MOTION SIMULATION OF A TWO-DIMENSIONAL BODY AT THE SURFACE OF A VISCOUS FLUID BY A FULLY COUPLED SOLVER

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INTRODUCTION

We present here an original solver [1] to compute two-dimensional free surface flows in viscous and incompressible fluid by a finite difference method. In most of the methods used nowadays for solving such problems, free surface elevation is updated at each time step by integration of the kinematic condition after computation of velocity and pressure fields [2] [6]. In these methods non-physical boundary conditions must be introduced to solve linear systems and non-linear free surface boundary conditions cannot be computed accurately. In the method presented here exact non-linear free surface boundary conditions are implemented on the real position of the free surface. At each time step the totally coupled linear system for velocity, pressure and free surface elevation unknowns is solved by a CGSTAB algorithm. Results for a free surface-piercing cylinder in forced heave, sway or roll motion are presented.

EQUATIONS AND NUMERICAL RESOLUTION

Navier-Stokes equations for laminar flows are written under convective form in a cartesian system (x^1, x^2) defining the physical fluid domain. The dependant unknowns are the cartesian components (u^1, u^2) of velocity, the dynamic pressure $p = P + \rho g x^2$ including gravitational effects and the free surface elevation h. A curvilinear system $(\varepsilon^1, \varepsilon^2)$ is used to simplify the implementation of boundary conditions. Here $\varepsilon^1 = 0$ is the equation of the immersed part of the body and $\varepsilon^2 = 0$ the equation of the free surface. A partial transformation of the moving physical space in a fixed curvilinear computational space is then defined.
In classical uncoupled methods a linear system issued from discretisation of transport and continuity equations is solved by weakly-coupled algorithms such as PISO or SIMPLER. Thus new velocity and pressure fields are obtained at each time step. The free surface elevation is updated by integration of the kinematic condition. This method leads to several theorical or numerical problems :
- a free surface boundary condition for velocities is lacking because of the use of kinematic condition for free surface elevation calculation. A supplementary non-physical condition must be used and does not allow an accurate calculation of viscous or surface tension effects. Moreover the normal dynamic condition is used as a Dirichlet condition for the pressure what leads to a poor mass conservation just under the free surface.
- the singularity of the kinematic condition at the intersection of free surface and solid body can be solved by introducing a meniscus. For very refined grids in the vicinity of the body this meniscus can become too important and lead to numerical divergence of the computation.
- the use of the SIMPLER algorithm gives a poor convergence of non-linear residuals (fig. 1) and it is a serious problem to compute unsteady flows.

In the new method proposed here the kinematic condition is used as a free surface boundary condition for velocity. The tangential dynamic condition is the other condition on the free surface for velocities (as in the uncoupled method). The discrete pressure unknowns are yet located at the centre of the cells (velocity unknowns are located at the nodes of the mesh) and no pressure boundary conditions are required. With these choices we have only physical boundaries conditions on the free surface. The normal dynamic condition gives a relation between pressure and free surface and will be used to compute the free surface elevation.
A totally-coupled solution is chosen to ensure mass conservation.
The mass conservation is represented by a pressure equation which is discretised by a Rhie and

Chow procedure to avoid checkerboard oscillations. This procedure is generalised for cells located near the free surface to take free surface effects into account and to make the pressure block invertible. At each iteration the following linear system for discrete velocity, pressure and free surface elevation unknowns (respectively called U, P and H) is solved and inverted by the iterative CGSTAB algorithm [8]:

$$\begin{array}{r} \text{transport equations} \rightarrow \\ \text{pressure equation} \rightarrow \\ \text{normal dynamic condition} \rightarrow \end{array} \left[\begin{array}{c|c|c} M_u & M_p & \\ \hline M_{du} & M_{dp} & \\ \hline & M_{sp} & M_{sh} \end{array}\right] \begin{pmatrix} U \\ P \\ H \end{pmatrix} = \begin{pmatrix} f_u \\ f_p \\ f_h \end{pmatrix}$$

With the coupled method the convergence of non-linear residuals is very good (fig. 1) compared to the convergence of the uncoupled method. Moreover the total CPU time is decreased by the totally coupled method (two or three twice as fast than the uncoupled method for the same global simulation time).

