Unsteady bow wave field and added resistance of ships in short waves

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Introduction

An application of ray theory is presented to compute ray pattern and wave amplitude distribution around the bow region of a blunt ship hull form. The solution of an eikonal equation gives the ray pattern. Wave amplitudes are solved from a transport equation along rays. Free-surface boundary condition has a considerable effect on the results. A study on these effects is included in the work. The wave action conservation law that applies to the wave amplitude computation in a non-uniform current is also applied here. Added resistance is computed using the pressure integral approach, and wave amplitude distribution around the bow is computed using both incident and reflected rays, and taking their interaction into account.

The extended ray theory

The ray theory formulation by Hermans (1993) is applied and extended in the present paper. The total velocity potential is represented as a sum of a double-body potential ϕ_r, steady wave potential ϕ_0, and unsteady potential ϕ. A ray expansion

$$\phi = a(x,y,z;k)e^{ikS(x,y,z)-i\omega t} \tag{1}$$

is introduced to represent the unsteady potential, where a is the amplitude function and S is the eikonal function. x, y, and z are Cartesian coordinates, of which z is oriented in vertical direction, positive upwards. ω is the encounter frequency, $k = \omega^2/g$, and t is time. Incident wave length λ is assumed to be so small that wave induced ship motions are very small and can be neglected. The free-surface condition derived by Sakamoto & Baba (1986)

$$\left(\frac{\partial}{\partial t} - u_r\frac{\partial}{\partial x} - v_r\frac{\partial}{\partial y}\right)^2 \phi + g\frac{\partial\phi}{\partial z} = 0 \tag{2}$$

is applied for ϕ, where u_r and v_r are the horizontal scalar components of the double body velocity $\nabla\phi_r$. This free-surface condition and Laplace equation are used to derive the two basic equations in ray theory: the eikonal equation and the transport equation. They are solved using the method of characteristics. In the solution of the transport equation, second order spatial derivatives of the eikonal function S are needed. In the present approach they are solved also along rays, which requires that the second order spatial derivatives of the velocity components u_r and v_r are evaluated. They are computed by numerical differentiation.

The original ray method was applied by Hermans only for simple bodies, for which analytical evaluation of u_r and v_r is possible. In this work, the ray method is extended to practical ship hull forms in two ways. In the first way, the velocity field is computed by a two-dimensional panel method using only the geometry of the waterline of a body. This corresponds to a ship with a very large draft. In the second way, the velocity field is computed by a three-dimensional panel method, where the total underwater part of the ship hull is modelled, and finite draft effects are correctly taken into account in computing u_r and v_r. The three-dimensional approach was accomplished by using the velocity field output from the Shipflow computer program package (Larsson et al., 1990).

Free surface boundary condition

The above free surface condition, Eq. 2, makes a pair with the free surface condition in the low speed theory for steady ship motion. Other free surface conditions can in principle also be used. One example is the so-called low speed free surface condition that can be derived from slightly different assumptions than used by Sakamoto & Baba (1986) for the unsteady potential ϕ. By assuming that the wave length of the unsteady motion is larger than for the steady motion, namely

$$\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\phi = O(U^{-1}\phi), \tag{3}$$

where U is speed of advance of the ship, and taking into account only linear terms in ϕ, the following low speed free surface condition on z = 0 can be derived

$$\frac{\partial^2\phi}{\partial t^2}+2u_r\phi_{xt}+2v_r\phi_{yt}+g\frac{\partial\phi}{\partial z}=0\ . \tag{4}$$

When Eq. 4 is compared with Eq. 2, it can be seen that the non-linear terms in $\nabla\phi_r$ are ignored. This free-surface condition yields a slightly different ray pattern than the other free-surface condition. Added resistance values differ more, especially when the reduced frequency $\tau = U\omega/g$ increases.
Wave amplitude can be also computed by using the wave action conservation law that can be applied in the steady form in the present case. It can be shown that the application of the wave action conservation law reduces to the transport equation derived by Hermans (1993) when a two-dimensional double body flow approximation is used. This corresponds to an infinite draft assumption. When a more realistic three-dimensional double body flow approximation is used, an additional term remains in the wave action equation compared with Hermans' transport equation. This additional term affects the wave amplitude and added resistance results but not the ray pattern.

