# WATER ENTRY OF A WEDGE INTO A CHANNEL

Odd M. Faltinsen[1] and Rong Zhao[2]
[1] Norw. Univ. of Science and Techn., N-7034 Trondheim, Norway
[2]MARINTEK, 7002 Trondheim, Norway

Full scale experiments of wetdeck slamming on a high-speed catamaran have demonstrated that significant local stresses can occur [1]. The wetdeck has a wedge-like cross-section. The deadrise angles are between $5^\circ$ and $15^\circ$ in the impact areas. This implies that local hydroelastic effects are not dominant. Strains in longitudinal stiffeners between transverse stiffeners were measured. What is important for the local strains is not the peak pressures by themselves, but representative spatially averaged pressures during the impact. Significant global accelerations of the vessel occurred during the impact. A consequence is reduced water entry velocities.There are several uncertainties associated with estimation of the experimental water entry velocities. An example is determination of the incident flow velocities to the wetdeck. This is affected by the side hulls. The objective is to develop a theoretical method that can estimate the side-hull effect. Since the local rise up of the water at the wetdeck is important, the local flow at the intersection between the water and the wetdeck must be accurately described.

Consider a cross-section of a catamaran with wedge-formed side hulls and wetdeck. During water entry of the wetdeck the free surface condition $\phi=0$ is satisfied at horizontal lines from the intersection points between the free surface and the wetdeck. Cross-flow past the side hulls is neglected. The channel flow presented in Fig. 1 describes the instantaneous flow at the wetdeck. The line $A_\infty E_\infty$ represents the centerline of the catamaran cross-section. The straight line from $L_\infty$ to $F_\infty$ is the center line of a side hull. The free surface condition $\phi=0$ is satisfied at the line *CH*. The velocity *V* at the far ends of the channel can be interpreted as the instantaneous water entry velocity. We will first limit ourselves to $\gamma=0$, i.e. the side hulls are vertical plates. The deadrise angle $\beta$ of the wetdeck is assumed small. $\beta$ can be approximated as zero in the solution of the instantaneous flow. Analytical expressions can to a large extent be derived.

By defining z as the physical complex plane and introducing $\zeta$ as an auxilliary complex plane, it follows by Schwartz-Christoffel transformation that

$$dz/d\zeta=-i(\ell/\pi)(\zeta+c)\zeta^{-1}(\zeta+1)^{-1/2}(\zeta+c^2)^{-1/2} \qquad (1)$$

The complex velocity potential is

$$w=\phi+i\psi=-V(\ell/\pi)\log(-\zeta/c) \qquad (2)$$

Here $\zeta=-1$, $-c$ and $-c^2$ correspond to respectively *B, C* and *D* in Fig. 1. Further $\zeta=0$ represents $E_\infty F_\infty$. i is the complex unit and $\ell$ is the maximum breadth of the channel. Eq. (1) gives for x between *B* and *C*

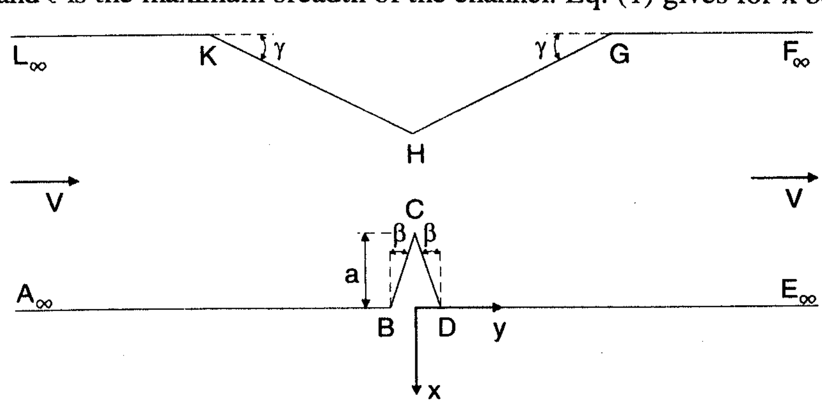


Fig. 1 Channel flow representing instantaneous flow during water entry of the wetdeck.

