Resonant Interactions of Kelvin Ship Waves With Ambient Ocean Waves

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1 Summary

In recent field experiments, Brown *et al.* (1989) observed that inside the Kelvin wake of a moving ship, there can be two soliton-like envelopes which extend to a long distance behind the ship. It is known that such solitary waves are not predicted by the steady ship-wave theory even with the inclusion of free-surface nonlinearity (*e.g.*, Newman 1971; Akylas 1987). Upon including unsteady waves, Mei (1991) suggested that oscillations of an advancing ship may be the possible source for the generation of oblique solitary waves in the Kelvin wake. Despite a speculation of high-frequency heave and pitch oscillations of a ship moving in ambient sea waves, his theory gives qualitative predictions of only partial solitary features observed in field experiments.

In this work, we provide an alternate mechanism for the generation of observed solitary waves in the ship wake by considering nonlinear resonant interactions of steady Kelvin waves and unsteady ambient waves in the spirit of Phillips (1960). We show that a new progressive wave along a particular ray inside the ship wake can be created due to third-order resonant interactions of Kelvin waves with an ambient incident wave. The wavenumber and propagation direction of the generated wave and the resonance-ray location, which depend on ship speed and ambient-wave wavenumber, are determined by the resonant conditions which we derive explicitly. To understand the development of the new wave, we derive the evolution equations for interaction wave components by a multiple-scale analysis. It is found that the generated wave has a solitary envelope in the lateral direction, and the envelope itself grows with the distance from the ship in the near wake while slowly oscillates in the far wake. Such theoretical predictions are confirmed in the near wake by direct time-domain simulations of a moving point source in an ambient incident wave field.

2 Resonant conditions

For a ship moving in an ambient incident wave field, let us examine the possibility of resonant interactions between the Kelvin ship wave and the incident wave. For convenience, let vector wavenumber $\mathbf{k_s}$ represent the Kelvin ship wave and $\mathbf{k_i}$ the incident wave. According to Phillips (1960), third-order interaction of the Kelvin wave and the incident wave, with the incident wave counted twice, can become resonant so that a new progressive wave can be generated if the following condition is satisfied:

$$\cos(\theta_i - \theta_s) = 2\eta^{-1/2} + 8\eta^{1/2} - 3\eta - 6 \tag{1}$$

where $\eta = |\mathbf{k_i}|/|\mathbf{k_s}|$ and θ_i and θ_s respectively denote propagation directions of the Kelvin wave and the incident wave relative to the ship velocity U. The generated free wave possesses a wavenumber $\mathbf{k_r}=2\mathbf{k_i} - \mathbf{k_s}$ and a frequency $\omega_r=2\omega_i - \omega_s$ with ω_i and ω_s representing frequencies of the incident wave and the Kelven wave respectively.

Under the condition (1), the resonant interaction between the Kelvin wave and the incident wave occurs along a particular ray inside the ship wake since Kelvin wave wavenumber $\mathbf{k_s}$ is constant on each ray. Once the new (free) wave is created, it propagates at its own group velocity, $\mathbf{V_g}=\omega_r\mathbf{k_r}/2|\mathbf{k_r}|^2$. Meanwhile, the ray itself moves forward at speed $\mathbf{U}$ as the ship advances. In general, the new wave and the resonance ray will propagate away each other so that no significant energy built-up for the new free wave can be resulted except for the case where the normal component of $\mathbf{V_g}$ on the resonance ray is identical to the normal velocity of the resonance ray, i.e.

$$\mathbf{V_g} \cdot \mathbf{n} = \mathbf{U} \cdot \mathbf{n} \tag{2}$$

where $\mathbf{n}$ denotes the unit normal of the resonance ray. Under the condition (2), the generated free wave propagates along the resonance ray and remains on the ray at any time as the ship moves forward. This leads to an energy built-up for the generated free wave along the resonance ray.

We remark that unlike resonant interactions among plain waves for which only the condition (1) is required, the occurrence of (third-order) resonant interactions between the Kelvin ship wave and an plain incident wave requires the satisfaction of both conditions (1) and (2). For a given incident wave ($\mathbf{k_i}$), we can determine the ship speed ($\mathbf{U}$) for the occurrence of resonances and the orientation of the resonance ray (α), from (1) and (2). Figure 1 shows the result as a function of the propagation angle (θ_i) of ambient incident waves (for the interaction with the transverse wave).

Note that if $\mathbf{k_s}$ and $\mathbf{k_i}$ exchange their positions in (1), it follows that the resonance may also occur for third-order interactions with the Kelvin wave counted twice.

