A Fully 3-d Rankine Method for Ship Seakeeping

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We will present here a 'fully' three-dimensional Rankine panel method, capturing both the steady and the time-harmonic potentials three-dimensionally. Our method captures all forward-speed effects, namely – in addition to the change in encounter frequency – dynamic trim and sinkage, steady wave profile (average wetted surface) and the steady wave elevation on the free surface, and local the steady flow field

We consider a ship with average speed U in a regular wave of small amplitude h. The boundary conditions will be linearized with respect to h. The method will only be outlined here. Bertram (1996,1998) give more details. We limit ourselves to cases where $\tau > 0.25$. Then only downstream waves will be created by the ship. We solve the problem in the frequency domain using an indirect Rankine singularity method, i.e. solving for source strengths as unknowns. The elements are first-order elements (plane and constant strength).

The problem is formulated in right-handed Cartesian coordinate systems. The inertial $Oxyz$ system moves uniformly with velocity U. x points forward, z downwards. The $\underline{Oxyz}$ system is fixed at the ship and follows its motions. When the ship is at rest position, $\underline{x}$, $\underline{y}$, $\underline{z}$ coincide with x, y, z. The rigid body motion expressed in the motion vector $\vec{u} = \{u_1, u_2, u_3\}^T$ and the rotational motion vector $\vec{\alpha} = \{u_4, u_5, u_6\}^T = \{\alpha_1, \alpha_2, \alpha_3\}^T$. All motions are assumed to be of first order small.

A perturbation formulation for the potential is used omitting higher-order terms:

$$\phi^t = \phi^{(0)} + \phi^{(1)} \tag{1}$$

$\phi^{(0)}$ is the steady contribution and $\phi^{(1)}$ the time-harmonic contribution proportional to h. We describe the elevation of the free surface ζ in a similar form as the potential where we specify explicitly that quantities are time-harmonic in encounter frequency ω_e:

$$\phi^t(x,y,z;t) = \phi^{(0)}(x,y,z) + \phi^{(1)}(x,y,z;t) = \phi^{(0)}(x,y,z) + Re(\hat{\phi}^{(1)}(x,y,z)e^{i\omega_e t}) \tag{2}$$

$$\zeta^t(x,y;t) = \zeta^{(0)}(x,y) + \zeta^{(1)}(x,y;t) = \zeta^{(0)}(x,y) + Re(\hat{\zeta}^{(1)}(x,y)e^{i\omega_e t}) \tag{3}$$

The symbol $\hat{}$ denotes generally the complex amplitude of a time-harmonic quantity. The harmonic potential $\phi^{(1)}$ is decomposed into the potential of the incident wave ϕ^w, the diffraction potential ϕ^d, and 6 radiation potentials:

$$\phi^{(1)} = \phi^d + \phi^w + \sum_{i=1}^{6} \phi^i u_i \tag{4}$$

The steady potential $\phi^{(0)}$ can be computed by a 'fully nonlinear' wave resistance program which yields also second derivatives of the potential using higher-order panels on the hull. The potential of the incident wave ϕ^w is also known as usual. So the remaining unknowns are the diffraction and (unit motion) radiation potentials. These are determined by solving the Laplace equation subject to the boundary conditions:

At the average free surface ($z = \zeta^{(0)}$):

$$(-\omega_e^2 + Bi\omega_e)\hat{\phi}^{(1)} + ((2i\omega_e + B)\nabla\phi^{(0)} + \vec{a}^{(0)} + \vec{a}^g)\nabla\hat{\phi}^{(1)} + \nabla\phi^{(0)}(\nabla\phi^{(0)}\nabla)\nabla\hat{\phi}^{(1)} = 0 \tag{5}$$

$\vec{a} = (\nabla\phi^{(0)}\nabla)\nabla\phi^{(0)}$, $\vec{a}^g = \vec{a} - \{0,0,g\}^T$, $B = -(1/a_3^g)\frac{\partial}{\partial z}(\nabla\phi^{(0)}\vec{a}^g)$.

On the ship hull $\underline{S}(\underline{\vec{x}}) = 0$, using $\vec{m} = (\underline{\vec{n}}\nabla)\nabla\phi^{(0)}$:

$$\underline{\vec{n}}\nabla\hat{\phi}^{(1)} + \hat{\vec{u}}(\vec{m} - i\omega_e\underline{\vec{n}}) + \hat{\vec{\alpha}}(\underline{\vec{x}} \times (\vec{m} - i\omega_e\underline{\vec{n}}) + \underline{\vec{n}} \times \nabla\phi^{(0)}) = 0 \tag{6}$$

The 'shifting' technique developed originally for the steady wave-resistance case can be adapted without problems to the time-harmonic problem and fulfills also automatically the open-boundary condition (no reflection at the downstream boundary).

