Runup on a Body in Waves and Current. Fully Non-Linear and Finite Order Calculations.

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Introduction

At the previous workshop last year in Marseille, a comparison was proposed between a fully non-linear Boundary Element Model (BEM) by Ferrant (1997) and a second order BEM by Skourup *et al.* (1997). This paper is dedicated to such a comparison.

Water waves are basically a non-linear phenomenon, and in recent years the interaction between waves, currents and structures in the sea has been given much attention. The fully non-linear BEMs tend to be very demanding with respect to computational time, and to reduce the computational time needed to solve such problems, finite order BEMs based on perturbation theory have been developed both in the frequency domain and in the time domain. Linear frequency domain models for strong and weak current have been developed by Nakos (1990) and by Nossen *et al.* (1991), or Malenica *et al.* (1995), while lower order time domain models with current have been developed by *e.g.* Kring (1994), Cheung *et al.* (1996) and Sierevogel and Hermans (1996).

By the very nature of the perturbation procedure about the still water level, the lower order models are restricted to the wave steepness not being too large. In order to find the range of validity of lower order models with respect to incoming wave steepness and Froude number, comparison with results from a fully nonlinear model is especially useful.

This abstract concerns the comparison of two particular BEMs, namely the fully non-linear BEM ANSWAVE by Ferrant (1997) and the lower order BEM WAVETANK by Skourup *et al.* (1997). This comparison will serve both as validation of both models in the low Froude number and low wave steepness regime, and as a method for defining the domain of validity of the finite order model.

Mathematical formulations

The problems considered fall in the frame of potential theory. The governing equation for the velocity potential, ϕ, is the Laplace equation. Using Gauss' theorem this equation can be transformed into an integral equation on the boundary of the domain.

A collocation procedure is used employing linear and continuous basis functions over triangular or quadrilateral elements and collocation points (nodes) at the element vertices. In points where the boundary has discontinuous derivative, multiple nodes are placed at the same geometrical position satisfying one boundary condition per normal direction. Thus the Boundary Integral Equation is reduced to a dense linear system of equations to be solved for the normal velocity at the free surface and the potential on the remaining boundaries. The resulting linear system of equations depends only on the boundary geometry.

The potential and surface elevation, η, are divided into an incident field, which is unaffected by the structure, and a scattered field, which radiates from the body of interest, and the numerical problem is solved for the scattered field alone. The fully non-linear model uses stream function theory to describe the incident wave field, whereas the lower order model uses a formulation for second order Stokes waves riding on a uniform current.

To time integrate the potential and the free surface elevation the fully non-linear model follows a semi-Lagrangian formulation of the kinematic and dynamic free surface boundary conditions, nodes being allowed to move in the vertical direction only. Neumann conditions are implemented to model both the impermeable boundary at the body and truncation boundaries. At each time step the boundary conditions are used to update ϕ and η on the free surface and $\partial\phi/\partial n$ on the rest of the boundaries. Time integration is made using a 4th order Runge-Kutta method with frozen coefficients. The boundary integral equations are solved to obtain the rest of the unknowns. For further details see Ferrant (1996). The lower order model apply Taylor series of the free surface conditions and perturbation expansions of the variables to reduce the problem to finite order at a time invariant geometry. In the present formulation terms are kept to second order with respect to the wave

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steepness perturbation parameter and to first order with respect to the current speed perturbation parameter. Time integration is accomplished using the Adams-Bashforth-Moulton method. Further details can be found in Büchmann *et al.* (1997).

Since the boundary integral equations depends only on the geometry, the finite order model can apply LU-decomposition of the linear system at the onset and then use back-substitution to solve at each time step. This procedure represents an initial $O(N^3)$ cost, where N is the number of collocation points, and an $O(N^2)$ cost per time step, where the $O(N^2)$ cost dominates for the values of N considered.

