Transient Motion of a Floating Body in Steep Water Waves

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To predict the transient motion of a floating body on the free surface requires knowledge of the force acting on it by the fluid. The force is usually found by integrating the pressure obtained from the Bernoulli equation over the body surface. The difficulty, however, is the calculation of the derivative of the potential with respect to time, or $\partial\phi/\partial t$. Several methods have been used in various publications. Lin, Newman and Yue (1984) for example obtained $\partial\phi/\partial t$ by calculating $d\phi/dt$. But this method has several limitations. One is that the same fluid particle has to be followed, which is particularly problematic when remeshing is applied. An alternative has been adopted by Cointe et al (1990) and Cao, Beck & Schultz (1994). They obtained $\partial\phi/\partial t$ by solving a boundary value problem which is similar to that for the potential itself. The difficulty with this method is that it requires the acceleration of the body as part of its body surface boundary condition, which in turn requires the pressure and therefore $\partial\phi/\partial t$. This suggests that iterations may be required for a floating body. Another technique is to combine the boundary value problem for $\partial\phi/\partial t$ with the equation of motion (Van Daalen 1993, Tanizawa 1995). This will avoid iterations and the acceleration of the body can be found directly. It will also give the new velocity and new position of the body, which will then be used for the calculation at the next time step.

It appears that the method proposed by Van Daalen (1993) and Tanizawa (1995) has several advantages. In this paper, we shall adopt the principle of their method. But instead of combining the governing equation for $\partial\phi/\partial t$ with the equation of motion, we shall define an artificial function to avoid the need for direct solution of $\partial\phi/\partial t$. The method is very similar to that widely used for the second order diffraction force (Lighthill 1979, Molin 1979) and has been proved extremely useful. Below we shall derive the equations based on this method for a three dimensional floating body.

We define a Cartesian coordinate system O-xyz such that the origin is on the mean free surface and z points vertically upwards. The function ϕ_t then satisfies the Laplace equation

$$\nabla^2\phi_t = 0 \tag{1}$$

in the fluid domain R. On the free surface we have

$$\frac{\partial\phi}{\partial t} = -\frac{1}{2}\nabla\phi\nabla\phi - gz \tag{2}$$

since the pressure is zero. On the surface of a moving body, the boundary condition is (Wu & Eatock Taylor 1996)

$$\frac{\partial}{\partial n}(\phi_t) = [\dot{U} + \dot{\Omega}\times r]\cdot n - U\cdot\frac{\partial\nabla\phi}{\partial n} + \Omega\cdot\frac{\partial}{\partial n}[r\times(U-\nabla\phi)] \tag{3}$$

where n is the inward normal of the body surface, U is the translational velocity, Ω is the rotational velocity and r is the position vector from the point on the body where U is measured. On the bottom and at infinity, ϕ_t satisfies the same conditions as the potential itself.

Once the solution for ϕ_t had been found, the pressure could be obtained from the Bernoulli equation

$$p = -\rho\frac{\partial\phi}{\partial t} - \frac{1}{2}\rho\nabla\phi\nabla\phi - \rho g z \tag{4}$$

where ρ is the density of the fluid and g is the acceleration due to gravitation. The force **F** and the moment **M** on the body could be obtained by integrating the pressure over its wetted surface S_0

$$F = \int_{S_0} p n \mathrm{d}S \tag{5a}$$

$$M = \int_{S_0} p(r \times n)\mathrm{d}S \tag{5b}$$

The difficulty here is that the acceleration in equation (3) is unknown before the force has been found, which in turn depends on the solution of $\partial\phi/\partial t$. To overcome this difficulty, we use the equation of motion. From Newton's Law we have

$$[M_b][a] = [F] + [F_e] \tag{6}$$

where $[M_b]$ is the body mass matrix, $[a]$ is the acceleration matrix with three translational components ($\dot{U}$)and three rotational components ($\dot{\Omega}$), $[F]$ is a column with three force components and three rotational moment components due to hydrodynamic loading and $[F_e]$ is due to other external forces By combining the boundary value problem for $\partial\phi/\partial t$ [eqs. (1)-(3)] and the equation of motion, acceleration can be eliminated, and the solution for $\partial\phi/\partial t$ can be found directly. This is the method used by Van Daalen (1993) and Tanizawa (1995).

