Hydroelastic Response of a Floating Thin Plate in Very Short Waves

M. Ohkusu & Y.Nanba
Research Institute for Applied Mechanics, Kyushu University, Japan

1.Introduction

A thin membrane with small bending rigidity floating on the water surface is a model of a floating structure with huge horizontal size as large as several kilometers and very small draft of a few meters; this configuration is a recent conceptual design of floating airport. We present a theoretical method to predict hydroelastic response of such membrane to the wave action of incident wave at a constant frequency ω.

2.Formulation

The dynamic condition and the kinematic condition for the velocity potential $\phi(x,y,z)e^{i\omega t}$ of the flow to be satisfied underneath the membrane occupying the part of $z=0$ surface represented by Ω_M in Fig.1 are :

$$\left[\frac{D}{\rho g}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2-\frac{\omega^2}{g}d+1\right]\frac{\partial \phi}{\partial z}-\frac{\omega^2}{g}\phi=0 \quad \text{at} \quad z=0 \quad \text{on} \quad \Omega_M \tag{1}$$

$$w=\frac{1}{i\omega}\frac{\partial \phi}{\partial z} \qquad \text{at} \quad z=0 \quad \text{on} \quad \Omega_M \tag{2}$$

where D is the bending rigidity, d the draft of the membrane, ρ the density of water and $w(x,y)e^{i\omega t}$ the vertical displacement of the membrane from the equilibrium position (Ohkusu & Namba (1996)). Notice that those conditions are imposed at $z=0$ because the draft d is negligibly small. Obviously the free surface condition on the water surface Ω_W, $z=0$ plane other than Ω_M, is given by

$$\frac{\partial \phi}{\partial z}-\frac{\omega^2}{g}\phi=0 \quad \text{at} \quad z=0 \quad \text{on} \quad \Omega_W \tag{3}$$

Our problem is to solve a boundary value problem of

$$\nabla^2\phi=0 \tag{4}$$

with the boundary conditions (1) and (3) and other condtions such as radiation condition and the edge condition of the membrane when a wave is incident on the membrane. Once ϕ is known we compute the deflection w of the membrane by using (1) and (2).

This problem may be considered as a problem of the wave propagation on two different media Ω_M and Ω_W, whose characteristics are represented by a quasi-free surface condition (1) and a free surface condtion (3) respectively. The "wave" elevation on the surface Ω_M is given by equation (2). The wave elevation on the real water surface Ω_W is also written similar way.

Dispersive relation corresponding to the quasi-free surface condition (1) will be written in the form

$$\left(\frac{D}{\rho g}k_*^4-\frac{\omega^2}{g}d+1\right)k_*-\frac{\omega^2}{g}=0 \quad \text{on} \quad \Omega_M \tag{5}$$

where k_* is the wave number of the "waves" occuring in the region Ω_M. One of five roots of equation (5) is a real number k_Λ. Two of other roots are complex numbers corresponding to the evanescent waves prevailing at the edge of the membrane. Another two roots have negative real part and not legitimate for our problem of deep water because the wave motion increases infinitely as z approches $-\infty$.

The wave number $k=\omega^2/g$ on the Ω_W is not equal to k_Λ on the Ω_M. This means the waves incident on the membrane are refracted following Snell's law when they propgate from the water surface into the membrane surface. Since $k>k_\Lambda$ generally, the incident waves do not penetrate into the membrane when the incident angle is less than a critical angle.

Let us consider the case when the wave length of the incident waves coming from the positive x is very small compared with B the breadth and L the length of the membraneas. It is a natural situation because the horizontal size of the structure is huge. So we assume $kL >> O(1)$ and $kB >> O(1)$. The waves penetrating through the edge EF at $x = 0$ into the region Ω_M at $0 << y << B$ will be approximately two dimensional waves uniform to the y direction, which would occur if the membrane extended from $y = -\infty$ to $y = +\infty$ without any edge. This is because the waves in that location many wavelengths away ffrom the edges is hardly affected by the existence of the edges at $y = 0, B$. Ohkusu & Nanba (1996) gave this 2D solution.

The velocity potential ϕ_{2D} of the 2D wave with uniform crest along the y direction and propagating into the positive x will be written in the form

$$\phi_{2D} = \bar{\phi}_{2D}(x, z)e^{-ik_\Lambda x} \tag{6}$$

when it propagates deep at $x >> O(1)$ into Ω_M. We can assume k_Λ is large and

$$\frac{\partial \bar{\phi}_{2D}}{\partial x} << k_\Lambda \tag{7}$$

in this location. The wave elevation near the edge FG ($y \sim 0$) and HE ($y \sim B$) on Ω_M must have a component matched with the form (6), which will be expressed in the form

$$\phi_\Lambda = \psi_\Lambda(x, y, z)e^{-ik_\Lambda x} \tag{8}$$

Another component of the wave elevation to occur near $y = 0$ or $y = B$ will be

$$\phi_0 = \psi_0(x, y, z)e^{-ikx} \tag{9}$$

This will be a penetrated wave through the edge FG at $y = 0$ or HE at $y = B$ from the water surface into the Ω_M. Nevertheless the incident waves e^{-ikx} are travelling into the direction paralel to both the edges and their incidence angle to the edges is zero, much less than the critical angle; since the progressing waves can not penetrate into the region Ω_M and the wave (9) must be the evanescent wave significant only near the edges, at $y = O(k^{-1})$ or $y = B - O(k^{-1})$.

