WAVE IMPACT ON ELASTIC PLATES

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The plane unsteady problem of wave impact onto an elastic beam of finite length is considered. Initially a wave crest touches the beam at its central point (central impact) or at its edge (edge impact). Then the liquid hits the beam from below at a constant velocity. The impact process can be divided into two stages: in the first stage (impact stage) the beam is wetted only partially, in the second stage (penetration stage) the beam is totally wetted and continues to interact with the liquid. The impact stage is considered here only. At this stage the hydrodynamic loads are very high and are dependent on both the velocity of contact region expansion and the beam deflection. The problem is coupled. The dimension of the contact region is unknown in advance and has to be determined together with the liquid flow and the beam deflection. Here the beam deflection is of main interest that is why the numerical method to treat the problem is designed in such a way that the elastic characteristics can be effectively evaluated, but not the hydrodynamic ones.

We shall determine the beam deflection, the bending stresses in the beam and the duration of the impact stage under the following assumptions: (1) the beam deflection is governed by the Euler beam equation; (2) the beam is connected with the main structure bu rotatory springs at the beam ends; (3) the liquid is ideal and incompressible; (4) the air influence on the impact, and both external mass forces and surface tension, are negligibly small; (5) the wave profile near impact point can be approximated by parabolic contour with the initial radius of curvature at the top R; (6) the beam length $2L$ is much less then R; (7) dimension of the contact region grows with time. Assumption (6) implies that the deformations of both the wave profile and the beam are of $O(\varepsilon)$ as $\varepsilon = L/R \to 0$ at the impact stage. Moreover, in the leading order the boundary conditions and the equations of the liquid motion can be linearized with the relative accuracy $O(\varepsilon)$.

1 Formulation of the problem

The central impact is considered in this section only, the edge impact is treated in a similar way. In order to formalize the derivation of the model describing the first stage of the impact, the following scales are introduced: L as the length scale, $L^2/(RV)$ as the time scale, L^2/R as the displacement scale, V as the velocity scale, $\rho V^2(R/L)$ as the pressure scale, where ρ is the liquid density. The original equation of liquid flow, the boundary and initial conditions and the Euler beam equation, which are written in the non-dimensional variables, contain three parameters $\varepsilon, \alpha, \beta$ where $\alpha = M_B/(\rho L)$, $\beta = (EJ)/(\rho L R^2 V^2)$. Here M_B is the beam mass per unit length, E is the elasticity modulus, J is the inertia momentum of the beam cross-section. The parameter ε can be referred to as the parameter of linearization.

Taking formally $\varepsilon = 0$ in the original equations and the boundary conditions, we obtain the following boundary-value problem with respect to velocity potential $\varphi(x,y,t)$ and the beam deflection $w(x,t)$:

$$\varphi_{xx} + \varphi_{yy} = 0 \qquad (y < 0), \tag{1}$$

$$\varphi_y = -1 + w_t(x,t) \qquad (y = 0, |x| < c(t)), \tag{2}$$

$$\varphi = 0 \qquad (y = 0, |x| > c(t)), \tag{3}$$

$$\varphi \to 0 \qquad (x^2 + y^2 \to \infty), \tag{4}$$

$$p(x,y,t) = -\varphi_t(x,y,t), \tag{5}$$

$$\alpha \frac{\partial^2 w}{\partial t^2} + \beta \frac{\partial^4 w}{\partial x^4} = p(x,0,t) \qquad (|x| < 1, t > 0), \tag{6}$$

$$w = 0, \qquad w_{xx} \pm k w_x = 0 \qquad (x = \pm 1), \tag{7}$$

$$w = w_t = 0 \qquad (|x| < 1, t = 0), \tag{8}$$

The bending stress distribution $\sigma(x,t)$ is given in the dimensionless variables as $\sigma(x,t) = w_{xx}(x,t)$, with its scale $Eh/(2R)$, where h is the maximal thickness of the beam. The positions of the contact points are described in the symmetrical case by the only function $c(t)$. Despite the fact that both the equations of motion and the boundary conditions are linearized, the problem remains nonlinear as $c(t)$ is unknown. The 'spring' conditions (7) were introduced by Kvålsvold and Faltinsen (1993), k is the nondimensional spring stiffness.

