On new mode of wave generation by moving pressure disturbance.

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The problem of waves generation by moving surface pressure disturbance is well known and well enough studied in the works of many authors. Here we present the results of numerical simulation of 2D plain waves generation by negative pressure disturbance, which revealed a new regime when moving with a critical speed disturbance doesn't generate upstreame-advancing solitons.

1. Mathematical model

Numerical model used in computations is the so-called discrete nonlinear-dispersive shallow water model [1]. The essence of this approach is that an incompressible flow with a free surface is simulated by a finite mechanical system of material particles with some holonomic constraints which represent an incompressibility condition. The governing equations are obtained then from Hamilton principle. The main advantage of such models is that they provide exact conservation of mass, momentum, angular momentum and energy even for coarse spatial discretization. So they give, in particular, the numerical solutions which are real solitary waves advancing with constant amplitude, shape and phase speed without any numerical dissipation or radiation. This property seems to be important for long-time calculations and played, in particular, significant role in solving Mach reflection problem [2].

In a shallow water case some additional simplifications can be made which, as in case of usual fluid motion equations, allow to reduce the dimension of a model. A detailed description of the model and the results of numerical testing it's accuracy are given in [1]. There was shown also that for even bottom this model gives a finite-dimensional approximation of well known Green-Naghdi (1976) equations. As far as the problem under consideration deals only with even bottom, one may assume that the discrete model used here is just a kind of difference scheme for Green-Naghdi equations in lagrangian variables, which conserves exactly the horizontal momentum and total energy.

2. Numerical results

An infinite fluid layer of constant depth H with a free surface is considered. The moving surface pressure disturbance is given by

$$p = \begin{cases} p_0 \cos^2(\pi\xi/2L), & \text{if } |\xi| \le L \\ 0, & \text{if } |\xi| > L, \end{cases}$$

where $\xi = x - x_0 - Ut$. Hereafter we assume the depth H, fluid density ρ and gravity acceleration g being unit, and the pressure disturbance being moved with a critical speed $U = 1$. The computational domain moves step by step after disturbance with the boundary conditions corresponding to the quiescent liquid at the right end and a kind of "open" condition at the left one. It is known that for $p_0 > 0$ such disturbance, pushing a fluid ahead, generates periodically upstream-advancing solitons. The wave resistance coefficient always remains positive oscillating near some mean value.

When $p_0 < 0$ the solitons are also generated but the mechanism is different. This rarefaction region first pulls out a wave of large amplitude which, having a large speed, quickly overtakes the pressure source and losing an amplitude becomes a soliton. The wave resistance coefficient periodically changes sign, but the mean value, as far as a radiation of solitons takes place, is usually nonzero. It appeared, however, that the halfwidth L of a rarefaction region can be specially chosen so that a new periodic mode arises when no upstream-advancing soliton are generated. This effect first was observed for the solitons of large amplitude, wich exist only in shallow water approximation but break down in reality. Then it was found to be the case for all amplitudes.

In fig.1 an example of such mode for $p_0 = -0.12$ is given. It is well seen that the pressure disturbance pulls out a wave, which coming to the right end of pressure region loses it's amplitude and dissapears there completely. At this time the next wave appears near the left end of pressure region and so on. The wave resistance coefficient C_d versus time t plot shows periodic behaviour with almost zero mean value. In our calculations this periodic mode took place for some 8-10 periods and then a soliton of small amplitude was emitted (see fig.1 (f)), which probably means that this mode is unstable. Still it seems to be interesting as an example of new nonlinear and nontrivial solutions.

In fig.2 the values of L versus p_0 are plotted for which this mode is realized. Fig.3 shows the maximal values of wave amplitudes which arise in this regime. It can be seen that this mode exists for moderate amplitudes and so it is possible trying to observe it in experiment.

1. *Frank A.M.* Discrete nonlinear-dispersion shallow water model. J. of Appl. Mech. and Techn. Phys.(transl.),1994,v.35,No.1,p.34-42.
2. *Serebrennikova O.A., Frank A.M.* Numerical modeling of Mach reflection for solitary waves. J. of Appl. Mech. and Techn. Phys.(transl.),1993,v.34,No.5, p.610-618.