Incident wave and diffraction potentials are decomposed into symmetrical and antimetrical components. Boundary conditions (4) and (5) then form systems of linear equations in the source strengths for the 8 unknown potentials ($\hat{\phi}^{d,s}$, $\hat{\phi}^{d,a}$, $\hat{\phi}^{i}$, $i = 1...6$). The four symmetrical (likewise the four antimetrical) systems of equations share the same coefficient matrix with only the r.h.s. being different. All four cases are solved simultaneously using Gauss elimination. Then the computation of all potentials and their derivatives at all collocation points is straight-forward. But for the total potential, the so-far unknown motion amplitudes still need to be determined. The expressions for this final step are derived in principle from 'force = mass · acceleration' to:

$$m(\ddot{\vec{u}} + \ddot{\vec{\alpha}} \times \underline{\vec{x}}_g) = -\vec{\alpha} \times \vec{G} + \int_{\underline{S}^{(0)}} (p^{(1)} - \rho(\vec{u}\vec{a}^g + \vec{\alpha}(\underline{\vec{x}} \times \vec{a}^g)))\vec{n}\, d\underline{S} \tag{7}$$

$$m(\underline{\vec{x}}_g \times \ddot{\vec{u}}) + I\ddot{\vec{\alpha}} = -\underline{\vec{x}}_g \times (\vec{\alpha} \times \vec{G}) + \int_{\underline{S}^{(0)}} (p^{(1)} - \rho(\vec{u}\vec{a}^g + \vec{\alpha}(\underline{\vec{x}} \times \vec{a}^g)))(\underline{\vec{x}} \times \vec{n})\, d\underline{S} \tag{8}$$

$p^{(1)}$ is the total unsteady pressure. $\vec{G} = \{0, 0, mg\}^T$ is the ship's weight, m the displacement mass, $\underline{\vec{x}}_g$ the center of gravity, I the matrix of mass inertia moments with respect to the origin of the ship-fixed system. Eqs.(7) and (8) yield a system of linear equations for u_i ($i = 1, ..., 6$) which is quickly solved by Gauss elimination.

We added recently an ad hoc correction to account for the propulsion characteristics. Thrust and resistance forces acting on the ship are affected by the motions. One could include thrust and resistance vectors similar to the weight vector $\vec{G}$ to account for all motions. However, the main effect comes from surge motions in long waves and this allows a somewhat simpler treatment. Surge motions change the longitudinal velocity of the ship. Correspondingly changed resistance and propulsion characteristics of the ship will induce considerable damping of surge motions especially for long waves. Also the local orbital velocity of the waves may have some influence. Inclusion of these effects yields, Bertram and Thiart (1998):

$$(1-t)T_h - R = ((1-t)(1-w)T_h' - R')\dot{u}_1 - (1-t)(1-w)T_h'(\phi_x^w(\underline{\vec{x}}_p) + \phi_x^d(\underline{\vec{x}}_p)) + \bar{v}_{dif}R' \tag{9}$$

T_h', R' are derivatives of thrust and resistance with respect to speed, t thrust deduction fraction t, w Taylor wake fraction. These are approximated by empirical formulas. $\bar{v}_{dif}$ approximates the influence of the orbital velocity averaging over the wetted surface of the ship:

$$\bar{v}_{dif} = \frac{1}{\underline{S}^{(0)}} \int_{\underline{S}^{(0)}} (\phi_x^w + \phi_x^d)\, dS \tag{10}$$

Eq.(9) is added as correction on the r.h.s. of the first component of vector eq.(7). The $\dot{u}_1$ term can be interpreted as surge damping, the remaining terms contribute to the exciting surge force.

