# Wave Breaking Simulation Around a Lens-shaped Mast By a V.O.F. Method

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Abstract

Using the CFD code EOLE, described in [3], numerical computations of steady waves around a partially immersed vertical lens-shaped mast have been done. These computations were realized using a "Volume of Fluid" method coupled with the Euler or Reynolds-averaged Navier-Stokes equations. The V.O.F. technique is often used to modelize unsteady phenomena. In [3], this technique was adapted to treat steady problems as non-linear waves induced by a submerged hydrofoil placed in a uniform flow. The purpose of this paper is to expose the way the V.O.F method was extended to compute fluid flows in body fitted grids, and to highlight the interesting features of this method applied to a strong non linear wave pattern around a 3D lens-shaped mast.

## 1 Introduction

Nowadays, as shown in [5], two major groups of methods are used to determine the wave resistance of structures moving at a constant speed. First, potential methods, based on a Dawson technique are more and more used by shipyards or tank facilities as industrial tools. However, the limits of the potential theory do not allow to treat viscous interactions, nor high non linear free surface. Therefore, important attention is payed to implement free surface algorithm in general Navier-Stokes or Euler solvers. A common feature of these methods is to follow the free surface evolutions by mean of mesh deformation. As the general evolution of ship goes to high speed vehicule, this method may be unadapted to deal with bow waves where wave breaking occur or whith transom stern flows. In the EOLE code , developed since 1990, the resolution of Euler or Navier-Stokes equations is coupled with the "Volume of Fluid" method for the tracking of the interface. This technique is very efficient for very complicated free surface unsteady phenomena like jets, bubble collapse, sloshing, cavitation. It is expected that this method may be used efficiently for steady problems like the wave resistance one. The algorithm, initialy developed for unsteady flows has been adapted to steady problems and its application in curvilinear coordinates is presented. A first serie of results around a vertical lens-shaped mast moving at Froude numbers between 0.4 and 1.2 is presented, and shows the interest of the method.

## 2 Theory

The steady Euler equations for incompressible fluids are solved using a pseudo-compressibility method [2]. This is an iterative method which consists in introducing derivatives with respect to a fictitious time called pseudo-time $\tau$, into the continuity and the momentum equations as follows :

$$\frac{\partial \tilde{\rho}}{\partial \tau} + \rho \operatorname{div} \vec{u} = 0, \qquad \frac{\partial \tilde{\rho} \vec{u}}{\partial \tau} + \operatorname{div} \left( \rho \vec{u} \otimes \vec{u} + p \bar{I} \right) = \vec{0}$$

Where $\tilde{\rho}$ is a pseudo-density, $\rho$ the fluid density, $\vec{u}$ the fluid velocity, $p$ the pressure and $\bar{I}$ the identity tensor. This system is closed using a relation linking the pressure and the pseudo-density and called pseudo-law of state : $p = G(\tilde{\rho})$. The steady solution is obtain as the asymptotic limit of the pseudo-transient solution of this pseudo-unsteady system (PUS) when the pseudo-time goes toward infinity. The main features of the numerical method for pseudo-time integration are a finite-volume method based on a space-centered scheme, second and fourth order artificial viscosity terms, five stages Runge-Kutta pseudo-time stepping and implicit residual smoothing.

## 2.1 Steady VOF method

The V.O.F. method implemented in the EOLE code is based on the technique, previously proposed by Hirt and Nichols [4]. The fraction of fluid in each cell of the discretization mesh is represented by a function $F$ whose value can vary from zero to one while the cell is respectively empty or full of fluid. The free surface is contained by the cells with $F$ values between zero and one. For a steady problem, the evolution of the $F$ field is governed by the following transport equation in which the time is replaced by the pseudo-time $\tau$ :

$$\frac{\partial F}{\partial \tau} + \frac{\partial uF}{\partial x} + \frac{\partial vF}{\partial y} + \frac{\partial wF}{\partial z} = 0 \tag{1}$$

where $(x, y, z)$ is the cartesian system, $(u, v, w)$ the cartesian components of the velocity. The evolution of the $F$ function is made from fluxes calculations (based on "donor-acceptor method" [4]) through all the faces of each cell. This algorithm is implemented in curvilinear system $(\xi, \eta, \zeta)$ via a coordinates transformation of jacobian $J = \frac{\partial(\xi,\eta,\zeta)}{\partial(x,y,z)}$. The last equation (1), for a cell $\Omega$ (of faces $\delta\Omega_{\xi+}$, $\delta\Omega_{\xi-}$, $\delta\Omega_{\eta+}$, $\delta\Omega_{\eta-}$, $\delta\Omega_{\zeta+}$, $\delta\Omega_{\zeta-}$ and volume $V_\Omega$) in the new coordinates system $(\xi, \eta, \zeta)$ is given by :