3 Multiple-scale analysis

To understand the development of the new progressive wave in the ship wake, we carry out a multiple-scale perturbation analysis to obtain evolution equations for envelopes of interaction wave components in the vicinity of the resonance ray. Referring to a coordinate system fixed with the ship, the evolution equation at steady state for the amplitude of the generated wave, A_r, for example, can be written in the form:

$$\sigma_1 \frac{\partial A_r}{\partial \xi} + i\left(\sigma_2 \frac{\partial^2}{\partial \xi^2} + \sigma_3 \frac{\partial^2}{\partial \xi \partial \zeta} + \sigma_4 \frac{\partial^2}{\partial \zeta^2}\right) A_r + i\left(C_1 A_s A_s^* + C_2 A_i A_i^* + C_3 A_r A_r^*\right) A_r = C_4 A_s^* A_i^2$$

where $*$ denotes the complex conjugate, and A_s and A_i are amplitudes of the Kelvin wave and the incident wave, respectively. Here ξ are ζ are coordinates along and perpendicular to the resonance ray. The coefficients $\sigma_{1,2,3,4}$ and $C_{1,2,3,4}$ are given in terms of wavenumbers and frequencies of the interaction waves.

After imposing proper boundary conditions in the lateral direction, the evolution equations can be easily solved by the use of numerical integration. Figure 2 shows a typical result for the envelope of the generated wave near the resonance ray. As expected, the generated wave grows by absorbing energy from the primary waves in the near wake while slowly oscillates due to an energy exchange with the primary waves in the far wake.

4 Direct numerical simulation

To verify the theory, we perform a nonlinear time-domain simulation of a moving point source in an ambient incident wave field using a high-order spectral method (Liu, Dommermuth & Yue 1992). Simulation results confirm the theoretical prediction that nonlinear resonant interaction of the Kelvin wave and the incident wave can generate a soliton-like free wave in the ship wake. In particular, quantitative comparisons for the growth rate of the solitary wave in the near wake are obtained, which are shown in figure 3.

5 References

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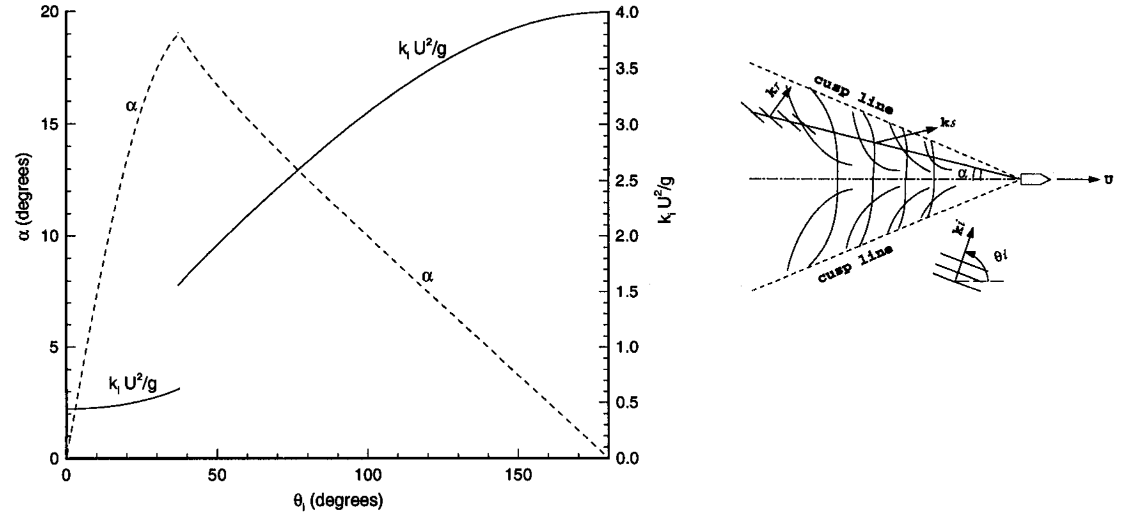

Figure 1: The resonance-ray location (α), and ship forward speed (U) and incident wave wavenumber ($\mathbf{k_i}$) for wave resonances as a function of incident angle (θ_i).

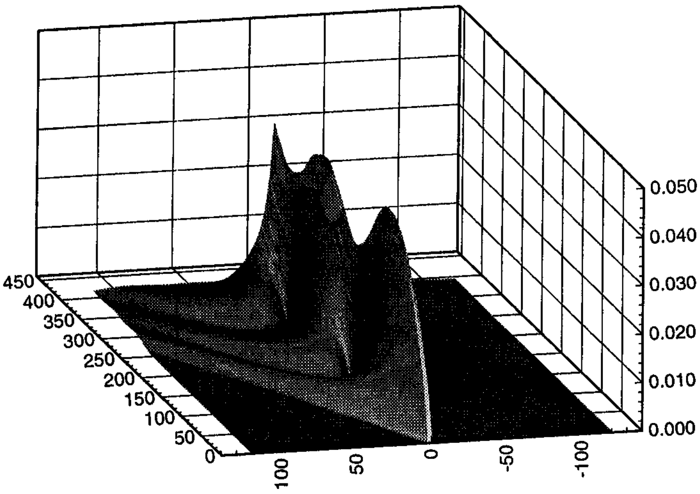

Figure 2: The envelope of the generated progressive wave in the neighbourhood of the resonance ray (α=7.5^o) in the Kelvin wake. (Incident-wave steepness $|\mathbf{k_i}|A_i$=0.15; incident angle θ_i=120^o; ship forward speed U=1.86$(g/|\mathbf{k_i}|)^{1/2}$; and Kelvin-wave steepness $|\mathbf{k_s}|A_s$=0.75$U(gr)^{-1/2}$ with r the distance from the ship.)

