Application of pressure-impulse theory to water wave impact beneath a deck and on a vertical cylinder.

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Introduction

The impact of water waves on structures can result in very violent motion, in particular if the waves are breaking or near breaking. Many of the studies of wave impact are of impact on a vertical wall or breakwater structure. Here we present results for two slightly different geometrical shapes. Many coastal and offshore structures have openings, overhangs and projections which are open to impact by incident water waves. The first case we consider is the wave impact on the underside of a projecting surface. The example discussed is that of a flat deck close to the mean water level. Secondly, many structures at sea, are supported by circular cylinders, hence we consider wave impact on a circular cylinder. A pressure-impulse approach is used for both cases.

Bagnold (1939) in considering wave impact on a wall, was the first to note that although pressure measurements show great variability between nominally identical wave impacts the integral of pressure over the duration of the impact, the pressure-impulse, is a more consistent measure of wave impact. Cooker and Peregrine (1990 a,b, 1992, 1995) exploited this theoretically to show that the pressure-impulse and its distribution is insensitive to the shape of the impacting water except in a region very close to the structure. Chan (1994) and Losada, Martin and Medina (1995) show good agreement with experiment for wave impact on a wall.

The pressure-impulse satisfies Laplace's equation, with relatively simple boundary conditions. Thus for simple shapes there are standard solution methods which can be used. Further, once one solution has been found the pressure-impulse contours give solutions for other related 'wave' shapes.

Pressure-impulse beneath a deck

We present pressure-impulse calculations for an impact on a horizontal surface near the surface of water of finite depth. For convenience we refer to the rigid surface as a deck. For simplicity, the deck is taken to be at water level and the water is chosen to hit the deck with a uniform vertical velocity component.

The boundary conditions for this problem have a square root singularity where the end of the deck meets the free surface. This singularity causes problems for many solution methods. However, one way to eliminate the problem of the singularity is to map the original problem using conformal maps as follows. First map to a half-space, then use another conformal map to perform a shift and stretch so that by using a final conformal map we can bend the problem back to a semi-infinite strip but with the boundary conditions shifted round to a convenient position, i.e. shift the boundary conditions on the deck round to the vertical wall. This means that the singularity is now eliminated by being mapped to a corner. As we only use conformal maps the pressure-impulse, P, continues to satisfy Laplace's equation, and so we solve using separation of variables. We have made the problem dimensionless by choosing units for which the length of the plate and velocity of the body of water before impact, are both unitary. Figure 1 shows the distribution of pressure-impulse in the water beneath the deck for depth to deck width ratio of 2.0 where the density, deck length and velocity of impact define the units.

The value of total impulse on the deck as a function of a, (water depth)/(deck length), is given in figure 2. This trend is for the impulse from impact of a given velocity and area to increase as

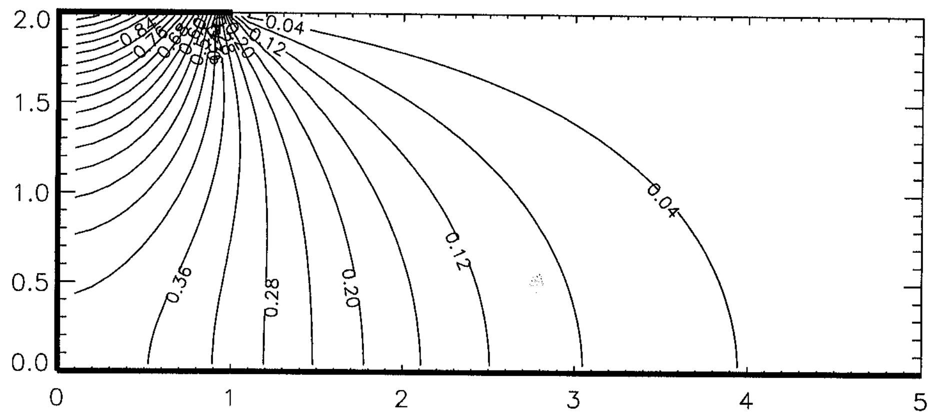

Figure 1: Pressure-impulse contours with a = 2.0. Total pressure-impulse on the deck and wall respectively are 0.81 and 1.02

the body of impacting water becomes more confined. The same trend is described by Cooker and Peregrine (1995) for impact on the wall of a rectangular box and by Topliss (1994) for impact within a circular cylinder. Consideration of flow in the most confined circumstances, as a becomes small, has given the concept of 'filling flows' (Peregrine and Kalliadasis, 1995), which is more appropriate for large overhangs. Further, in Peregrine and Thais (1996), an estimate of how the compressibility of dispersed air bubbles, such as those entrained in waves during breaking, may soften wave impact is given.