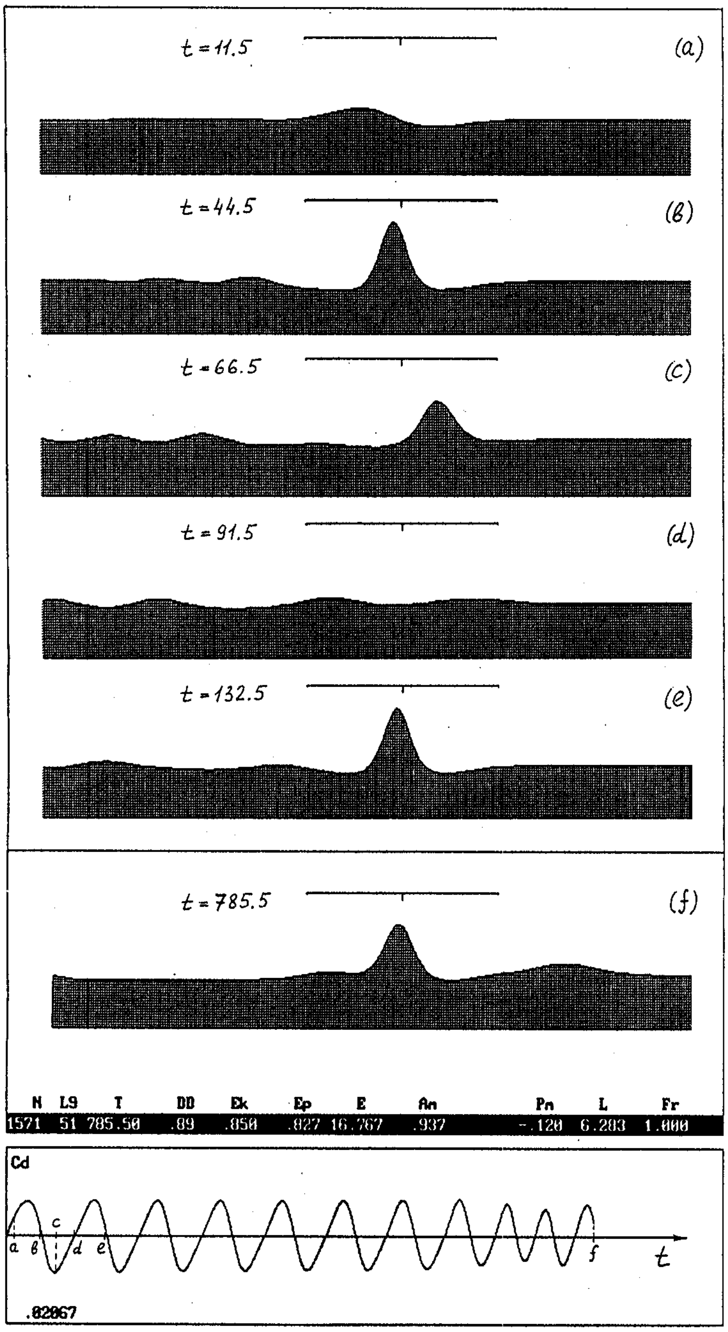

Fig. 1

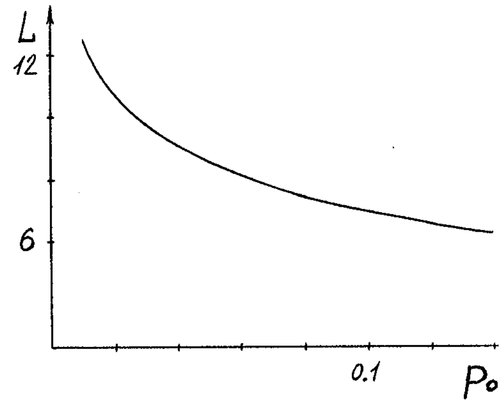

Fig. 2

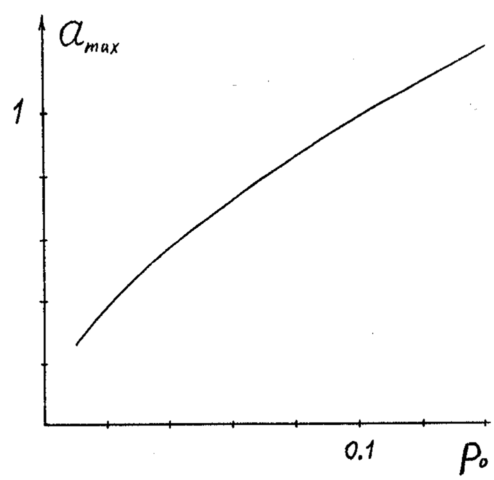

Fig. 3