A time domain method to compute transient non linear hydrodynamic flows

M. BA, A.FARCY (ENSMA) and M. GUILBAUD, (CEAT, Université de Poitiers)
Laboratoire d'Etudes Aérodynamiques-URA CNRS n°191,
43 rue de l'Aérodrome, 86036 Poitiers CEDEX, FRANCE

Introduction

Today, most of the numerical codes for the computations of ship seakeeping or for the diffraction-radiation motions for platforms are solved in the frequency domain using a linear form of the free surface boundary conditions, called the Neumann-Kelvin approach. The water can be considered as incompressible and inviscid and the flow around the body as irrotationnal except on some lines or surfaces, so the Laplace equation is valid in the fluid domain. These problems can be solved by panel methods using either Rankine (aerodynamic) or Kelvin singularities. For more complicated (non harmonic) motions, the time domain has to be chosen instead of the frequency one and in his case, the Green's function is so complicated (Newman, 1995, Mas et Clément, 1995) that no computational codes have been developed up to day. But these linearized approaches are limited to small harmonic motions with mean constant forward speed and the body condition has to be satisfied on the mean position of the exact body surface. For motions with larger amplitudes, this simplification is no more possible and the body condition has to be satisfied of the body exact position, implying also that the free surface conditions cannot more be linearized. So these previous problems are fully non linear and the flow analysis is more easily done in the time domain.

If less developed than the computations in the frequency domain, the calculations using the time domain (cf. Beck ,1994 for review) become more popular with the development of computers. We present here the first results obtained with a non linear method to compute transient free surface flows. To reduce the computational time, the surface source distribution on the free surface and on the body are replaced by source points desingularized, as proposed by Cao et al.(1990). To check the validity of the method, computations are presented on the transient flow around a submerged source with impulsive start. The results are compared with those of linearised computations. Finally some results on a submerged ellipsoid are also presented.

Formulation of the non linear problem

The flow of an ideal and incompressible fluid of infinite depth is considered with the undisturbed free surface located in the plane z=0. The frame of co-ordinates uses the z-axis positive upwards and the x-axis pointing in the direction of the mean velocity of the body. The surface tension is neglected. As the problem starts from rest, the flow is irrotational implying the existence of a velocity potential ϕ, satisfying the Laplace equation in the fluid domain. This potential must also satisfy the body condition on the surface S_B of the body :

$$\frac{\partial \phi}{\partial n} = \vec{V}_E \cdot \vec{n} \text{ on } S_B \quad (1),$$

where $\vec{n}$ is the unit normal vector directed into the fluid and $\vec{V}_E$ is the local velocity of the body. A condition of non perturbation when the depth of immersion goes to infinity must also be satisfied. On the instantaneous free surface, the potential must also satisfy both the kinematic and the dynamic boundary conditions ; if the free surface elevation is given by z=E(x,y,t), those conditions are given by :

$$\frac{\partial E}{\partial t} - \frac{\partial \phi}{\partial z} + \frac{\partial E}{\partial x}\frac{\partial \phi}{\partial x} + \frac{\partial E}{\partial y}\frac{\partial \phi}{\partial y} = 0 \text{ on } z = E(x,y,z,t), \quad (2)$$

$$\frac{\partial\phi}{\partial t}+\frac{1}{2}\left|\overrightarrow{grad}\,\phi\right|^2+gE=0 \text{ on } z=E(x,y,t) \qquad (3).$$

Finally, the fluid disturbance must vanish at infinity, and the following initial conditions have also to be satisfied :

$$\phi=0 \text{ for } t\leq 0 \text{ in the whole fluid domain, and } E(x,y,t)=0 \text{ for } t\leq 0 \quad (4).$$

The two previous conditions (2) and (3) can be written using the material derivatives, enabling to compute the variation of a physical quantity following a fluid particle and leading to the kinematic condition as :

$$\frac{D\vec{X}_p}{Dt}=\overrightarrow{grad}\,\phi \quad (5),$$

where $\vec{X}_p(x(t),y(t),z(t),t)$ is the location of a fluid particle on the free surface. The dynamic condition can be written as :

$$\frac{D\phi}{Dt}=-gE+\frac{1}{2}\overrightarrow{grad}\,\phi\cdot\overrightarrow{grad}\,\phi \qquad (6).$$

Method of resolution

At each time step, the potential is assumed to be known on the free surface, the real location of which being also known. Conditions (4) is used at the beginning of this time marching procedure. For the next time step, equation (5) is used to compute the new free surface elevation and equation (6) to obtain the new value of the potential on the free surface. So, at each time step, a new mixed problem with a Neumann condition on the body (known normal potential derivative) and a Dirichlet problem on the free surface (known potential) has to be solved. To satisfy the boundary conditions, the body is divided into quadrilateral panels and a part of the free surface, into rectangular panels. To reduce the computational time, point sources are distributed on the free surface instead of using surface source distribution on panels. The potential and the velocity induced by these point sources being singular when the collocation points is located on source positions, a desingularised technique has been followed (Cao et al., 1990 or Beck, 1994). So point sources are located into the body or above the free surface. The source displacement, with distance L_d, is done along the normal to the panel. The choice of L_d is difficult and the values do not be too large or too small in order to obtain correct results. Beck (1994) has proposed as optimum value $L_d=(S_F)^{0.25}$ where S_F is the area of the panel containing the source on the body or the mean value of the areas of the four panels surrounding a source on the free surface.