Using a fully non-linear model, the boundary geometry changes at each time step requiring both the construction and solution of a new linear system at each time step. Preconditioned GMRES is used to solve this system requiring $O(N^2)$ cost per time step. Even though both methods use $O(N^2)$ operations per time step, for the same number of nodes the lower order method is much faster. Also for the lower order model the wave steepness can be chosen in the post-processing procedure, and thus a whole range of wave steepnesses can be calculated in one computation. On the other hand the lower order model may require more nodes than the fully non-linear model to resolve the same physical problem. This is due to the fact that the scattered free second order waves may be much shorter than the incident waves, and is especially true for increasing Froude numbers. The increase in the number of nodes required in the finite order model is particular important since both models use $O(N^2)$ memory, and for the finite order model, with a lower cost per time step, it turns out that it is memory rather than CPU time that limits the problem size.

NUMERICAL RESULTS

The two models have been used to calculate the runup on a bottom mounted vertical circular cylinder in waves and current. Simulations have been made for $kh = ka = 1$, where k is the wave number, h is the water depth and a is the radius of the cylinder, and for incident wave heights H/h up to 0.300 (wave steepness H/L up to 0.0477, where $L = 2\pi/k$ is the wave length). For this value of kh Stokes second order wave theory is invalid for $H/h > 0.365$ ($H/L > 0.058$). This means that the incident wave height should be well below this limit when the lower model is used. The runup profile has been found for a range of different Froude numbers, $Fr = U/\sqrt{gh}$, and wave heights, H.

The runup profile around the cylinder has been plotted on Figure 1 as function of the angle, β, for three different Froude numbers and two different wave heights. The agreement between the two models is very good for low Froude numbers (*e.g.* $Fr = 0.025$), while for larger Froude numbers (*e.g.* $Fr = 0.100$) some differences are observed. The analysis of these differences motivated the introduction of a correction to the finite order results, accounting for the steady wave elevation due to the current alone. Being of second or higher order in the current strength, steady waves due to the current were not taken into account in the original finite order formulation. Using the dynamic free surface boundary condition (the Bernoulli equation) the so-called "double body elevation" can be found to improve the results from the lower order model. The results from the lower order model with the double body elevations added are also shown on Figure 1. The correction due to "double body elevations" is seen to improve the agreement between the two models significantly. This is particularly true for low wave steepnesses, where the double body elevation is the dominant nonlinear contribution. For higher values of the wave steepness, both higher order diffraction effects and interactions between steady and oscillatory flows come into play. With the double body elevations added, however, the agreement between the models is good for a sensibly wider range of Froude numbers and wave steepnesses.

Even though the profiles compare well in all the cases shown, in Figure 1.d) some short wave features are observed in the lower order model results close to $\beta = 0.7\pi$. A convergence study has been made, and it has been shown that the second order solution is not fully converged in space. Also, it has not been possible to make a finer discretization without bringing the truncation boundaries too close to the cylinder. It should be noted also, that for waves in an opposing current of intermediate strength, say $Fr = -0.100$, the results from the two models differs significantly on the side of the cylinder near $\beta = \pi/2$.

Figure 2 shows the runup at the front of the cylinder, $R = \max \eta(\beta = 0)$, as function of wave height for different Froude numbers. This figure confirms the very good agreement of the models in the low Froude number – low wave steepness regime. For higher inputs, the increasing influence of nonlinear phenomena not included in the finite order model is clearly observed. Also it is clear from Figure 2 that the non-linear contributions to the runup are very significant.

CONCLUSIONS

A comparison has been made between the fully non-linear BEM by Ferrant (1997) and the lower order BEM by Büchmann *et al.* (1997) with the focus on calculating runup on a bottom mounted vertical circular cylinder

