A Comparison of Two Rankine-source Panel Methods for the Prediction of Free-surface Waves

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Introduction

Dispersion (wavelength error) and damping (amplitude error) have been investigated analytically for two Rankine-source panel methods in two dimensions. The flow is assumed steady incompressible and the Kelvin free-surface boundary condition is applied. The first method uses an upwind four-point operator (Dawson operator) on the free-surface for the velocity derivative in the streamwise direction and to enforce the radiation condition (Kim, K. J. 1989) and (Janson, C. E. 1996A), while the second method uses an analytical expression for the velocity derivative together with a collocation point shift one panel length upstream to satisfy the radiation condition (Jensen, P. S. 1987), (Jensen, G., et. al 1988), (Kim, B. K. 1990) and (Janson, C. E. 1996A). Both first order panels (flat panels, constant source strength) and higher order panels (parabolic panels, linearly varying source strength) have been investigated for the first method while the analysis is restricted to first order panels for the analytical method. The source panels are allowed to be positioned either on the free-surface (standard method) or at a distance above the free-surface (raised panel method). The collocation points on the free-surface may be located at the same longitudinal position as the panel centres (standard method) or they may be shifted upstream relative to the panel centres.

The two methods have also been compared numerically for a three-dimensional flow using the Series 60 CB=0.60 hull to verify the conclusions from the two-dimensional analysis. A grid dependence study for the free-surface was performed for non-linear computations and the grid convergence is compared for the wave profile at a longitudinal cut. The residual of the free-surface boundary condition is also compared for the two methods.

Analysis of Dispersion and Damping

Numerical dispersion and numerical damping occur when the continuous potential flow problem is discretized in a numerical method. Both the discretization of the free-surface source distribution and the introduction of numerical operators to compute the velocity derivative in the free-surface boundary condition introduce errors to the method. A systematic methodology for this type of analysis is described in detail in (Sclavounos, P. D. and Nakos, D. E. 1988) and in (Raven, H. C. 1996) and it is used for the present analysis. The method investigates the properties of the numerical method after transformation to the Fourier space.

A two-dimensional continuous source distribution is assumed at a distance z_{fs} above the undisturbed free-surface level and the Fourier transforms of the induced velocity and velocity derivative at a point on the undisturbed free-surface due to the source distribution are introduced into the Kelvin free-surface boundary condition. The Kelvin free-surface boundary condition can then be formulated to include an operator that relates the induced vertical velocity on the free-surface to the right hand side of the equation which is assumed to be known.

The source distribution is for the first order numerical method discretized into flat panels of uniform size Δx having a constant source strength and the velocity derivative in the streamwise direction is computed using an upwind four-point numerical operator. The source panels are located a distance $z_{fs} = \alpha\Delta x$ above the undisturbed free-surface level and all collocation point are allowed to be shifted a distance $\gamma\Delta x$ upstream of the panel centres. The Fourier transform of the induced velocities at a collocation point on the free-surface and the Fourier transform of the four-point operator are introduced into the Kelvin free-surface boundary condition and as in the continuous case an operator for the vertical velocity can be formulated.

For the first order analytical method the Fourier transform of the analytical expression for the velocity derivative in the streamwise direction replaces the Fourier transform of the four-point numerical operator in the Kelvin condition and an operator for the vertical velocity can again be formulated.

The operator for the higher order numerical method is similar to the first order numerical method but it includes contributions from the curvature of the panel and from the linear source variation.

The dispersion and damping for the discretized method can now be investigated from plots of the real and imaginary parts of a function $Lh(s)$ which is included in the non-dimensionalized form of the operator for the vertical velocity. The difference between the operators for the discretized methods and the operator for the continuous source distribution is shown as the difference between the non-dimensional wave number s and the function $Lh(s)$. The principle for the analysis is shown in figure 1A where on the abscissa $s = 0$ means an infinite number of panels per wavelength and $s = 0.5$ means two panels per wavelength. The intersection between a horizontal line $1/(2\pi \cdot Fn_{\Delta}{}^2)$ where Fn_{Δ} is the panel Froude number and the real part of the function $Lh(s)$ gives the principal far-field wave number found by the discretized method and the difference between this wave number and s gives the dispersion for the method. The intersection between the imaginary part of $Lh(s)$ and a vertical line at the principal far-field wave number indicates the damping of the discretized method. A spurious wave numbers may in some cases be found by the discretized method if a second intersection between the real part of $Lh(s)$ and the horizontal line exists.