$$\frac{dF}{d\tau} + \frac{1}{\Delta\xi\Delta\eta\Delta\zeta}\left(\int_{\delta\Omega_{\xi+}} f\hat{u}d\eta d\zeta - \int_{\delta\Omega_{\xi-}} f\hat{u}d\eta d\zeta + \int_{\delta\Omega_{\eta+}} f\hat{v}d\xi d\zeta - \int_{\delta\Omega_{\eta-}} f\hat{v}d\xi d\zeta + \int_{\delta\Omega_{\zeta+}} F\hat{w}d\xi d\eta - \int_{\delta\Omega_{\zeta-}} f\hat{w}d\xi d\eta\right) = 0 \tag{2}$$

where $f(\xi, \eta, \zeta, \tau)$ is a continue fonction defined at each point $(\xi, \eta, \zeta, \tau)$ of the fluid domaine and whose values are contained between 0 and 1. $\hat{u}$, $\hat{v}$ and $\hat{w}$ are the modified contravariant velocity components which can be expressed as follow :

$$\hat{u} = \vec{U}.gr\vec{a}d\xi\frac{J}{\bar{J}}\ , \quad \hat{v} = \vec{U}.gr\vec{a}d\eta\frac{J}{\bar{J}}\ , \quad \hat{w} = \vec{U}.gr\vec{a}d\zeta\frac{J}{\bar{J}}\ .$$

where $\bar{J}$ is the jacobian of the coordinates transformation estimated at each face of the cell. For example at the face $\delta\Omega_{\xi+}$ :

$$\vec{n}\,dS = \frac{1}{\bar{J}}\overrightarrow{\text{grad}}\xi d\eta d\xi$$

where $\vec{n}dS$ is the normal vector of the face in the cartesian coordinates. The expression for the calculation of the fluxes originaly proposed by Hirt and Nichols [4] is now written, in the curvilinear system, as follow :

$$\Delta F = \min(F_{AD}|\hat{U}|\Delta\tau + CF, F_D V_\Omega) \quad \text{with} \quad CF = \max[(1 - F_{AD})|\hat{U}|\Delta\tau - (1 - F_D)V_\Omega, 0]$$

where $F_A$ and $F_D$ are the volumes of fluid contained in the "acceptor" and the "donor" cell respectively (see figure 1). $F_{AD}$ can be both $F_A$ or $F_D$ depending of the mode "donor" or "acceptor" determined by the slope of the free surface which is calculated in the curvilinear system using the gradient of the V.O.F. The "acceptor" mode is adapted to the case of a free surface moving parallel to its normal vector, and the "donnor" mode is adapted to the case of a free surface moving perpendicular to its normal vector. Writing all the V.O.F. algorithm in curvilinear coordinates allow fluid computations to be realized in body-fitted grids.

As the scheme used to discretize the previous equation (2) is explicit in pseudo-time, a $CFL$ criteria is added to the one of the conservative equations (continuity and momentum equations) to force the free surface not move throught more than a part of a cell during a pseudo-time step :

$$\frac{U_j\Delta\tau}{\Delta x_j} < CFL_{Vof}$$

where $x_j$ and $U_j$ are respectively $x$, $y$, $z$ and $u$, $v$, $w$ for $j = 1, 2, 3$. The pseudo-time step value is determined by taking the minimum value between the one given by the previous relation and the value calculated with the $CFL$ criteria on the conservative equations.

# 3 Numerical results

Some numerical results about the wave generation induced by a 3D lens-shaped vertical mast (length : 0.3375 m and maximum thickness : 0.045 m) partially immersed (0.4 m) are presented. Computations were realized for several values of the Froude number from 0.4 to 1.4 in order to observe the evolution of the wave resistance, and to make comparisons with existing results. On figure (2) the values of the wave resistance obtained using EOLE, are compared with the ones measured during the experiments (performed at "Ecole Centrale de Nantes" [1]) and those computed with the Dawson method (REVA code, potential linear and non linear theory) by Delhommeau *et al* [1]. At intermediate values of the Froude Number (between 0.4 and 1) EOLE's results are closer to the experimental ones than those given by the REVA code. The better representation of the non-linear free surface arround the body is the explanation. For higher values of the Froude number (between 1 and 1.4) the wave resistance values of both codes are in agreement with the experiments because the free surface deformation along the body is smoother. For the free surface location, the experiments show that breaking appears when the Froude number value is larger than 0.5. Such phenomenon can be qualitatively represented by steady Volume of Fluid computations. On figures 3, 4 and 5, the free surface position along the body and the plane of symmetry obtained with EOLE are compared to the one given by REVA for the values of the Froude number equal to 0.4, 0.6 and 1 respectively. Note that the free surface calculated by the V.O.F. method is represented by the line V.O.F. = 0.5, using the post-prossessing logiciel Tecplot (Amtec Engineering, Inc.). A more precise representation should be preferable. The extrema computed with EOLE have a larger amplitude than the ones given by REVA but their locations along the body are in agreement with each other. As the pseudo-time is a non-physical iterative variable, the free surface instabilities (like droplets) appearing on the figures (4) and (5) give qualitative informations about the breaking (for example the Froude number of transition), but can not represent the unsteady evolution of the interface. To be more accurate in representing such phenomenon some unsteady computations can be performed, but they need more CPU time. The calculations presented here, were done on half a domain (because of symmetry reasons) discretized by a 230000 cells mesh on 2000 pseudo-time iterations. This kind of computations have been realized on a Digital DEC ALPHA 600/266 station (428 SPECfp92) in 37 hours.