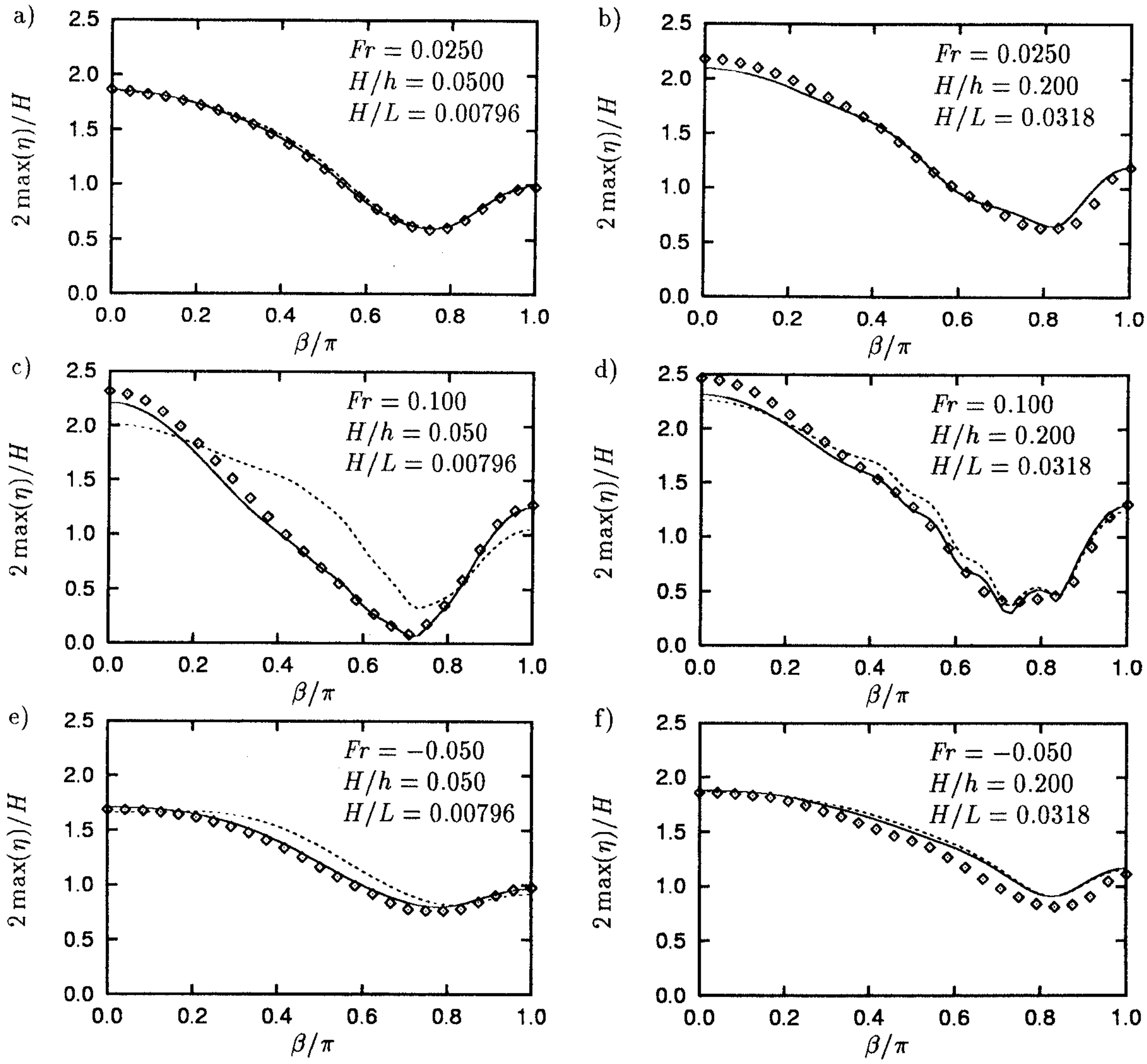

Figure 1: Examples of runup profiles on a cylinder for $kh = ka = 1$. Results from the fully non-linear model (◇), the lower order model (····) and the lower order model with double body elevations added (—).

in waves and current. Runup results from these two models agree very well for low Froude numbers and up to medium wave steepness. For large wave steepness and Froude numbers the difference between the results from the two models increases. However, the correction due to "double body elevations" is seen to improve the agreement between the two models significantly.

Thus for low Froude numbers and small to medium wave steepness the lower order method represents an accurate and computationally fast alternative to the fully nonlinear approach, at least for the present geometry and wavenumber. For increasing Froude numbers the difference between the results from the two models becomes larger and the lower order method also becomes less efficient due to a demand of finer discretization than the non-linear method. For higher Froude numbers and wave steepnesses the fully non-linear approach should be used. Note also the versatility of the fully nonlinear model which can be applied to a variety of problems among which is wave-current interaction as presented here and calculation of higher order forces as presented by Ferrant (1996).