Applications

Wave field due to an submerged doublet with a constant mean forward speed

The flow generated by the impulsive start of a doublet (source and sink of same intensity located 0.1m apart in the x direction) from rest. The forward speed and doublet intensity are quickly set to their steady values U_∞ and σ_0 using the following relations:

$$V(t)=U_\infty\left(1-e^{-4t}\right) \ \ and \ \ \sigma(t)=\sigma_0\left(1-e^{-4t}\right) \quad (7).$$

This doublet travels along an axis parallel to the x axis, 1mm deep under the free surface. The initial mesh on the free surface is located at $0\leq y\leq 20$m and $-7.5\leq x\leq 7.5$m and is subdivided into a mesh of 40x30 panels(figure 1). The nodes are equidistant in the x direction but in the y one, the distance between two nodes increases of 10%, both in the positive and negative y direction. The doublet intensity, σ_0 is assumed to be known, the unknowns for this problem are the intensities of the point sources on the free surface. These values are computed by writing that the potential is given on the free surface. For each time step, a new location of the free surface and the new distribution of the potential for the next time step are obtained from equations (5) and (6). In these equations, the right hand sides are analytically computed because point sources are used; the time derivatives in the left hand sides are computed by a fourth order Runge-Kutta method.

On figure 2, the free surface elevation above the doublet is plotted for 3 values of σ_0 (0.05-0.75 and 0.9) for a fully converged computations (t=40s), with a time step Δt=0.2s. The results obtained with the use of the steady forward speed Green's function, so corresponding to a linear and steady computation, are also plotted. It can be observed on this plot that, as the doublet intensity increases, the non linear wave amplitudes become greater than that the ones computed by the linear method. The difference is maximum for the first crest above the doublet. The evolution of the free surface is presented on the figure 3 (σ_0=0.05) and for four values of the time t=4-10-16 and 25s, showing the evolution of the unsteady solution towards the steady one.

Wave field due to a submerged ellipsoid starting from rest

Computations have been done on a ellipsoid with horizontal axis a=5m and lateral one, b=1m. The horizontal axis is located at the distance h=1.586m under the undisturbed free surface. In this case, the mesh on the free surface is made of square panels (40 in the x direction and 20 in the lateral one). The singularity intensities on the body are obtained from the body condition (eq. 2). The expression proposed by Beck(1994) for the desingularisation distance has been modified at both longitudinal ends of the ellipsoid. The evolution of the free surface with time is presented on figure 4 for a Froude number based on the depth of immersion h, $F = U_\infty / \sqrt{gh} = 1.26$, for 4 time values. After the impulsive start, t=1.8s, a crest can be observed on front part of the body and the level decreases on the rear part. As the time increases, the wave becomes steepest and a second crest appears immediately upstream of the body. At t=12s, the shape of the first wave become smoother and the second wave propagates with a V-shape; at the same time, a second trough and a third crest appears. A second V-shape wave appears and become important at t=30s, but its amplitude is weaker than the one of the first wave. Finally at t=45s, a quasi-steady state is obtained.

Conclusion

First results obtained in the time domain using a non linear method to compute transient flows close to a free surface are presented. The method uses desingularized source points, on the free surface and on the body, modified at both ends of bodies, to avoid numerical difficulties, keeping relatively low the computational time. The converged results have been first checked for a submerged doublet with known strength by comparing with steady calculations achieved with a panel method using the linear steady Green's function, showing good agreement. The evolution of the free surface with time has been also studied. The second application presented concerns a submerged ellipsoid. Work is on progress to optimise the computational time and to extent the validity of the method to surface-piercing bodies.

References

Beck R.F., "Time-domain computations for floating bodies", Applied Ocean Research, 16, 267-282, 1994.

Cao Y., Schultz, W.W. and Beck R.F., "Three-dimensional desingularised boundary integral methods for potential problems", Int. J. for Num. Methods in Fluids, Vol. 12, 785-803, 1991.

Mas S. and Clément A., "Computation of the finite time-domain Green function in the large time rang", 10th Int. Work. on Water Waves and Floating bodies", Oxford, 1995.

Newman J.N., "The approximation of free-surface Green functions", Fritz Ursell retirement meeting, 1990.