The formulation of the problem (1) - (8) is not complete. It must be added by an equation for the dimension of the contact region. Usually the equation derived by Wagner (1932) is used, but this equation is difficult to incorporate into a numerical scheme. We use here the equation suggested by Korobkin (1996). The equation is, in fact, a modification of the classical Wagner condition. It is

$$\int_0^{\pi/2} y_b[c(t)\sin\theta, t]d\theta = 0, \tag{9}$$

where the function $y_b(x,t)$ describes the shape of the beam with respect to the initial position of the free surface. In the present case, $y_b(x,t) = x^2/2 - t + w(x,t)$, equation (9) gives

$$t = \frac{1}{4}c^2 + \frac{2}{\pi}\int_0^{\pi/2} w[c(t)\sin\theta, t]d\theta. \tag{10}$$

The problem (1) - (10) is solved with the help of the normal mode method. This method leads to infinite system of ordinary differential equations with respect to the principal coordinates of the beam deflection $w(x,t)$.

2 Normal mode method

Within the framework of this method the beam deflection $w(x,t)$ is sought in the form

$$w(x,t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(x). \tag{11}$$

Here $\psi_n(x)$ are non-trivial solutions of the homogeneous boundary-value problem

$$\frac{d^4\psi_n}{dx^4} = \lambda_n^4 \psi_n \qquad (|x| < 1),$$

$$\frac{d^2\psi_n}{dx^2}(\pm 1) \pm k\frac{d\psi_n}{dx}(\pm 1) = 0, \qquad \psi_n(\pm 1) = 0,$$

where λ_n are the corresponding eigenvalues. Moreover, the eigenfunctions $\psi_n(x)$ satisfy the orthogonality condition

$$\int_{-1}^{1} \psi_n(x)\psi_m(x)\, dx = \delta_{nm},$$

where $\delta_{nm} = 0$ when $n \neq m$ and $\delta_{nn} = 1$. Substitution of equations (11) into (6) - (8) and solution of the hydrodynamical part of the problem (1) - (5) provide the following system of ordinary differential equations

$$\frac{d\vec{a}}{dt} = (\alpha I + \kappa S)^{-1}(\beta D\vec{d} + \vec{f}), \tag{12}$$

$$\frac{d\vec{d}}{dt} = -\vec{a}. \tag{13}$$

Here $\vec{a} = (a_1, a_2, a_3, \ldots)^T$, $\vec{d}$ is the vector $\vec{d} = (d_1, d_2, d_3, \ldots)^T$, $d_n = (\beta\lambda_n^4)^{-1}(\alpha\dot{a}_n + b_n)$, $\vec{f} = (f_1(c), f_2(c), f_3(c), \ldots)^T$, I is the unit matrix, D is the diagonal matrix, $D = \text{diag}\{\lambda_1^4, \lambda_2^4, \lambda_3^4, \ldots\}$. The right-hand side of the system (12), (13) depends on $\vec{a}$, $\vec{d}$, c, but not on t. Therefore, it is convenient to take c as a new independent variable, $0 \leq c \leq 1$. Differential equation for $t = t(c)$ follows from (10) and has the form

$$\frac{dt}{dc} = Q(c, \vec{a}, \dot{\vec{a}}), \tag{14}$$

where

$$Q(c, \vec{a}, \dot{\vec{a}}) = \frac{c + (4\kappa/\pi)(\vec{a}, \vec{\Gamma}_c(c))}{2 - (4\kappa/\pi)(\dot{\vec{a}}, \vec{\Gamma}(c))}, \tag{15}$$

$$\Gamma_n(c) = \int_0^{\pi/2} \psi_n(c\sin\theta)d\theta, \qquad \Gamma_{nc}(c) = \int_0^{\pi/2} \psi_n'(c\sin\theta)\sin\theta d\theta.$$