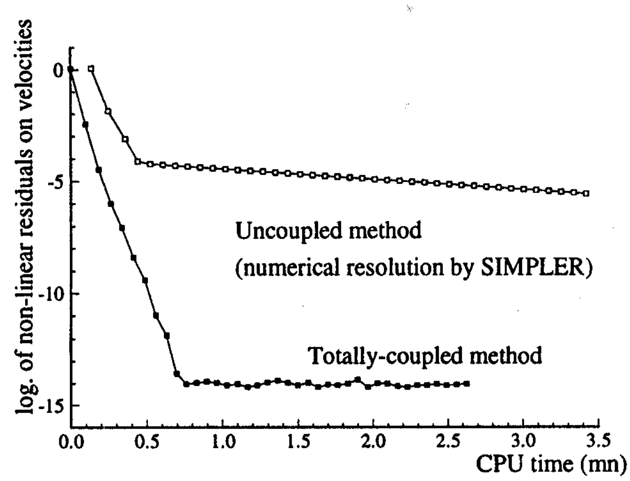

Fig. 1 : Convergence of non-linear residuals with the present method and an uncoupled method

RESULTS

The monoblock structured grids used here are generated by an direct algebraic method.
Heave forced motion has been first computed for a circular cylinder. The flow is supposed to be symmetric and simulated only in half the fluid domain.

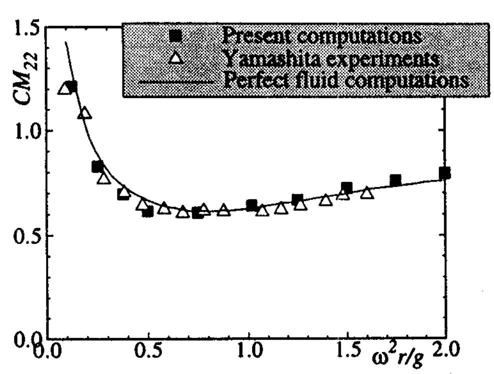

Fig. 2 : Added mass in heaving

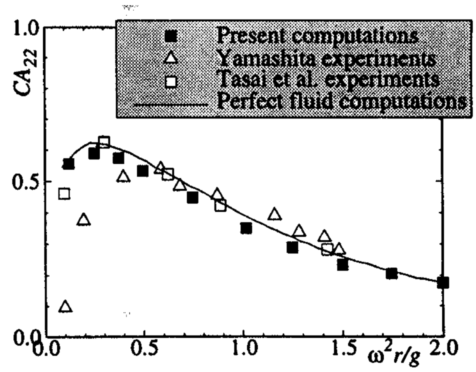

Fig. 3 : Damping coefficient in heaving

A Fourier transform of the computed time series of the hydrodynamic forces acting on the body leads to non-dimensional hydrodynamic coefficients.
Numerical results are in good agrement with Yamashita [10] and Tasaï *et al.* [7] experiments even for the 3rd-order force amplitude [3] and perfect flow computations [5]. Added mass and damping

coefficients are presented on figures 2 and 3.
For numerical simulations of a rectangular cylinder in forced sway or roll motion [4] the fluid domain comprises two free surface boundaries which are not connected. These two interfaces are defined by the equations $\varepsilon^2 = 0$ and $\varepsilon^2 = \varepsilon^2_{max} = 1$ (see fig. 4).

