Steep Faraday Waves

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We extend a recently submitted manuscript (Lei, et al. 1995) and observe experimentally unpredicted steep standing waves, with flat and dimpled crests for larger wave amplitude. These steep waves are accompanied by evident symmetry breaking in time. Calculated standing waves of large amplitude are also strongly asymmetric in time, but have sharper crest than predictions. These different wave forms both are the result of 2:1:2 internal resonance, analogous to "quartet" interactions for the traveling waves. In the physical experiments, we have observed different route to breaking standing waves. With increasing forcing amplitude, the standing waves with dimpled crest grow steeper, and form overturning plungers to each side of the crest. The steepness for these wave forms exceeds the theoretical predictions for gravity standing waves. When the forcing amplitude goes higher, three different modes emerge: 1) wave with flat/dimpled crest, 2) wave similar to the numerical simulation in profile, and 3) wave similar to the Penney & Price (1952) standing wave solution. The alternation of three modes forms a three-period modulation, while additional breaking occurs in mode 2 with a sharp jet at the crest. The wave elevation in mode 2 is twice as high as those waves with dimpled crest and double plungers on each sides. This phenomena is believed to be intimately related to the behavior of a dynamical system when multiple equilibrium states with same energy levels exist.

The study of parametrically excited linear subharmonic instabilities started with Faraday (1831). The study of Faraday waves was significantly advanced by Benjamin & Ursell (1954) solving the linearized inviscid problem after deriving the Mathieu equation.

Our motivation for investigating standing waves is to better interpret remotely sensed ocean surfaces, for example, those obtained by SAR (Synthetic Aperture Radar). SAR, based on the backscattering of microwaves by surface features, is sensitive to the curvature and periodicity of the sea surface. It is generally acknowledged that standing waves can have sharper curvature than their progressive wave counterparts. Therefore, standing waves, especially capillary-gravity standing waves with length scales on the order of the Bragg wavelength, have a strong influence on both the SAR return and its interpretation. Local standing wave systems occur through reflection from coastlines and structures. Standing waves can also be created by the interaction of ship-generated and ambient waves, by waves generated along the edge of a mesoscale current system, and by the vertical or horizontal oscillation of basins or containers.

We have investigated analytically, numerically, and experimentally several aspects of standing waves. As recognized in the literature (Henderson & Miles, 1990, hereafter referred to as H&M), damping-rate models are inadequate, so measured values are used in the weakly-nonlinear theory and the numerical predictions. Measurements show the relationship between the phases of the forcing displacement signal and the Faraday wave. A physical explanation is given for the relative phase of the Faraday waves — the trough and crest phases of the subharmonic waves are in-phase with consecutive troughs of the forcing signal.

An examination of the p–q stability diagram for the Mathieu equation reveals that the contact line increases the viscous natural frequency significantly. ($p_{min} \geq 1$ for dissipative system without contact-line effects becomes $p_{min} < 1$.) Thus, the neutral-stability curve is shifted downward in the p-q diagram. Effects of the contact line are shown two ways: by conducting experiments in rectangular containers of increased aspect ratios and by the addition of Photo Flo; both reduce the contact-line effects dramatically. With Photo Flo as a contact-line lubricating agent, the viscous effect dominates the threshold forcing amplitude and frequency, and the neutral-stability curve agrees with the theory of H&M.

The limit-cycle diagram exhibits strong "soft spring" nonlinearity through hysteresis, revealed by varying the forcing frequency in discrete steps with a fixed forcing amplitude. In particular, if the experiments are initiated from the lower-frequency side of the backbone curve, there is an abrupt change in the limit-cycle amplitude, while the experiments initiated from the higher-frequency side make a gradual transition, eventually with wave breaking once the frequency is lowered sufficiently. Experiments with and without Photo Flo show that this hysteresis depends strongly on the forcing amplitude and the contact-line effect. The limit-cycle amplitudes of the Faraday waves are in agreement with our numerical simulation and the theory of H&M.

Large wave modulations are observed in the experiments in the high-frequency forcing regime. The modulations in the physical experiments are explained by the influence of ambient noise due to the contact line, or noise in the forcing signal. The asymptotic estimate of the "sideband resonance" through forcing noise compares favorably with controlled experiments and numerical simulations. The addition of Photo Flo reduces the ambient noise level caused by the contact line; however, intentional sideband forcing precipitates a return of the modulations even with Photo Flo.

We show one manner in which two-dimensional Faraday waves break — overturning plungers to each side of the crest. The maximum steepness exceeds the theoretical predictions for gravity standing waves; however, the pure standing wave is not observed.

There are several unanswered issues that may have a great influence on Faraday wave resonance, the standing wave form, and the role of surface tension in surface waves. First, a more quantitative study is necessary on the exact contact-line mechanism responsible for the viscous frequency shift and modulations, and the role of Photo Flo. Research similar to Ting & Perlin (1995) should provide additional insight on this issue. Although there have been many fruitful studies on the weakly nonlinear theory and experiments on Faraday waves, e.g. Henderson & Miles (1990), the possible 2:1 internal resonance in the subharmonic case (coexistence of the subharmonic and synchronic modes for a forcing frequency) as studied by Gu & Sethna (1987) requires more detailed study. Our experiments uncovered complicated steep standing forms that cannot be described by the present weakly nonlinear theory.

There are differences between dissipation-introduced asymmetry and the new standing wave forms. (Analytical descriptions of the new wave form should be interesting.) Our conjecture is that the new wave form is typical of a steep forced gravity-capillary wave with broken temporal symmetry. Further experiments quantifying the effect of surface tension would increase understanding of the new wave form.

The applicability of our observed wave forms to those on the open ocean and their application to more applied problems such as SAR properties should be pursued. We plan to extend the experiments to three dimensions (using smaller aspect ratio). Further study is required on the characteristics of the spectral Cauchy integral method, particularly, the origin of the higher-frequency modes and the effect of various dissipation functions.

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