All the results presented here, show the ability of the V.O.F. method to describe non linear free surface phenomena. Further developments, especially to improve the representation of the free surface, will be carried on.

# 4 Acknowledgments

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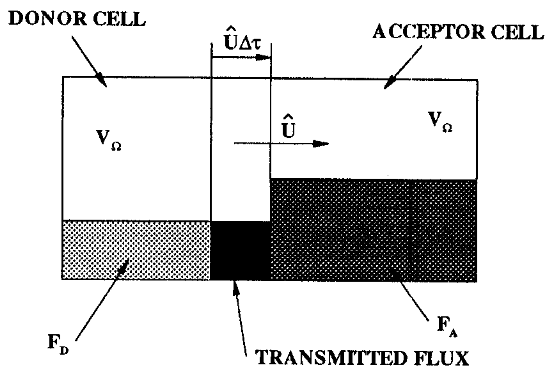


Figure 1: V.O.F. method : donor and acceptor cells.

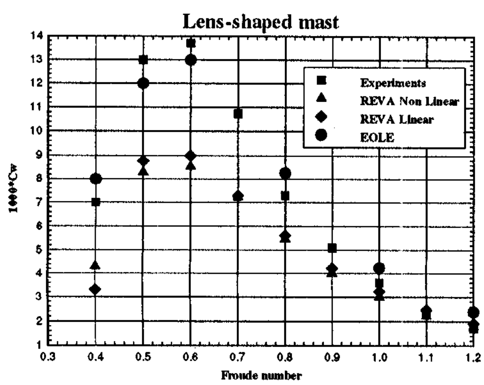


Figure 2: Lens-shaped mast : wave resistance coefficients.

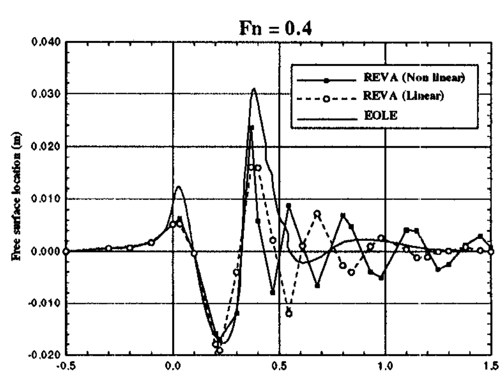


Figure 3: Lens-shaped mast : free surface elevation $Fn = 0.4$.

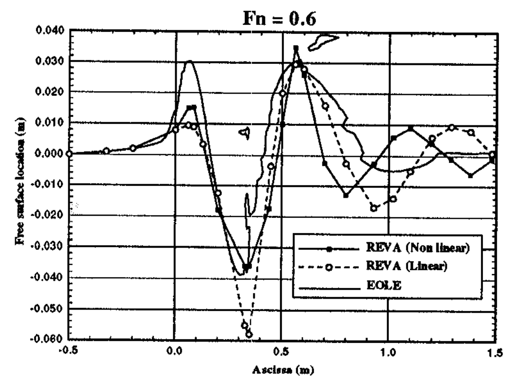


Figure 4: Lens-shaped mast : free surface elevation $Fn = 0.6$.

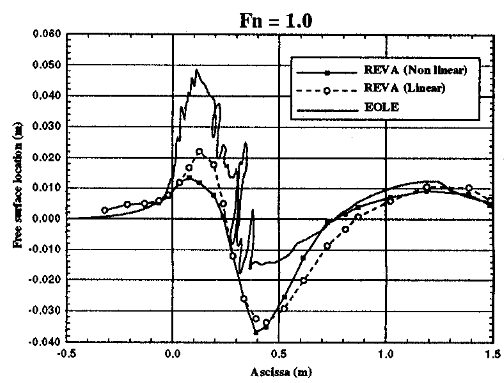


Figure 5: Lens-shaped mast : free surface elevation $Fn = 1$.

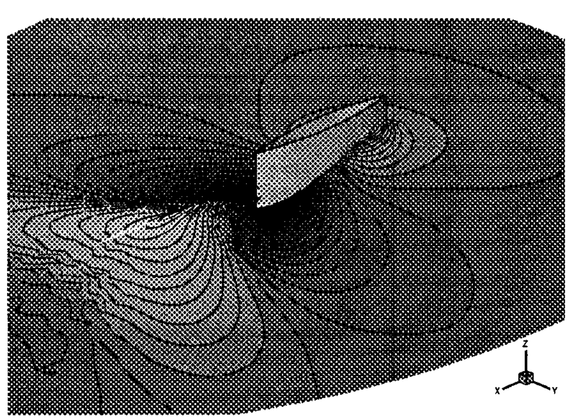

Figure 6: Lens-shaped mast : wave pattern $Fn = 0.6$