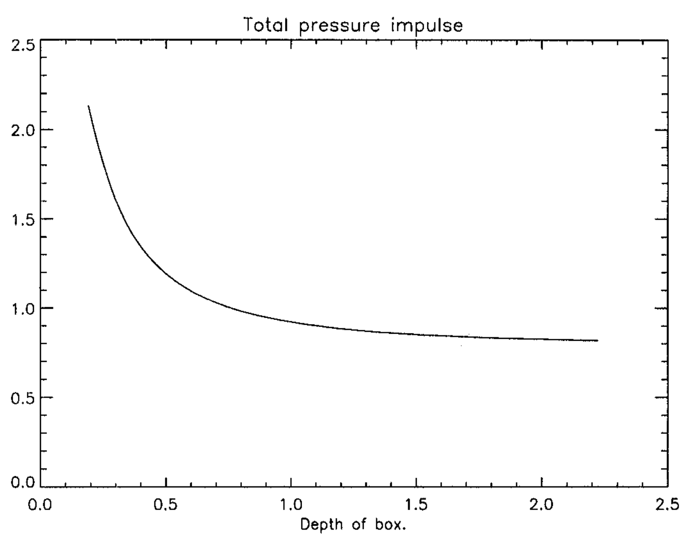

Figure 2: Total impulse on deck against depth a, where a = (water depth)/(deck length).

Impact on a cylinder

We now consider the pressure-impulse acting on a vertical cylindrical structure. For a first approximation in impact problems the water-hammer pressure-impulse is sometimes used. However, Cooker and Peregrine (1990) showed that predicted pressures are reduced significantly due to the effect of the free surface, where the pressure is atmospheric. Cooker and Peregrine (1995) noted that unless the thickness of the impacting water is quite small the actual shape of the wave away from the impact region is relatively unimportant. Hence, for simplicity, the shape of a wave impacting on a cylinder is considered to have a horizontal free surface.

Again we use pressure-impulse theory, and solve Laplace's equation, this time in cylindrical coordinates, by separation of variables. Figure 3 shows the pressure-impulse contours on the cylinder in an infinite body of water with the impact on a patch of the cylinder just below water level. Figure 4 shows the pressure-impulse contours in a wall of water impacting on the cylinder. In both these cases, which are chosen for mathematical convenience, the impact velocity is taken to be unidirectional. The angle ϕ is angle in radians, where $\phi = 0$ is at the centre of the impact region. The velocity V will vary with position. However, for demonstration, it is adequate to choose a simple velocity field. For example, a typical velocity in a breaking wave in deep water would be about $1.4c$, where c is the phase velocity of the linear wave of the same period. For simplicity of analysis the velocity of impact is scaled to unity in the present analysis.