$$x=\frac{\ell}{\pi}\left\{\sin^{-1}\left(\frac{-\zeta-0.5(1+c^2)}{0.5(1-c^2)}\right)-\sin^{-1}\left(\frac{0.5(1+c^2)+c^2/\zeta}{0.5(1-c^2)}\right)\right\}\ ,\quad -1<\zeta<-c \tag{3}$$

The corresponding velocity potential is given by Eq. (2) . It follows from Eq. (3) that the half beam ***a*** of the wetdeck at the instantaneous water line can be related to $c$ by

$$c=(1-\sin(0.5\pi a/\ell))/(1+\sin(0.5\pi a/\ell)) \tag{4}$$

We need the vertical velocity v at the free surface in predicting the instantaneous water line. The water line is at $\zeta=ce^{i\theta}$ with $0<\theta<\pi$. We find from Eqs. (1) and (2) that

$$v=V(c^2+2c\cos\theta+1)^{1/2}(2c(\cos\theta+1))^{-1/2} \tag{5}$$

$$\cos\theta=-\{0.25(1-c)^2/c+\sin(\pi(x+a)/\ell)(1-c)\ c^{-1/2}+(1-0.25(1-c)^2/c)\cos(\pi(x+a)/\ell)\} \tag{6}$$

Eq. (5) is singular at x=-a. When x~-a,

$$v=V(0.5(1-c)\ \ell/c^{1/2}/\pi)^{1/2}\ (-(x+a))^{-1/2} \tag{7}$$

The intersection point x=-a as a function of time is found similarly as in [2]. One first determine the intersection points and then find the time it takes for the free surface to move from one intersection point to the next. In the very near vicinity of x=-a, Eq. (7) is used to integrate partly analytically the path of a free surface particle.

The solution is inconvenient if $\ell/a$ is large. Instead a far-field approach is followed when $\ell/a>3$ and used as a starting condition for the complete soluti on. The far-field solution considers an image system of a horizontal plate with length 2a and centre x=0, y=0 about the vertical walls $x=\pm\ell$. This means cross-flow past an infinite number of horizontal plates in the free surface. The individual plates have equal length 2a. There is a constant horizontal distance $2\ell$ between the centres of two adjacent plates. The vertical velocity at the centre x=0, y=0 of the real plate induced by the image plates can be represented as an infinite series with sum $V(0.5\pi a/\ell)^2/6$. We concentrate now on the real plate and follow Wagner's analysis for water entry of a wedge in infinite free surface (Ch. 9 in [3]). We write $V=V_0+V_1t$, where $V_1$ represents the acceleration and $t=0$ corresponds to initial impact time. The integral equation that determines the intersection point x=-a between free surface and body surface is

$$\tan\beta|x|=\int_0^{|x|}\frac{|x|\mu(a)da}{\sqrt{x^2-a^2}} \tag{8}$$

Further

$$\mu(a)=V(1+(0.5\pi a/\ell)^2/6)dt/da \tag{9}$$

Since solution of Eq. (8) is $\mu(a)=2\tan\beta/\pi$, solution of Eq. (9) is

$$2k\tan\beta\tan^{-1}(a/k)=\pi(V_0t+0.5V_1t^2) \tag{10}$$

where $k=2\sqrt{6}\ell/\pi$. Eq. (10) determines ***a*** as a function of time. The free surface elevation $\eta$ and vertical free surface velocity v are

$$\eta=2|x|\tan\beta\sin^{-1}(a/|x|)/\pi \tag{11}$$

$$v=V(1+(0.5\pi a/\ell)^2/6)|x|(x^2-a^2)^{-1/2} \tag{12}$$

When the complete theory is started, Eqs. (11) and (12) are used as initial conditions.

The pressure $p$ on the wedge is found from $\rho\partial\phi/\partial t$, where $\rho$ is the mass density of water. The far-field solution gives

$$p/\rho = V_1(V_i/V)(a^2-x^2)^{1/2} + V(da/dt)\,a((0.5\pi a/\ell)^2(a^2-x^2)^{1/2}/3 + (V_i/V)(a^2-x^2)^{-1/2}) \qquad (13)$$

where $V_i/V = 1 + (0.5\pi a/\ell)^2/6$. Eq. (13) can be analytically integrated to obtain space-averaged pressures and total force. The space-averaged pressure in the complete theory is obtained by first numerically integrating $\phi$ from $x_i$ to $x_{i+1}$ for each time step and then numerically time differentiate this expression. The total vertical force $F$ can also be expressed as $d(a_{22}V)/dt$ where $a_{22}$ is the vertical added mass of the plate. $a_{22}$ can be analytically expressed [4]. It follows that