Our final solution for the velocity potential ϕ is a summation of k_Λ component (8) and k component (9).

3. k_Λ-component

ϕ_Λ of (8) will be derived as follows. $\partial\psi_\Lambda/\partial x << k_\Lambda$ transforms $\nabla^2\phi_\Lambda$ to

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - k_\Lambda^2\right)\psi_\Lambda = 0 \tag{10}$$

The condition (1) on Ω_M is approximated by

$$\left[\frac{D}{\rho g}\left(-k_\Lambda^2 + \frac{\partial^2}{\partial y^2}\right)^2 - \frac{\omega^2}{g}d + 1\right]\frac{\partial \psi_\Lambda}{\partial z} - \frac{\omega^2}{g}\psi_\Lambda = 0 \quad \text{at} \quad z = 0 \quad \text{on} \quad \Omega_M \tag{11}$$

The edge conditions of the membrane will be given by

$$\left.\begin{aligned} \left(\frac{\partial^3}{\partial y^3} - (2-\nu)k_\Lambda^2\frac{\partial}{\partial y}\right)\frac{\partial\psi_\Lambda}{\partial z} &= 0 \\ \left(\frac{\partial^2}{\partial y^2} - k_\Lambda^2\nu\right)\frac{\partial\psi_\Lambda}{\partial z} &= 0 \end{aligned}\right\} \quad \text{at} \quad y = 0 \tag{12}$$

Here ν is Poisson's ratio but we assume $\nu = 0$ in the article for simlicity.

The free surface condition on Ω_W is equation (3). It is straightforward to find radiation condition on the water surface side, which is

$$\psi_\Lambda \sim Ae^{ik\sqrt{1-(k_\Lambda/k)^2}y} \quad \text{at} \quad y \to -\infty \tag{13}$$

The solution should match with (6) on the Ω_M side when it is away from the edge of $y=0$:

$$\psi_\Lambda \sim \bar{\phi}_{2D}(x,z) \quad \text{at} \quad y=Y \quad (0 << Y << B) \tag{14}$$

The solution ψ satisfying all the conditions (10) to (13) is given by

$$\psi_\Lambda(x,y,0) = f(x)\bar{\psi}_\Lambda(y,0) \tag{15}$$

Here $\bar{\psi}_\Lambda$ is a solution of a linear Fredholm integral equation

$$\bar{\psi}_\Lambda(y,z) = e^{k_\Lambda z} + k_\Lambda \int_0^Y \left(\bar{\psi}_\Lambda(y',z) - \frac{k}{k_\Lambda}\int_0^Y g(y',\eta)\bar{\psi}_\Lambda(\eta,0)d\eta \right) S(y,z;y',0)dy' \quad \text{at} \quad z=0 \tag{16}$$

where $S(y,z;y',z')$ is wave source function of the Helmoholtz equation satisfying (10) and the radiation (13) which has been extensively studied. One expression of the wave source function is

$$S(y,0;y',0) = -\frac{k_\Lambda}{\pi}\int_0^\infty \frac{\mu K_1(k\sqrt{(y-y')^2+\mu^2})}{\sqrt{(y-y')^2+\mu^2}}d\mu + \frac{i}{\sqrt{1-(k_\Lambda/k)^2}}e^{-ik_\Lambda|y-y'|} \tag{17}$$

The Green function $g(',\eta)$ in the equation (16) is a solution of

$$\left(\frac{D}{\rho g}\frac{d^4}{dy^4} - 2\frac{D}{\rho g}k_\Lambda^2\frac{d^2}{dy^2} + \frac{D}{\rho g}k_\Lambda^4 - \frac{\omega^2}{g}d + 1 \right) g(y,y') = \delta(y-y') \tag{18}$$

with the boundary conditions (12) ($\nu=0$ is assumed) and

$$g = \frac{\partial g}{\partial y} = 0 \quad \text{at} \quad y=Y \tag{19}$$

The condition (11) is readily transformed into an integral form using $g(y,y')$ as

$$\frac{\partial \bar{\psi}_\Lambda}{\partial z} = k\int_0^Y g(y,y')\bar{\psi}_\Lambda(y',0)dy' \tag{20}$$

which was used in deriving the integral equation (16) from the Green's second identity.