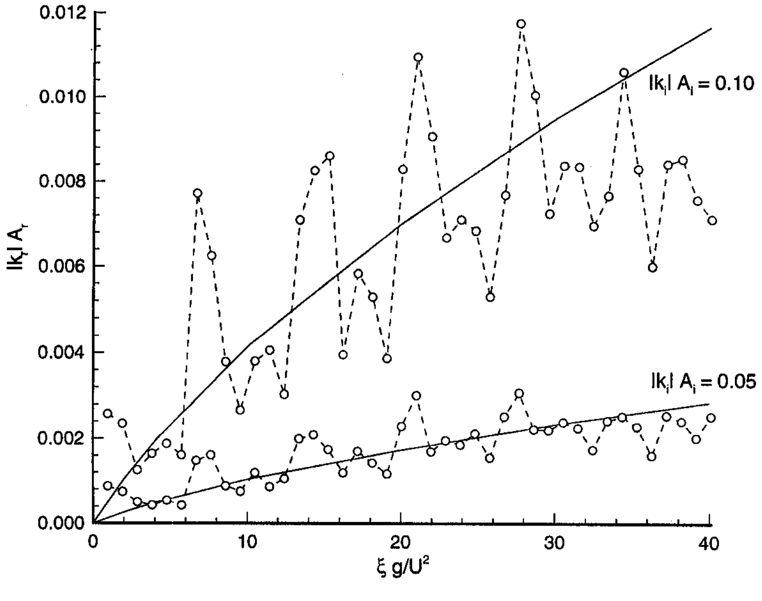

Figure 3: The comparisons between the theoretical solution by a multiple-scale analysis (———) and the direct simulation result (- -⊙- –) for the initial growth of the generated progressive wave on the resonance ray (α=7.5^o) in the Kelvin wake of a moving point source. (θ_i=120^o; U=1.86$(g/|\mathbf{k_i}|)^{1/2}$; and $|\mathbf{k_s}|A_s$=0.75$U(gr)^{-1/2}$.)

DISCUSSION

Peregrine H.:
a) This talk assumes considerable coherence in the ambient waves;
b) Another explanation is that these features are due to the waves from the ship's stern passing through the wake and shear layer due to boundary layers (Peregrine, 1971 J.F.M.)

Zhu Q., Liu Y., Yue K.P.:
a) Our theory shows that third-order quartet resonant interactions between steady ship waves and unsteady ambient incident waves can generate soliton-like progressive waves inside the Kelvin wake of a ship. Accordingly, the generated resonant wave becomes comparable in magnitude to the local ship wave over an interaction distance $L \sim O\left(\varepsilon^{-2}\right)$, where ε is the steepness of the incident or steady ship waves. This is confirmed by our numerical results. For the case of Brown *et al.* (1989), L is estimated to be $O(150 \sim 200)$ m (corresponding to the ship forward speed of $\sim 7.7 m/s$). Such distances are not unreasonably large to assume some coherence of the dominant ambient wave component which we assume to be present.
b) Peregrine (1971) found that as a result of the presence of a ship wake, the stern waves diverge at an angle to the center-line of the ship smaller than that for the bow waves. However, this is unlikely to be the mechanism responsible for the observation of Brown *et al.* (1989) since the stern waves decay with distance R from the ship as $R^{-1/2}$ which is much faster than the observed solitary wave decay rate (which is slower than $R^{-1/3}$).

Schultz W.: Your solutions do not appear to be symmetric behind the ship. Are they? [answer: beach reflection, etc]. This seems to be more far fetched than the ambient waves exciting the ship and analyzing the unsteady wave pattern as in the last presentation by Chen and Noblesse. Any comments?

Zhu Q., Liu Y., Yue K.P.: For a single monochromatic incident wave, our theory predicts the generation of a soliton-like progressive wave on one side of the ship wake. In order for two soliton-like waves to be observed inside the Kelvin wake, the theory requires the presence an additional incident wave component. Since Brown *et al.* (1989) did not measure/report on the ambient wave environment of the field experiments, the presence of additional ambient wave components can neither be ruled out nor confirmed.

If the unsteady wave due to ship motions is considered, its cusp line angle, wavenumber and propagation direction depend on $\tau = \omega U/g$, where ω is the oscillation frequency and U the forward speed of the ship, and g the gravitational acceleration. From the aerial photograph of Brown *et al.* (1989) (cf. figure 2), the solitary wave appears at an angle of 13° ~ 14° (measured from the ship track) which requires τ to be 0.5 ~ 0.7. At these values of τ, the propagation direction and wavenumber of the unsteady wave should be close to those of the steady waves near the Kelvin cusp line. These are not supported by the field observations.