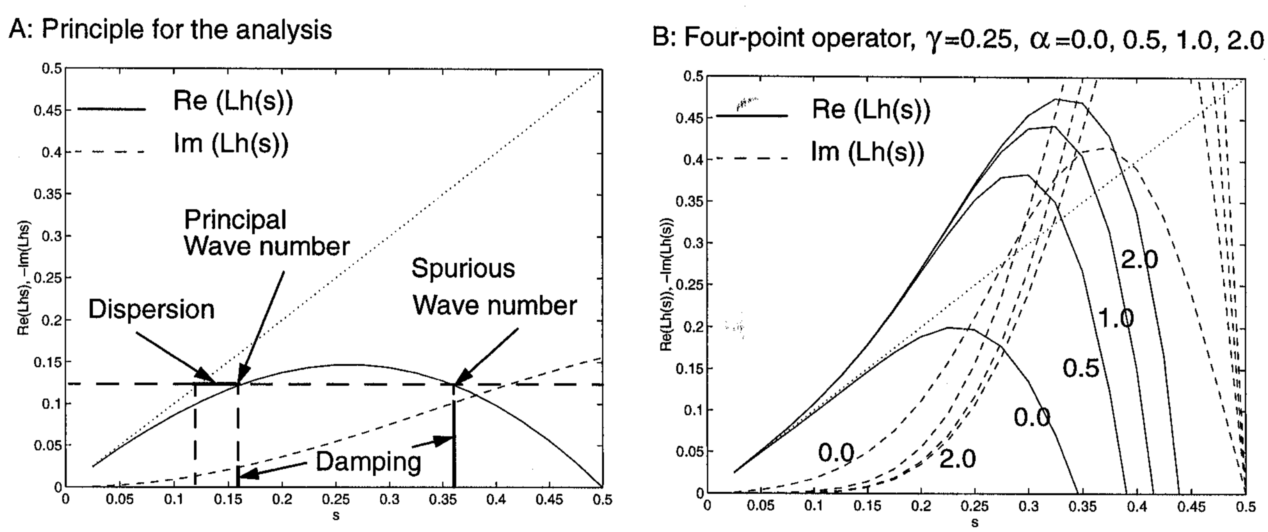

Figure 1 Interpretation of the real and imaginary parts of $Lh(s)$

Different positions of the source panels and the collocations points were investigated and as one example figure 1B shows the influence of the distance between the source panels and the free-surface, α, for a collocation point shift, γ, of one quarter of a panel length and the four-point operator. It can be seen that the damping is reduced as the distance, α, is increased and that the dispersion is small for the principal wave number. The analytical method, figure 2A, shows very small dispersion in a large wave number range as the distance, α, is increased. No damping is present for the analytical method. The analysis shows that there is only a very small difference between first and higher order panels if the source panels are raised a distance above the free-surface. Details of the present analysis are described in (Janson, C. E. 1996B).