Unsteady wave elevation around a blunt bow

Measured data exist for the unsteady wave elevation for the Series 60 $C_B = 0.8$ model near the bow region in short waves (Ohkusu, 1996). An approximate approach to compute also the wave elevation with ray theory is developed in this work. First, a set of points is selected where the total unsteady wave elevation is to be computed. For each point the values of the wave elevation and eikonal function of the nearest incident ray passing the point are saved. These values are stored also for the nearest passing reflected ray, and the total wave elevation at each point can be computed.
In ray theory, computation of wave elevation along a reflected ray is in many cases difficult and very small steps in integration of the ray and transport equations are required. In addition, there are caustic curves, where wave amplitudes tend to infinity, and no realistic wave amplitude results can be obtained. Caustics are defined as envelopes of reflected rays, and their location can be predicted numerically. Thus, the caustic curves can be avoided in the wave elevation computation.
In physical terms, waves become steeper and tend to break when they approach a caustic curve. Naito et al. (1987) explain that this breaking happens when a complex valued wave number is obtained as a solution of the local dispersion relation. In the present method, the eikonal equation is equivalent to the dispersion relation, and ray tracing gives the solution for the local wave number. When the reflected rays are traced with the present method, no complex valued solutions for the local wave number are obtained when an adaptive stepsize control is used in the integration. If too large values for the step are used, reflected rays do not bend enough, and complex valued wave numbers are obtained. This fact does not support the use of Naito's breaking criterion.

Hermans (1993) derived a uniform expansion for wave elevation computation in an essentially one-dimensional case, where he is able to reduce the transport equation to a simple form well-known in optics. A boundary-layer solution near the caustic point is presented, but only very few numerical results are given. It is difficult to extend this approach to a practical ship hull form.

Results and discussion

At present stage, results are available for the Series 60 C_B = 0.8 hull form at full and ballast draft and for a bulk carrier hull form OHS with a bulbous bow at ballast condition. Examples of result for ray patterns near ship bows are given in Figures 1 and 2.

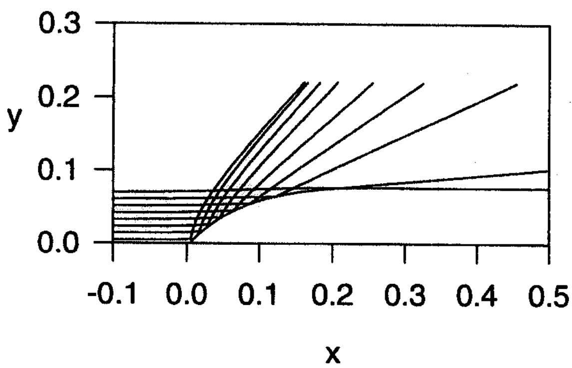

Figure 1. Ray pattern for Series 60 C_B=0.8 hull form at ballast draft. Fn=0.1, λ/L=0.5.

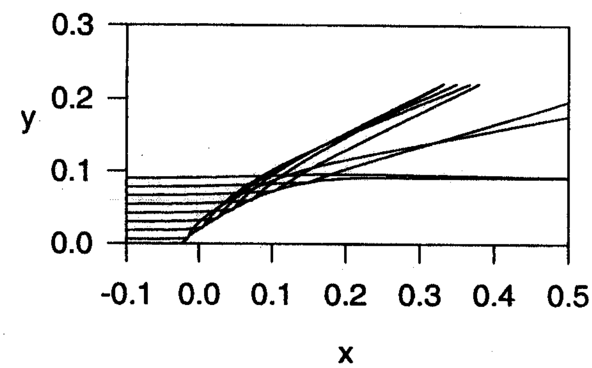

Figure 2. Ray pattern for the OHS hull form at ballast draft. Fn=0.17, λ/L=0.5.

Wave elevation results are most interesting in the region around the bow, where both indicent and reflected rays exist. This region lies between the ship hull and the caustic curve, and examples of caustic location computed numerically are shown in Figure 3 as a function of reduced frequency τ. Computed wave elevations with comparison of measured results are shown in Figure 4 for an example case. The results are divided with the far field amplitude values. The agreement is good especially near the ship hull and near the bow.

Added resistance is computed with the pressure integral approach using an equation derived by Hermans (1993). Results for the Series 60 C_B = 0.8 hull at Fn = 0.10 at full draft are shown in Figure 5. The results with a two-dimensional velocity field computation are shown in Figure 6 for the so-called blunt ship model (Nakamura et al., 1983), which has an extremely blunt bow form. In this case, results by the method of Sakamoto & Baba (1986) are also included. It can be concluded that the low speed free-surface condition, Eq. 4, can be applied for relatively small values of reduced frequency τ. When τ increases, overestimation of added resistance occurs. The use of wave action equation gives in most cases improved results especially as τ increases. For an extremely blunt bow form ray theory results are in good agreement with experimental results. Additional computations will be made to validate the method further.

References

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Larsson, L., Broberg, L., Kim, K.-J. & Zhang, D.-H. 1990. A Method for Resistance and Flow Prediction in Ship Design. Transactions of SNAME, 98 (1990) 495 - 535.

