a wave source function not increasing exponentially at $|y| \to \infty$ (*Ursell (1968)*).

Following the well-know procedure of the slender ship theory, unknown $F(x)$ is determined such that the outer approximation of (13) will match with the inner approximation of the outer potential. The matching condition is:

$$1 - \frac{1+i}{2\sqrt{\pi k}} \int_0^x \frac{d\xi Q(\xi)}{\sqrt{x-\xi}} = F(x) \tag{15}$$

$$Q(x) = F(x)k^2 \int_0^{B(x)} \left[(\psi_0(y,0:x)+1) - \int_0^{B(x)} f(y',\eta)(\psi_0(\eta,0:x)+1)d\eta \right] dy' \tag{16}$$

Deflection of the platform of realistic dimensions, which is predicted by numerical implementation of our approach, will be presented at the Workshop.

References

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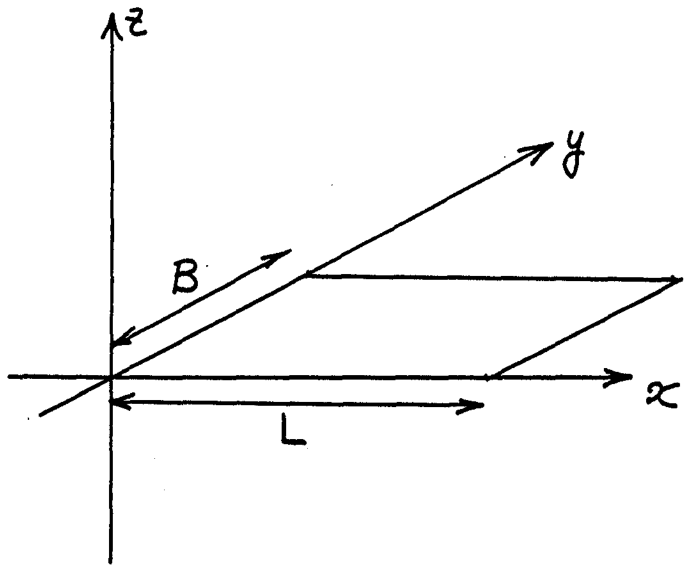

Fig. Coordinate system

DISCUSSION

Maniar: You showed plots where zero transmission occurred. Is it possible to have cases where both reflection and transmission are zero (theoretical)? (It would presumably correspond to a case of "total internal reflection").

Ohkusu and Nanba: Without appropriate damping provided from outside of the system which absorbs the wave ebnergy, it is not possible to make both reflection and transmission zero. Of course it is possible by selecting appropriate EI and other characteristics of the plate to make either of them zero.