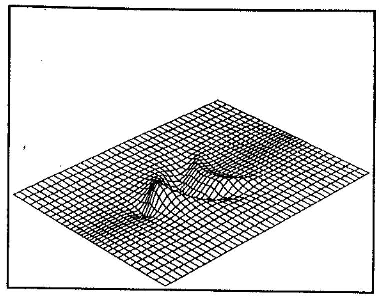

Fig. 1 : Mesh of the free surface

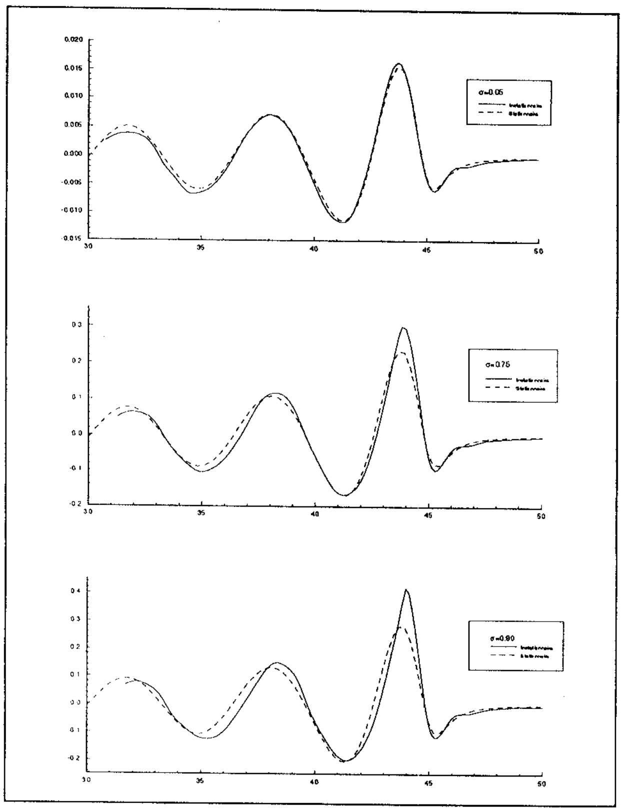

Fig. 2 : Free surface elevation at t=40s

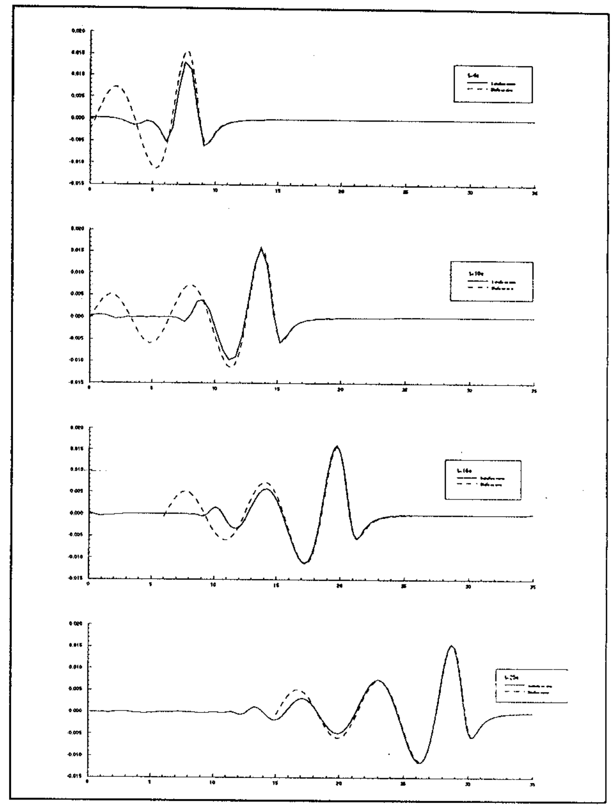

Fig. 3 : Evolution of the free surface

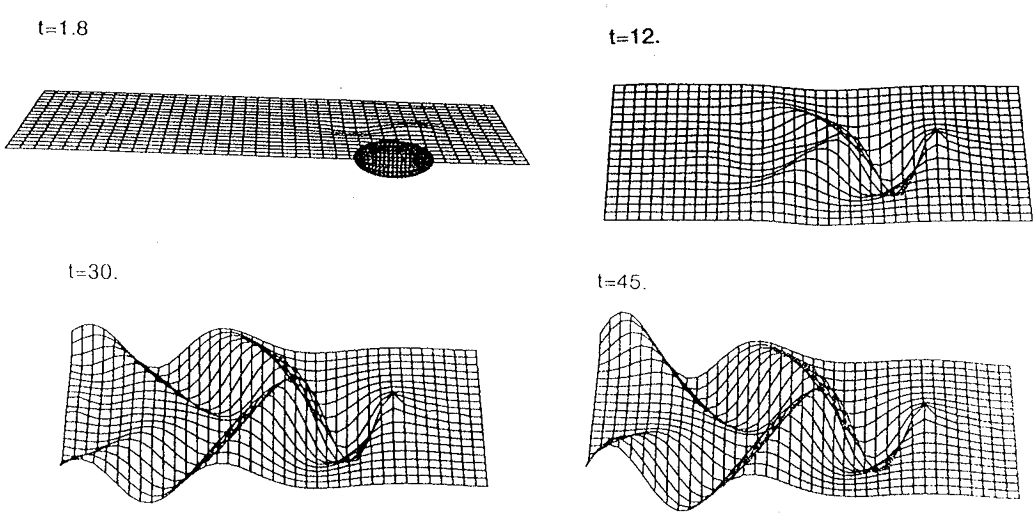

Fig. 4 : Wave fied due to a submerged ellipsoid starting from rest