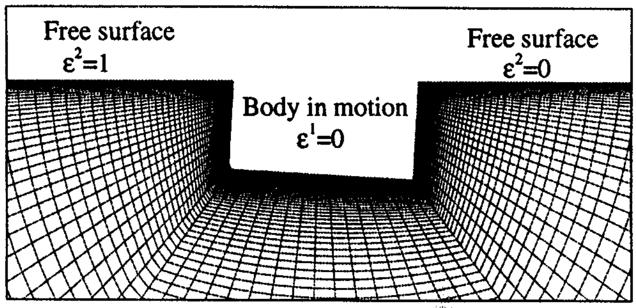

Fig. 4 : Shape of the grid during a computation in roll

For the sway motion the beam-to-draft ratio B/T was 2 (B=0.4 m in the present computation and in Vugts experiments [9]) and the forced motion of the form $y(t) = y_a \sin(\omega t)$ with y_a=0.02 m. The present method leads to a good accordance of the computed added mass with the experimental results or perfect fluid computations of Vugts (fig. 5) but under-estimates the damping coefficient for non-dimensional frequencies upper than 0.75 (fig. 6).

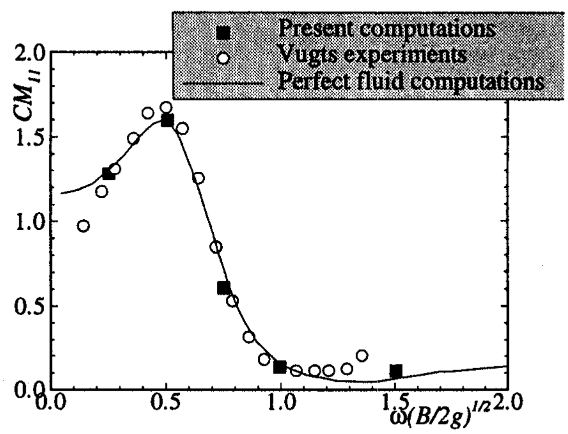

Fig. 5 : Added mass in swaying
(for a rectangle)

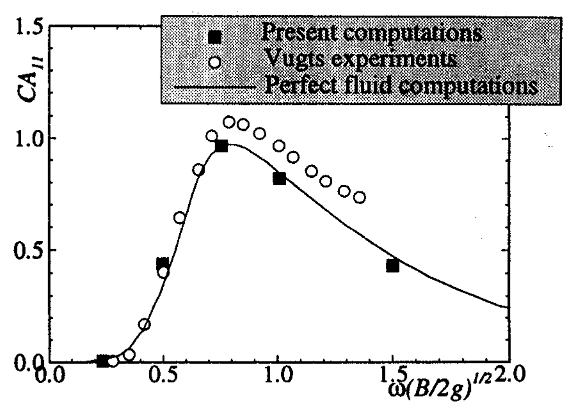

Fig. 6 : Damping coefficient in swaying
(for a rectangle)

For roll motion the beam-to-draft ratio B/T was 2 and the forced motion of the form $\phi(t) = \phi_a \sin(\omega t)$ with Φ_a=0.1 rad. Results are compared with viscous flow computations of Yeung *et al.* [11] (made with Φ_a=0.05 rad) based on the Free-Surface Random-Vortex Method and Vugts experiments with Φ_a=0.1 rad (non-dimensional experimental results for Φ_a=0.1 rad and Φ_a=0.05 rad are nearly the same) or inviscid flow computations.
The CM_{ij} and CA_{ij} with i different from j are the mass coupling and the damping coupling coefficients in the i-equation by motion in the j-mode respectively (with 1 for sway motion, 2 for heave motion and 3 for roll around an axis perpendicular to the plane of the flow). The hydrodynamic coefficients are non-dimensionalised according to $CM_{ij} = a_{ij} / \rho AB^2$ and $CA_{ij} = b_{ij}\sqrt{B/2g} / \rho AB^2$. A is the area coefficient. A 10000 nodes-grid (100 on the body) was used for most of the computations with a time step of 0.01 s. For lower motions frequencies a 23000 nodes-grid (230 nodes on the body) was required and the time step was 0.005 s.
Added roll moment of inertia (fig. 7) is well-predicted and close from Vugts experiments. However the damping coefficient in roll is highly over-predicted (fig. 8) for all motion frequencies. On the contrary the mass and damping coupling coefficients are in good agreement with Vugts experiments and perfect fluid computations (fig. 9 and 10) except for the mass coefficient for the lowest computed