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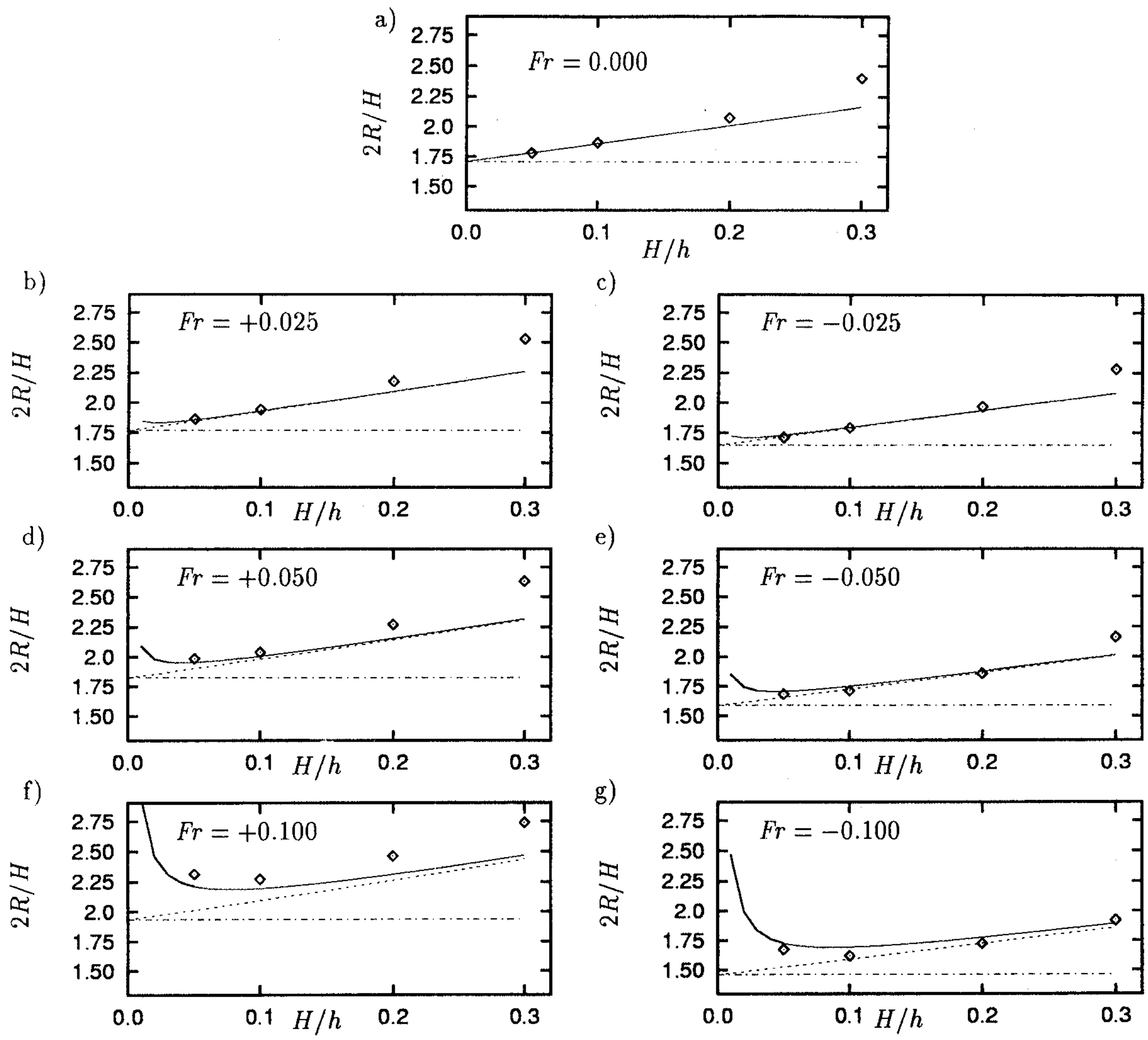

Figure 2: Runup at the front of the cylinder for $kh = ka = 1$. Results from the fully non-linear model (◇), the lower order model to first order in H/h (---), to second order in H/h (····) and to second order in H/h with double body elevations added (—).

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