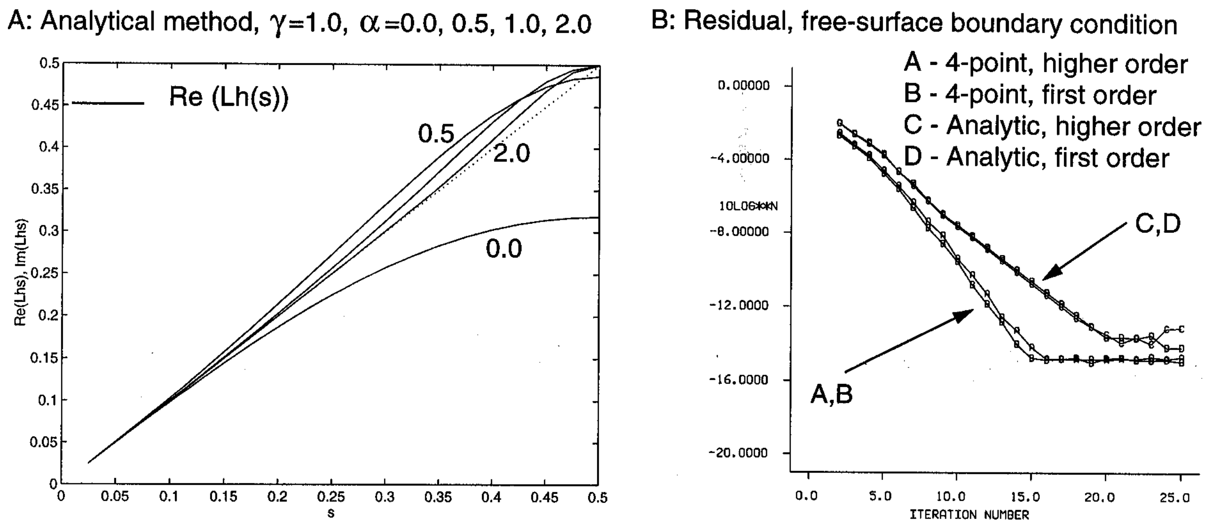

Figure 2

Numerical comparison for the Series 60 hull, Fn = 0.316

Non-linear computations were carried out for the Series 60 hull using both the four-point operator and the analytical method. The solution method for the non-linear problem is to linearize the free-surface boundary condition around a known base solution and solve the problem in an iterative manner. In each iteration the problem is linearized with respect to the solution from the previous iteration. The first iteration is started from a zero Froude number flow where a Neuman condition is applied on the free-surface. In the first linear solution the linearized free-surface boundary conditions are applied on the undisturbed free-surface and are in the following iterations moved to the wavy free-surface computed in the previous iteration. The source panels were raised about one panel length above the wavy free-surface.

The iteration history of the max residual for the combined free-surface boundary condition is in figure 2B shown for the four-point operator and the analytical method both for first and higher order panels using 25 panels per fundamental wave length. It can be seen that the residuals are reduced to very small values for both methods but the analytical method shows a slightly slower convergence. Note the logarithmic scale for the residual. There is only a very small difference between first and higher order panels.

A grid dependence study was carried out for the number of panels on the free-surface using higher order panels. In figure 3 the wave profile is plotted at 0.0755*Lpp aside of the centre line for 5, 10, 15, 20, 25 and 30 panels per fundamental wave length and as can be seen the wave profile converges towards the measured profile as the number of panels is increased. A slightly faster convergence is noted for the analytical method but the solution is still not grid independent for 30 panels per wave length.

It is interesting to note that the same conclusions can be made from the Fourier analysis in two dimensions and the numerical computation in three dimensions. In both cases the analytical method shows smaller dispersion than the four-point operator and the amplitude converges faster due to smaller damping. But, for the large number of panels used in applied computations there is only a small difference between the analytical method and the four-point operator. Only a minor difference was obtained between first and higher order panels in the Fourier analysis and this very small difference occurred for the wave profile also in the numerical computations.

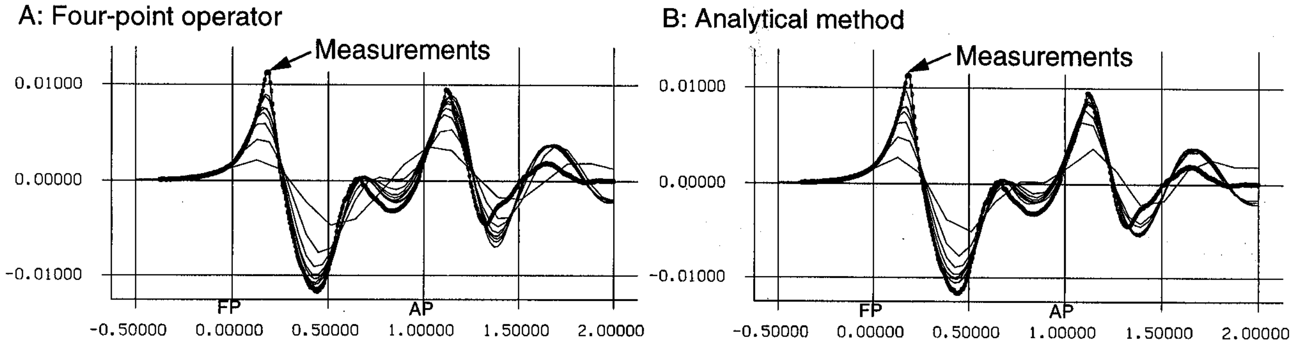

Figure 3 Wave profile 0.0755*Lpp aside of the centre line

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