frequency.
These first results are satisfying and show the interest of viscous-flow computations for such quite complex flows. However other computations must be undertaken particularly for the calculation of the damping coefficient in sway and roll. More refined grids in the vicinity of the body will be used to try to compute viscous effects (particularly vortices shedding near solid walls and corners of the body in motion) with more accuracy and should improve present results.

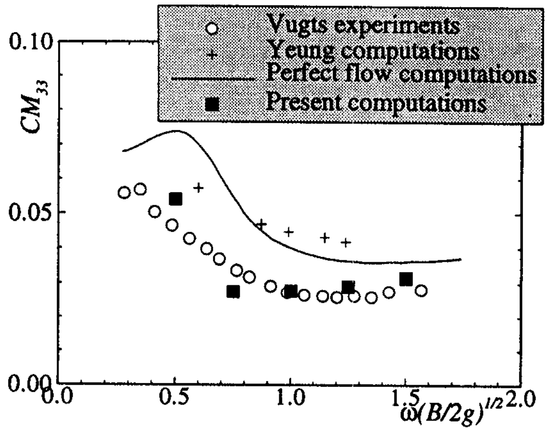

Fig. 7 : Added mass moment of inertia in roll (for a rectangle)

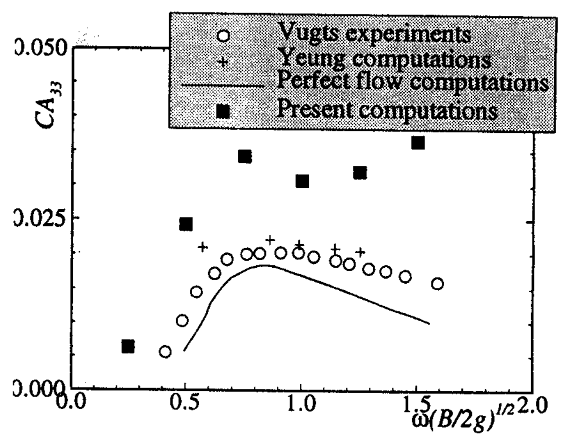

Fig. 8 : Damping coefficient in roll (for a rectangle)

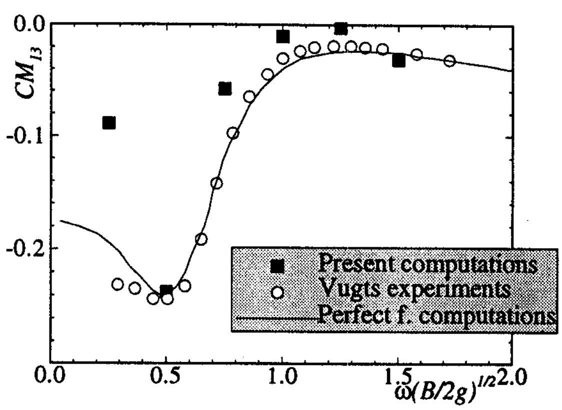

Fig. 9 : Mass coupling coefficient of roll in sway (for a rectangle in roll)

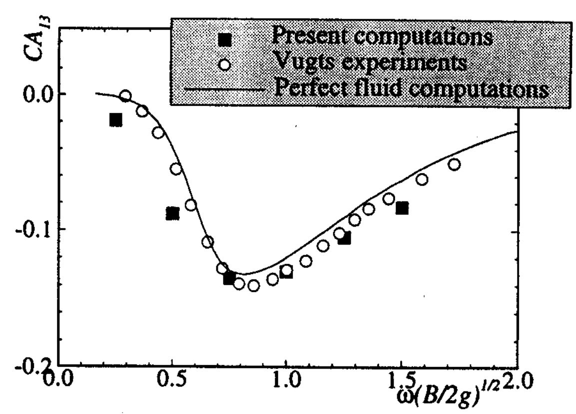

Fig. 10 : Damping coupling coefficient of roll in sway (for a rectangle in roll)