$$F/\rho = -V_1\,4\ell^2\pi^{-1}\ln\cos(0.5\pi a/\ell) + 2\ell V^2 C_w \tan(0.5\pi a/\ell)/\tan\beta \qquad (14)$$

where the wetting factor $C_w = a\cdot\tan\beta/d(t)$ and $d(t) = V_0 t + 0.5V_1 t^2$. Non-dimensional force, space-averaged pressure and time are introduced. $F\tan^2\beta/(\rho V^2 d(t))$ and $p_{av}\tan\beta/(0.5\rho V^2)$ are presented as function of $d(t)/(a_{max}\tan\beta)$. $a_{max}$ means maximum value of $\boldsymbol{a}$. When $V=V_0$, non-dimensional force and pressure will only depend on $\ell/a_{max}$. Results are shown in Fig. 2 for $\ell/a_{max}$ between 1.2 and 2.0

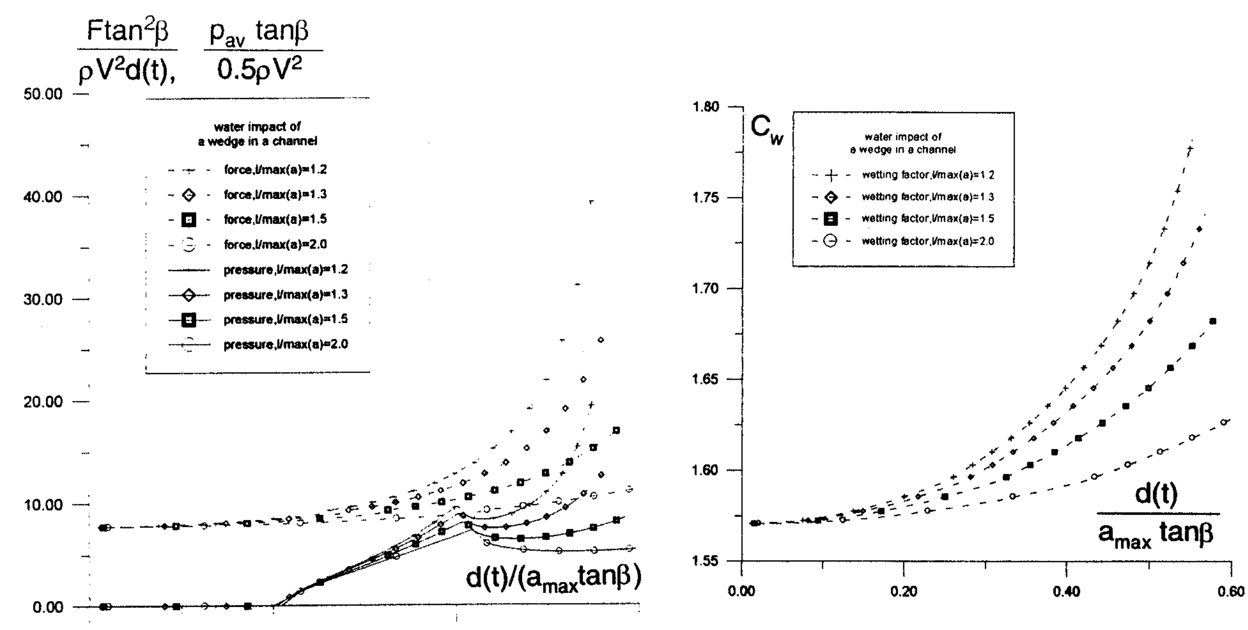


Fig. 2 Non-dimensional force $F$, space-averaged pressure $p_{av}$ and wetting factor $C_w$ as a function of non-dimensional time during water entry of a wedge. Constant entry velocity $V$.