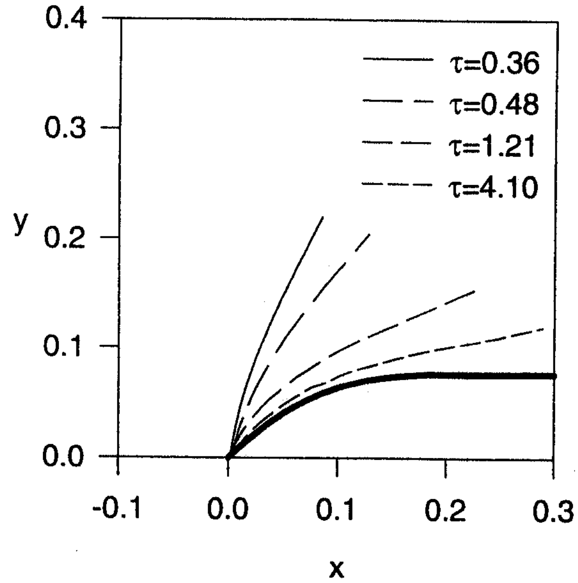

Figure 3. Caustic curves near the bow of the Series 60 $C_B = 0.8$ hull form at full draft.

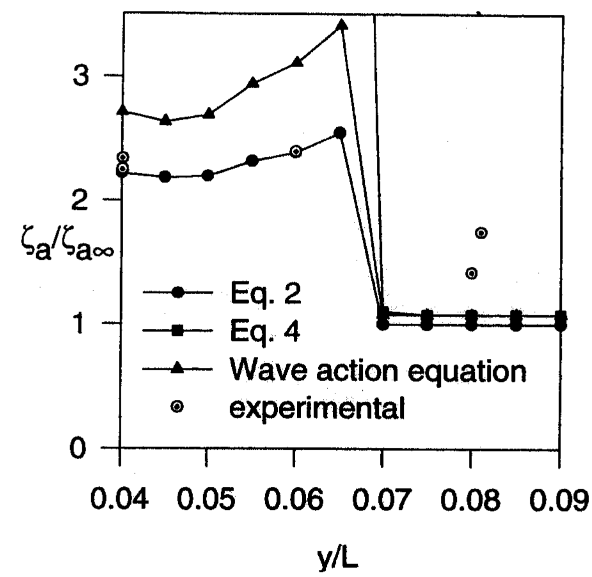

Figure 4. Total unsteady wave elevation for the Series 60 C_B=0.8 hull form at full draft. Fn = 0.2, $\lambda/L = 0.5$, $\tau = 1.2$, $x/L = 0.05$.

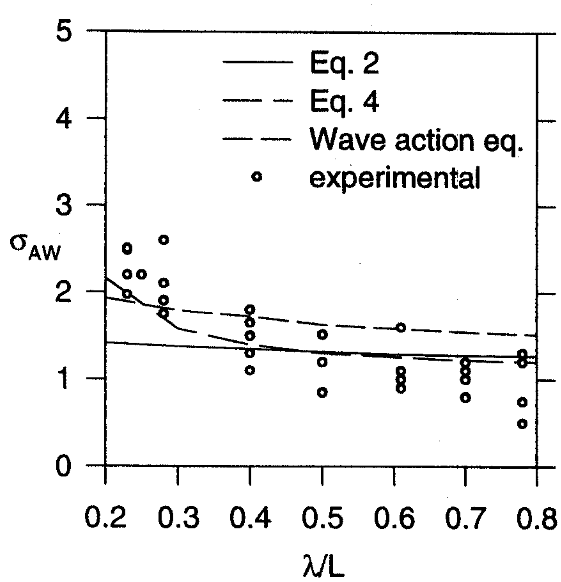

Figure 5. Added resistance for the Series 60 $C_B = 0.8$ hull at full draft. Fn = 0.10, $0.36 \le \tau \le 0.87$. $\sigma_{AW}=R_{AW}/(\rho g B^2/L)$.

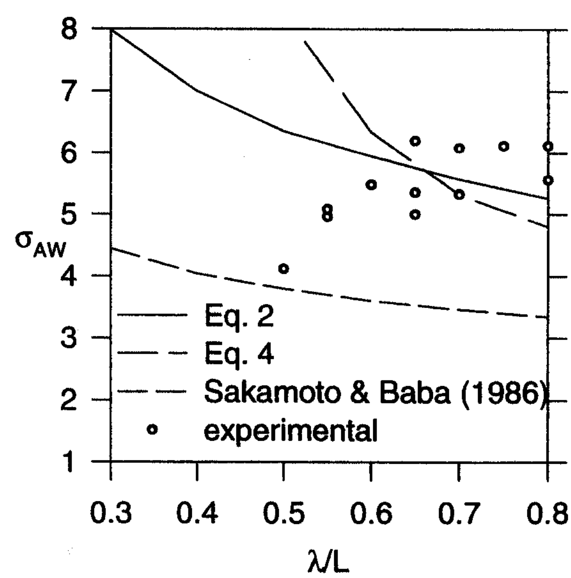

Figure 6. Added resistance for the blunt ship model with two-dimensional velocity computation. Fn = 0.15, $0.60 \le \tau \le 1.16$.

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