Here we propose a scheme which does not need the solution of $\partial\phi/\partial t$. We define a function ψ_i which satisfies the Laplace equation and the following boundary conditions

$$\frac{\partial\psi_i}{\partial n} = n_i \tag{7a}$$

on the body surface and

$$\psi_i = 0 \tag{7b}$$

on the free surface, where

$$n = (n_1, n_2, n_3), \qquad r \times n = (n_4, n_5, n_6) \tag{8}$$

The condition of ψ_i on the seabed is the same as that on the potential itself. By using Green's identity, we have

$$\int_{S_0+S_F+S_\infty}\left(\frac{\partial\phi}{\partial t}\frac{\partial\psi_i}{\partial n} - \psi_i\frac{\partial^2\phi}{\partial n\partial t}\right)\mathrm{d}S = 0$$

The integration at infinity and over the sea bed is zero. Also as $\psi_i = 0$ on the free surface, we have

$$\int_{S_0}\frac{\partial\phi}{\partial t}n_i\mathrm{d}S = \int_{S_0}\psi_i\frac{\partial^2\phi}{\partial n\partial t}\mathrm{d}S - \int_{S_F}\frac{\partial\phi}{\partial t}\frac{\partial\psi_i}{\partial n}\mathrm{d}S$$

Substituting equations (2) and (3) into this, we have

$$\int_{S_0}\frac{\partial\phi}{\partial t}n_i\mathrm{d}S = \int_{S_0}\psi_i\{[\dot{U} + \dot{\Omega}\times r]\cdot n - U\cdot\frac{\partial\nabla\phi}{\partial n} + \Omega\cdot\frac{\partial}{\partial n}[r\times(U - \nabla\phi)]\}\mathrm{d}S$$
$$+\int_{S_F}\left(\frac{1}{2}\nabla\phi\nabla\phi + gz\right)\frac{\partial\psi_i}{\partial n}\mathrm{d}S \tag{9}$$

Substituting equation (9) into (6), we have

$$([M_b] + [N])[a] = [Q] + [F_e] \tag{10}$$

where $[N]$ is a matrix whose coefficients are

$$N_{ij} = \rho\int_{S_0}\psi_i n_j \mathrm{d}S \tag{11}$$

and $[Q]$ is a column with

$$Q_i = -\int_{S_0} \psi_i\{-U\cdot\frac{\partial\nabla\phi}{\partial n} + \Omega\cdot\frac{\partial}{\partial n}[r\times(U-\nabla\phi)]\}\mathrm{d}S - \int_{S_f+S_0}\left(\frac{1}{2}\nabla\phi\nabla\phi + gz\right)\frac{\partial\psi_i}{\partial n}\mathrm{d}S \qquad (12)$$

Based on equation (10), $[N]$ can be defined as the added mass matrix. It is easy to show that it satisfies the well known identity for the added mass in the unbounded fluid domain: $N_{ij}=N_{ji}$. It can be seen that there are second order derivatives in equation (12). They are similar to the well known m-terms (Timman & Newman 1962). The direct computation of these terms require special attention (see for example Wu 1991, Zhao & Fatinsen 1989). Here we adopt the method used by Wu & Eatock Taylor (1987) based on the equation of Ogilvie and Tuck (1969). Using Stokes theorem, $\psi=0$ on the free surface and $\nabla^2\phi = 0$ we have

$$\int_{S_0}\psi\frac{\partial\phi_x}{\partial n}dS = \int_{S_0}\psi(\phi_{xx}n_x + \phi_{xy}n_y + \phi_{xz}n_z)dS$$

$$= \int_{S_0}[\psi\phi_{xx}n_x + \frac{\partial}{\partial x}(\psi\phi_y)n_y + \frac{\partial}{\partial x}(\psi\phi_z)n_z - \psi_x\phi_y n_y - \psi_x\phi_z n_z)]dS$$

$$= \int_{S_0}[\psi\phi_{xx}n_x + \frac{\partial}{\partial y}(\psi\phi_y)n_x + \frac{\partial}{\partial z}(\psi\phi_z)n_x - \psi_x\phi_y n_y - \psi_x\phi_z n_z)]dS$$

$$= \int_{S_0}(\nabla\psi\nabla\phi n_x - \frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial n})dS \qquad (13)$$

In general, it can be shown that

$$\int_S \psi\frac{\partial\phi_{x_i}}{\partial n}dS = \int_S(\nabla\psi\nabla\phi n_{x_i} - \frac{\partial\psi}{\partial x_i}\frac{\partial\phi}{\partial n})dS \qquad (14)$$

Substituting equation (14) into (12), we obtain

$$Q_i = \int_{S_0}\{\nabla\psi_i[(U+\Omega\times r)\cdot n][\nabla\phi - (U+\Omega\times r)] + \psi_i(\Omega\times U)\cdot n\}\mathrm{d}S$$
$$-\int_{S_0+S_f}\left(\frac{1}{2}\nabla\phi\nabla\phi + gz\right)\frac{\partial\psi_i}{\partial n}\mathrm{d}S \qquad (15)$$

This removes the need for the second order derivatives.