We present results for the S-175 ITTC containership at $F_n = 0.275$. The hull was discretized by 631 elements. The hull was modified in the aft region by integrating the rudder into the hull. For symmetric motions, this will have only negligible effect, but for antimetric motions this should capture the physics better than omitting the rudder totally. In a first step, the nonlinear wave-resistance problem was solved to determine the steady potential and its derivatives. The same discretized hull model was used for the seakeeping computations. The grid on the free surface had then about 1300 elements for each case. Fig.1 shows the RAOs for motions in head waves. The results of our panel method agree generally very well with experiments. The surge motions for low frequencies are still computed somewhat higher than measured. The reason is unclear, but could lie in nonlinear effects or margins of errors in the experiments. We also show results for the same grids, but with the classical steady-flow approximation, i.e. no trim and sinkage, flat free surface, uniform flow, and integration only to the calm-waterline. This approximation yields differences in the heave and pitch motions of up to 20% for medium wave lengths. Similar effects were observed for a Series-60 ship by Bertram (1997).

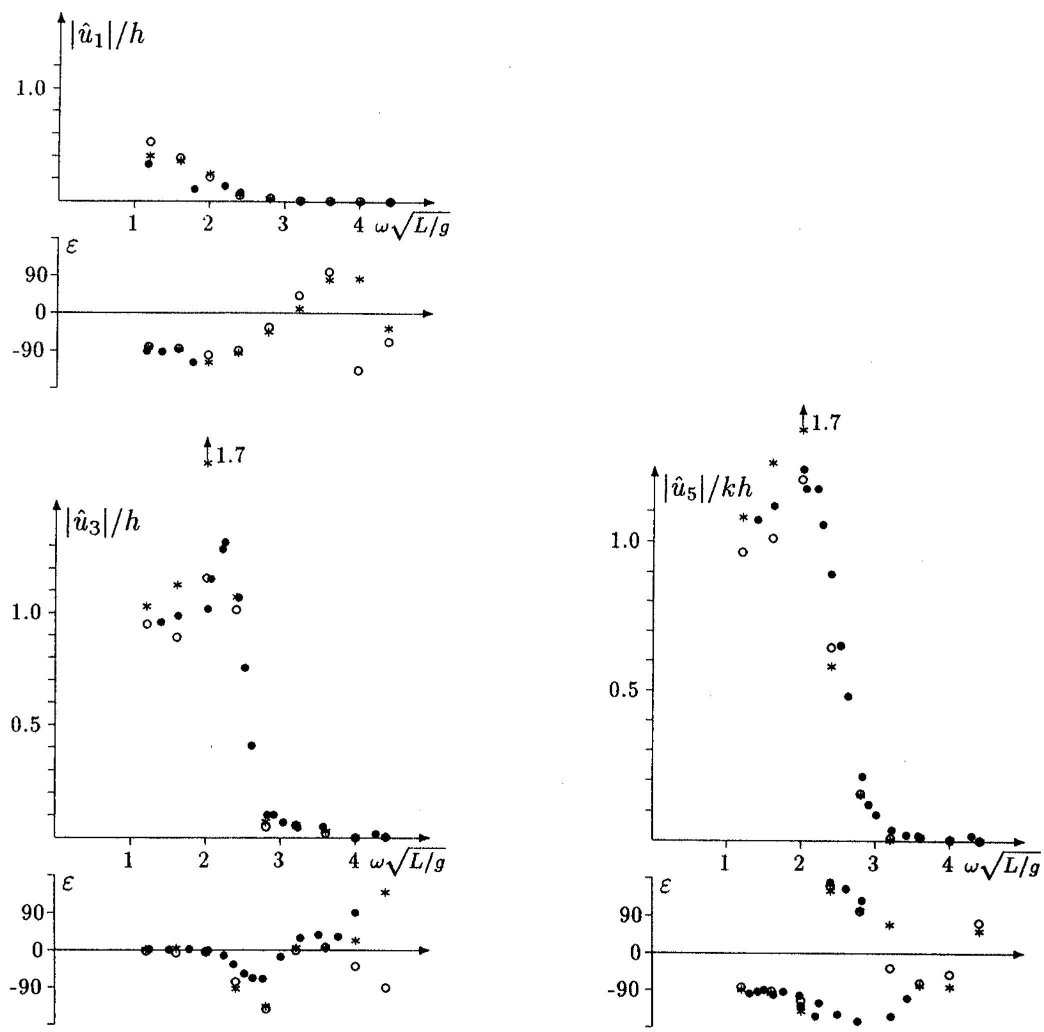

Fig.1: RAOs for S-175, $\mu = 180°$, $F_n = 0.275$; • experiment, ∘ Rankine panel method (RPM) with all forward-speed effects, $*$ RPM with classical uniform flow approximation.

References

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