Multiplying equations of system (12), (13) by dt/dc and taking (14) into account, we get

$$\frac{d\vec{a}}{dc} = \vec{F}(c, \vec{d})Q(c, \vec{a}, \vec{F}(c, \vec{d})), \tag{16}$$

$$\frac{d\vec{d}}{dc} = -\vec{a}Q(c, \vec{a}, \vec{F}(c, \vec{d})), \tag{17}$$

where $\vec{F}(c, \vec{d}) = (\alpha I + \kappa S(c))^{-1}(\beta D\vec{d} + \vec{f}(c))$. The initial conditions are

$$\vec{a} = 0, \qquad \vec{d} = 0, \qquad t = 0 \qquad (c = 0). \tag{18}$$

The system (16), (17) is suitable for numerical evaluation. Indeed, for small times we have $c(t) = O(t^{1/2})$, $w(x,t) = O(t^{3/2})$, $w_t = O(t^{1/2})$, $w_{tt} = O(t^{-1/2})$, and therefore, one cannot start numerical calculations for system (12) - (14) with homogeneous initial conditions. Difficulties with initial conditions for system of differential equations with respect to principal coordinate $a_n(t)$ and their derivatives $\dot{a}_n(t)$, where the time t is taken as the independent variable, are described by Kvålsvold end Faltinsen (1993). On the other hand, $t = O(c^2)$, $w = O(c^3)$, $w_t = O(c)$, $w_{tt} = O(c^{-1})$ as $c \to 0$, and there are no problems with initial conditions for system (14) - (17).

Initial value problem for the edge impact is similar to (14) - (18) but elements of the system are different. Moreover, the derivative dt/dc can become large (the speed of contact region expansion is small) at some moment t_1 of the impact stage. In this case we need to return to system (12), (13) as $t \approx t_1$.

3 Numerical results

The initial-value problem (14) - (18) is solved numerically by the fourth-order Runge-Kutta method with uniform step Δc. The condition that the numerical scheme is stable was derived. The step Δc has to decrease as $O(N^{-2})$ if the number of modes N taken into account increases.

Main part of the calculations were performed for simply supported beam ($k = 0$). Central impact was analysed for the case $L = 0.5$m, $R = 10$m, $h = 2$cm, $E = 21 \cdot 10^{10}$H/m^2, $V = 3$m/s, $\varrho = 1000$kg/m^3, $\varrho_b = 7850$kg/m^3, $b = 0.5$m, where ϱ_b is the beam density and b is the beam width. This gives $\alpha = 0.314$, $\beta = 0.311$. The number of 'dry' modes N taken into account is equal to 15. The speed of the contact region expansion was found to be positive and bounded as $c > 0$. Numerical results are compared with both the Wagner approach and the Karman approach for the estimation of the wetted size of the beam. It was found that the simplified approaches do not provide appropriate approximations of the speed of the contact region expansion. Bending stress peaks close to the end of the impact stage and its maximum value is 140N/mm^2. One-mode approximation, $N = 1$, does not give correct information about evolution of the bending stresses, but maces it possible to estimate their maximal value.

Edge impact is analysed for $\alpha = 0.157, \beta = 0.04$. It was revealed that the speed of the contact region expansion is not uniform and takes its minimal positive value at distance $1.2L$ from the impact point. After that the speed grows beyond all bounds before the beam is totally wetted. This means that acoustic effects have to be taken into account at the final phase of the impact stage. The hydrodynamic force tends to infinity as $dc/dt \to \infty$, where $c(t)$ is the dimension of the contact region. This effect, which was not revealed for central impact, is referred to as blockage. It is assumed that the parameter β, which is the dynamical rigidity of the beam, is responsible for this effect. The calculations were performed for $\beta = 0.02, \beta = 0.04$ and $\beta = 0.06$. It was found that small variations of β lead to significant changes of the process evolution. When a half of the beam is wetted, the speed dc/dt becomes negative and the wetted area starts to decrease for $\beta = 0.02$. This phenomenon may be responsible for cavitation effects and the beam ventilation. In the case $\beta = 0.06$ the speed dc/dt is positive and bounded as $c > 0$, and the hydrodynamic force is bounded at the impact stage. Comparison of central impact and edge impact for the same values of the parameters α, β shows that at the end of the impact stage the beam deflections and the distributions of the bending stresses differ significantly.

If $k \neq 0$ the calculations were performed for the central impact only. It is shown that the conditions of simply supported beam, $k = 0$, can be used to estimate bending stresses near the beam centre.

The numerical results demonstrate that at least five modes have to be taken into account to derive the initial data for the penetration stage of the impact. It should be noted that the present approach does not require supercomputers. The computer program, which was used for the calculations with 15 modes, takes about 30 minutes of computer time in a PC-486(66MHz) computer.

References

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