together with the wetting factor $C_w$. The pressure is averaged from $x=a_{max}/3$ to $2a_{max}/3$. There is a small jump in the force when the complete theory is started. This is not noticeable in the figure. The similar behaviour of the far-field solution and the complete theory represents a good verification test. When time goes to zero, the non-dimensional force is $\pi^3/4$. When $\ell/a_{max}\to 1$ and the water is at $a_{max}$, the force goes to infinity. This can be seen from the last term in Eq. (14). The factor $\tan(0.5\pi a/\ell)$ behaves like $(\ell-a)^{-1}$ when $a\to\ell$. The singular behaviour of the force will be stronger since $C_w$ increases when $a\to\ell$. The increase of $C_w$ is caused by increased fluid velocities at the free surface when $a\to\ell$. A large force on the wetdeck means a large acceleration of the catamaran and a subsequent drop in the impact velocities. This implies that the actual force is not infinite. The space-averaged pressure has a peak, then drops and increases again. The peak occurs when the spray root of the jet $(x=\pm a)$ is at $x=x_{i+1}$. The far-field solution is then a good approximation. The later increase is caused by decreased

$\ell/a$-values. The first peak of the non-dimensional pressure is of main concern in the structural analysis. The reasons are the decreasing entry velocity during the water entry and that maximum pressure can be approximated by the pressure term presented in Fig. 2. There is an additional pressure term proportional to the acceleration $\dot{V}_1$. Since $\dot{V}_1$ is negative, this "added mass" pressure causes a pressure reduction and can as time goes on provide a negative total pressure.

The presented expressions do not describe the jet flow (spray) at the body surface. However, since the velocity potential has a square root singularity, we can match with the inner 2-D jet flow solution by Wagner [5]. The far-field solution gives a jet thickness $\delta=0.5a\tan^2\beta/\pi$. The complete theory gives $\delta=(V/da/dt)^2(1-c)\ell/(8\sqrt{c})$. Fig. 2 gives $a$ and $da/dt$ from $C_w$. Eq. (5) determines $c$.

The previous procedure cannot be used for finite interior half angle $\gamma$ of the side hulls. $\gamma$ was about 25° at one of the tested impact areas. The side hull effect can be examined by setting $\beta=\pi/2$ in Fig. 1 and using a Schwartz-Christoffel transformation. $G$, $H$ and $K$ in Fig. 1 correspond to respectively $c^2$, $c$ and 1 in the $\zeta$-plane. On the free surface $\zeta=ce^{i\theta}(0<\theta<\pi)$, $dx/d\theta=(\ell/\pi)(2c(1-\cos\theta)/(c^2-2c\cos\theta+1))^{\gamma/\pi}$ and the vertical velocity $v=V(\ell/\pi)d\theta/dx$. $c$ can be related to the x-value of $H$ by integrating $dx/d\theta$ from $\theta=0$ to $\pi$. v at $x=0$ is a good measure of the incident velocity to the wetdeck. The instantaneous draft $D(t)$ of the side hull was about $\ell$ at initial time of impact. Results of $v_i/V$ are presented in Fig. 3 and show the importance of the side hulls for finite $\gamma$-values. $v_i$ means v at x=0. If the wetdeck is introduced into the analysis, the flow around the wetdeck is influenced by the side hulls. The results in the first part of the paper can be used to judge the importance of this effect. However, all effects can be combined simultaneously. The Schwartz-Christoffel transformation requires then four parameters that have to be related to physical coordinates. A complete solution will be presented and verified with the more analytically based results presented in this paper.

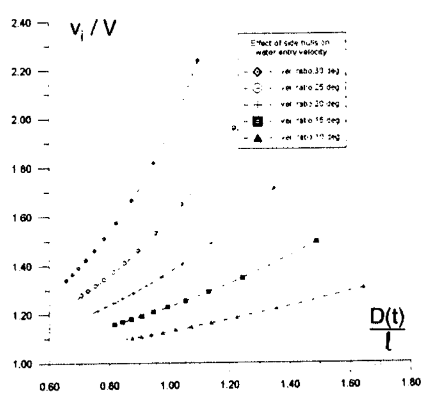


Fig. 3 Influence of side hulls on water entry velocity.

A simplified procedure where the instantaneous flow picture for $\gamma=0$, $\beta=0$ and finite $\gamma$, $\beta=\pi/2$ were combined, has been used to predict the structural loads in the wetdeck. A procedure like this can predict similar maximum strains as measured by [1]. However, since the flow was not measured, errors in estimates of incident wave velocities cause errors in water entry velocities and structural loads. A numerical method that should predict also the water entry velocities must include the non-linear effects due to side hulls and the wetdeck on global vessel accelerations.

## References


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