It can be seen that the core of the above method is the introduction of the artificial potential ψ_i . This potential satisfies a Neumann condition on the body surface and a Dirichlet condition on the free surface. They are the same conditions as those on the real potential ϕ itself. Also ψ_i does not contain ϕ in its boundary conditions. Thus it can be solved when ϕ itself is solved. This means that the present method does not require additional CPU and memory. Numerical results will be presented in the workshop.

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References

Cointe, R., Geyer, P., King, B., Molin, B and Tanoni, M (1990) Nonlinear and linear motions of a rectangle barge in a perfect fluid, *18th Symp. on Naval Hydrody.*, pp.85-98, The University of Michigan, Ann Arbor

Cao, Y., Beck, R. and Schultz, W.W. (1994) Nonlinear motions of floating bodies in incident waves, *9th Workshop on Water Waves and Floating Bodies*, Kuju, Oita

Lin, W.M., Newman, J.N. and Yue, D.K. (1984) Nonlinear forced motions of floating bodies, *15th symp. on Naval Hydrodynamics* , pp.33-47, Hamburg

Newman, J.N. (1977) *Marine Hydrodynamics*, MIT press

Van Daalen, E.F.G. (1993) Numerical and theoretical studies of water waves and floating bodies, Ph.D thesis, University of Twente, Enschede, The Netherlands

Lighthill, M.J. (1979) Waves ahd hydrodynamic loading, *Proc. of the second Int. Conf. Behaviour of Offshore Structures*, pp.1-40

Molin, B. (1979) Second order diffraction loads upon three-dimensional bodies, *Appl. Ocean Res.*, Vol.1, pp.193-202

Ogilvie, T.F. and Tuck, E.O. (1969) A rational strip theory for ship motion, Part 1. Repo. no. 012, Department of Naval Architecture and Marine Engineering, University of Michigan

Tanizawa, K. (1995) A nonlinear simulartion method of 3-D body motions in waves formulations with the acceleration potential, *10h Workshop on Water Waves and Floating Bodies*, Oxford

Timman, R. and Newman, J.N. (1962) The coupled damping coefficients of symmetric ship, *J. Ship Res.* Vol.5, pp.1-7

Wu, G. X. (1991) Numerical Scheme for calculating the m_j terms in wave-current-body interaction, *Appl. Ocean. Res*, Vol.13, pp.317-319

Wu,G.X. and Eatock Taylor, R. (1987) Hydrodynamic forces on submerged oscillating cylinders at foward speed, Proc. Roy. Soc. London, Vol. A414, pp.149-170

Wu, G.X. and Eatock Taylor, R. (1995) Time stepping solutions of the two dimensional non-linear wave radiation problem, *Ocean Engineering*. Vol.22, pp.785-798

Wu, G.X. and Eatock Taylor, R. (1996) Transient motion of a floating body in steep wåter waves, *To beSubmitted*

Zhao, R. and Faltensen, O.M. (1989) Interaction between current, wave and marine structures *5th Int. Conf. on Numerical Ship Hydrodynamics,* Hiroshima, Japan

DISCUSSION

Ferrant: You seem to infer that Cointe et al (1990) used iterations for the computation of the acceleration. This is not the case. They use a factorization of ϕ_t into elementary components corresponding to unit accelerations in each mode of motion. These ϕ_{it} strangely look like your ψ_i's. So I think that there is no difference between what you propose and what has been used by Cointe et al (among others), except for the use of Stokes theorem (eq. 13 and after in your paper). Please tell me whether I am right or not.

Wu & Taylor: It is evident that we have overlooked what was said in the paper by Cointe et al. Having read the paper carefully again, we now realize they did not use iteration in their scheme, as implied in our abstract. We hope that the authors will accept our apology.

Concerning the second point of the discusser, we have now also realised that there are some similarities between our scheme and that of Cointe et al., but there also appear to be some significant differences. Cointe et al. defined quantities ϕ_t corresponding to unit accelerations. However, from their paper it is not clear what free surface boundary condition is imposed on those quantities. As the free surface boundary condition (pressure being zero) contains the potential itself, the boundary value problem can be solved only after the potenttial has been found. In our scheme, however, the quantities ψ can be be solved at the same time as the potential itself.