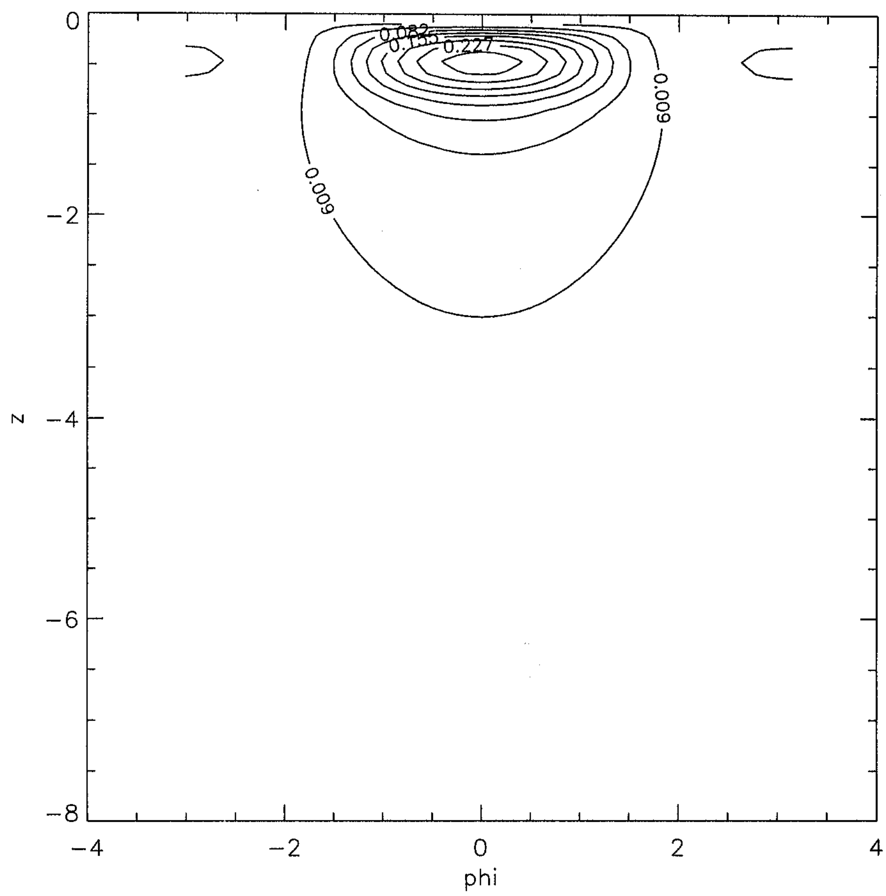

Figure 3: Distribution of pressure-impulse on a cylinder (unwrapped) with the wave impact on half of the top 10 % of the cylinder

There is a high pressure gradient towards the top of the cylinder/wedge, giving a strong impulse away from the point of high pressure acting on any projections. The method of direct solution of

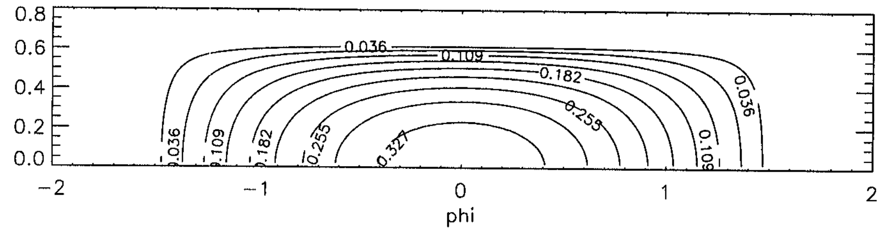

Figure 4: Distribution of pressure impulse on a cylinder (unwrapped) with the wave impact from a wall of water.

Laplace's equation by separation of variables used here is mathematically simple. However in practice it may be better to use a boundary integral method.

Conclusion

A readily evaluated solution is found for the pressure-impulse from waves hitting a deck from below. It is found that the impulse is greater if the water is shallow. Pressure-impulse theory can also give a model of impact on a cylinder. The total impulse for 2D impact on a wall of the same projected cross-sectional area is 1.016, on a cylinder it is 0.250 when the impact is surrounded by water and 0.263 for the 'wedge' of angle π bounded below by a fixed bed. We can compare this with the water hammer approach, which is dependent on how long the wave crest is. For example, if the wave crest has width l, with the unit density, velocity, and cylinder radius we are using, it gives an impulse due to momentum in the wave of $2l$. This is much bigger than the above case where $l = 1$=radius of the cylinder.

We intend to compare the pressure-impulse model of impact on a cylinder with experimental data given in Chaplin, Greated, Flintham and Skyner (1992).

Acknowledgements

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DISCUSSION

Grilli S.: Knowing the pressure impulse acting on a very rigid structure (i.e. for which elasticity can be neglected), how would you estimate the impact duration, in order to obtain pressures and forces?

Wood D., Peregrine H.: The pressure impulse technique is aimed at getting relatively simple solutions for complex events. Thus a simple time profile, a triangle, may be assumed, with an estimate of duration that depends on the actual physical scale. Wave impact studies indicate that, for the laboratory scale, durations around a milli-second occur for the most violent impacts. At larger scales 0.01 to (0.5) seconds may be appropriate. There are clear indications that at larger scales both air trapped by the impact, and air bubbles dispersed in the water are important.

Kim Y.: I think it is not difficult to include simple compressibility model to pressure Poisson equation. Do you have any idea to consider the compressibility of fluid with simplified method?

Wood D., Peregrine H.: The simplest model of air trapped by an impact is to include a rebound velocity. We have just started such work.
The effect of bubbles dispersed in the water as the cause of compressibility has been studied for a different but related flow in Peregrine & Thais (1996) "The effect of entrained air in violent water impacts", Journal of Fluid Mech. 325, pp 377-397.