REFERENCES

[1] B. Alessandrini, G. Delhommeau, *Preprints of 21st Symposium on Naval Hydrodynamics (Tuesday Sessions),* Trondheim, Norway, pp. 40-55, 1996.
[2] L. Gentaz, B. Alessandrini, G. Delhommeau, *10th WWWFB,* Oxford, pp. 77-80, 1995.
[3] L. Gentaz, B. Alessandrini, G. Delhommeau, *15th ICNMFD,* Monterey, pp. 112-113, 1996.
[4] P.M. Guillerm, *Rapport de DEA*, Ecole Centrale de Nantes, 1996.
[5] A. Papanikolaou, H. Nowacki, *13th Symp. on Naval Hydrodynamics*, pp. 303-331, 1984.
[6] *Proceedings of CFD Workshop,* Tokyo, 1994.
[7] F. Tasaï, W. Koterayama, *Reports of Research Inst. for Applied Mech.*, Kyushu Univ., vol. 23, no. 77, 1976.
[8] H.A. Van der Vorst, *J. Sci. Stat. Comp.*, vol. 13, 1992.
[9] J.H. Vugts, *International Shipbuilding Progress,* vol. 15, pp. 251-276, 1968.
[10] S. Yamashita, *Journal Society Naval Architecture of Japan*, vol. 141, pp. 61-69, 1977.
[11] R. Yeung, C. Cermelli, S.W. Liao, *Preprints of 21st Symposium on Naval Hydrodynamics (Tuesday Sessions),* Trondheim, Norway, pp. 69-86, 1996.

DISCUSSION

Yeung R.W.: These are encouraging calculations along a boundary-fitted coordinate full N-S solver of Yeung & Ananthakrishnan (1992, J. Eng. Math.). However, I do not believe the calculations you presented have sufficient precision. Fig. 3 shows that the total damping with viscosity is <u>less</u> than the <u>inviscid</u> solution. Our recent FSRVM method (Ref. 11) has been validated by comparing vorticity structures with DPIV measurements. As mentioned in Ref. [11], we have some doubts that Vugt's expt. data for added inertia is correct. Your calculations for CM_{33} may well be off by as much your damping is unreasonably high in Fig. 8. I hope you can be successful in tracking down the problems.

Gentaz L., Alessandrini B., Delhommeau G.: The originality of the present method consists in solving only one fully-coupled linear system for the velocity, pressure and free surface elevation unknowns at each iteration. This method first implemented by B. Alessandrini (1995, Numerical Method in Laminar and Turbulent Flows, Atlanta, vol. IX, part 1, pp. 1173-1184) allows complete free-surface boundary conditions to be taken into account and an efficient and fast convergence during nonlinear process (figure 2) contrary to weakly-coupled methods as this one described by Yeung and Ananthakrishnan in Journal Engineering Mathematics, 1992, or others (1994, proceedings of CFD Workshop, Tokyo).

We do not believe that differences between Navier-Stokes computations and experiments for the figure 3 (damping coefficient for a circular cylinder in forced heave motion) are significant. In this case, part of viscous forces seems negligible. In our opinion, these differences can be explained by Fourier analysis of forces acting on the body or gaps in the manual plot of experimental data but are not due to an insufficient precision in the computation.

Concerning hydrodynamic coefficients for a rectangular cylinder in forced roll motion, damping coefficient is actually largely overestimated and its computation must be improved. For the added inertia coefficient CM_{33} we hope you can provide other experimental data to confirm your hypothesis and computations.