The unknown $f(x)$ will be determined with the condition (14) and given by

$$f(x) = \bar{\phi}_{2D}(x,0)/\bar{\psi}_\Lambda(Y,0) \tag{21}$$

4. k-component

ϕ_0 of equation (9) is the effect due to the wave penetration through the edge $y=0$ from the water surface side ($y<0$) is significant only close to the edge. It is obtained as a solution when head seas are incident on a slender membrane; the method is given in Ohkusu & Nanba (1996). The solution is written as

$$\psi_0(x,y,z) = F(x)[e^{-kz} + \bar{\psi}_0(y,z)] \tag{22}$$

where $\bar{\psi}_0$ is a solution of an integral equation

$$\bar{\psi}_0(y,0) = \int_0^B kG(y,,0;y',0)\left[(\bar{\psi}_0(y,0)+1) - \int_0^B f(y',\eta)(\bar{\psi}_0(\eta,0)+1)d\eta\right]dy' \tag{23}$$

$G(y,z:y',z')$ is a wave source function not increasing exponentially at $|y| \to \infty$ given by Ursell (1968).

Unknown $F(x)$ is determined such that the outer approximation of (22) will match with the inner approximation of the outer potential. The matching condition is:

$$1 - \frac{1+i}{2\sqrt{\pi k}}\int_0^x \frac{d\xi Q(\xi)}{\sqrt{x-\xi}} = F(x) \tag{24}$$

$$Q(x) = F(x)k^2 \int_0^B \left[(\bar{\psi}_0(y,0)+1) - \int_0^B f(y',\eta)(\bar{\psi}_0(\eta,0)+1)d\eta \right] dy' \qquad (25)$$

in those expressions $f(y,y')$ is the Green function similar to $g(y,y')$. $f(y,y')$ is a solution of (18) with k_Λ replaced by k satisfying the boundary conditions (12) at $y=0$ as well as $y=B$ with k substituted in h_Λ.

5. Numerical example

One example of numerical results by the present method is illustrated in Fig.1. This picture shows the deflection of the membrane at one time instant. Feature of the combined effect due to k and k_Λ components is seen. Details of numerical calculation and another results will be presented at the Workshop.

References

Ohkusu,M. and Nanba,Y. (1996): Hydroelastic behavior of a very large floating platform in waves, 11th WWWFB, Hamburg

Ursell,F. (1968): On head seas travelling along a horizontal cylinder, J.of Inst.Maths. Applics. 4

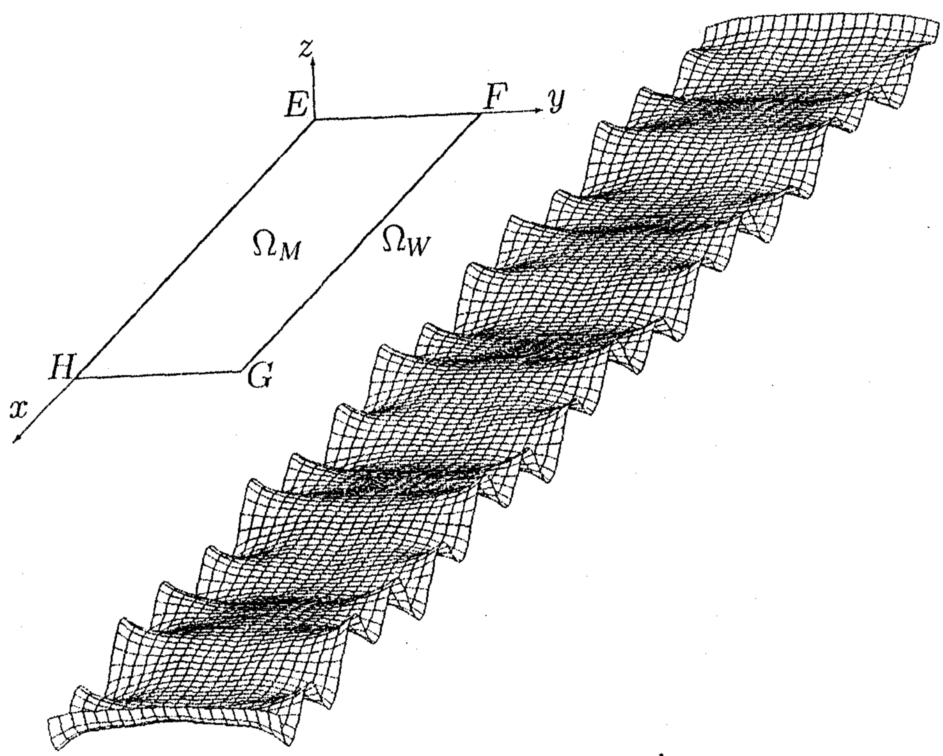

Fig.1 Numerical Example