A Finite-Depth Unified Theory of Ship Motion

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1 Introduction

In deep water strip theory has been refined to the unified theory of [1][2]. There are not many studies on the slender-body seakeeping in finite depth problem. In the present study, a new slender-body theory is introduced as the extension of unified theory to the finite-depth radiation problem with zero speed. Borresen[3] formulated a finite depth unified theory with forward speed, and he wrote the the kernel of integral equation as the double integral in the Fourier domain. However, he was not successful to get the hydrodynamic coefficients or motions. In the present study, the series form of the kernel is derived, and the ship motion RAO is obtained. The computation extends to the second-order mean drift forces following the same idea of Kim & Sclavounos[4]. The results are compared with WAMIT's.

2 Theoretical Background

2.1 The Far-Field Solution

The far-field solution of heave and pitch motion is written as a distribution of the three-dimensional Green function G_{3D} along the center line of a ship.

$$\phi(x,y,z) \;=\; \int_L q(\xi)G_{3D}(\xi-x,y,z)d\xi \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty} du e^{iux} q^*(u)G^*(u;y,z) \tag{1}$$

where $q(\xi)$ is the strength of Green function and

$$G^*(u;y,z) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} dv e^{ivy} \frac{cosh\{\sqrt{u^2+v^2}(z+h)\}}{cosh\{\sqrt{u^2+v^2}h\}[\sqrt{u^2+v^2}tanh\{\sqrt{u^2+v^2}h\}-\frac{\omega^2}{g}]} \tag{2}$$

The superscript * means the Fourier transform, and $G^*(u;y,z)$ recovers the infinite depth case of Ogilvie and Tuck when $h \to \infty$. When y is very small, this can be approximated as

$$\phi(x,y,z) \;\approx\; q(x)G_{2D}(y,z) + \int_L q(\xi)f(\xi-x,0,0)d\xi \tag{3}$$

with $f^*(u;y,z) = G^*(u;y,z) - G^*(0;y,z) = G^*(u;y,z) - G_{2D}(y,z)$.

2.2 The Near-Field Solution

The velocity potential of the near-field solution is written as the sum of a homogeneous solution ϕ_H and a particular solution ϕ_P.

$$\phi(x,y,z) = \phi_P(x,y,z) + C(x)\phi_H(x,y,z) \tag{4}$$

Adopting the same concept with the deep water theory, the outer expansion of the near-field solution can be written as

$$\phi(x,y,z) = \{\sigma(x) + C(x)[\sigma(x) + \bar{\sigma}(x)]\}G_{2D}(y,z) - C(x)\bar{\sigma}(x)[G_{2D}(y,z) - \bar{G}_{2D}(y,z)] \quad (5)$$

with

$$G_{2D}(y,z) - \bar{G}_{2D}(y,z) = 2i\frac{cosh\{m_o(z+h)\}}{cosh(m_oh)}\frac{m_oh}{\nu h + (\frac{m_oh}{cosh(m_oh)})^2}cos(m_oy) + O(\frac{1}{y}) \quad (6)$$

where $\sigma(x), \bar{\sigma}(x)$ are the sectional strength and its complex conjugate of the two-dimensional Green function, G_{2D}. These can be obtained after solving the two-dimensional boundary value problem at each section. Besides, $\nu = \frac{\omega^2}{g} = m_otanh(m_oh)$.

2.3 Matching

Two matching conditions can be founded from Eq.(3) and (4), and an integral equation is derived from them.

$$q(x) + \frac{\nu h + (\frac{m_oh}{cosh(m_oh)})^2}{2im_oh}[1 + \frac{\sigma(x)}{\bar{\sigma}(x)}]\int_L q(\xi)f(\xi - x, 0, 0)d\xi = \sigma(x) \quad (7)$$

This integral is the most important key in unified theory. To check the consistency of this integral equation with that of infinite depth, the kernel of the integral should be studied in more detail.

2.4 The Kernel of Integral Equation

The Fourier transformation of the kernel is written as follows:

$$\begin{aligned} f^*(u;y,z) &= G^*_{3D}(u;y,z) - G_{2D}(y,z) \\ &= -\frac{1}{2\pi}\int_{-\infty}^{\infty} dve^{ivy}[\frac{cosh\{\sqrt{u^2+v^2}(z+h)\}}{cosh\{\sqrt{u^2+v^2}h\}[\sqrt{u^2+v^2}tanh\{\sqrt{u^2+v^2}h\} - \nu]} \\ &\quad - \frac{cosh\{|v|(z+h)\}}{cosh(|v|h)[|v|tanh(|v|h) - \nu]}] \end{aligned} \quad (8)$$

The contour integral and inverse Fourier transformation lead to the series form of the kernel when $(y,z) \to (0,0)$,

$$\begin{aligned} f(x,0,0) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f^*(u;0,0)e^{iux}du \\ &= \frac{i}{2}\frac{m_o^2}{m_o^2h - \nu^2h + \nu}\{-iY_o(m_ox) + J_o(m_ox) - \frac{2}{m_o}\delta(x)\} \\ &\quad -\sum_{n=1}^{\infty}\frac{m_n^2}{m_n^2h + \nu^2h - \nu}\{\frac{1}{\pi}K_o(m_nx) - \frac{1}{m_n}\delta(x)\} \end{aligned} \quad (9)$$

where $-\nu = m_ntan(m_nh)$. The $\frac{1}{r}$ singularity of the three-dimensional Green function cancels out with the logarithmic singularity of the two-dimensional Green function when it is integrated with respect to x. Also the Bessel function has a logarithmic singularity which is integrable. Fig.1 shows the Fourier-transformed kernel of the integral equation, and the kernel for finite depth approaches that of infinite depth as the depth becomes large.

2.5 Hydrodynamic Coefficients and Motions

The three dimensional correction terms on the added mass and damping coefficient can be added when the integral equation is solved[1][2]. To compute the wave excitation forces and moments, the far-field formula is applied in the present study[4]. Unified theory is applicable to heave-pitch coupled motion, but the finite-depth strip theory has to be used for sway-yaw-roll coupled motion.

2.6 The Second-Order Mean Drift Forces

The finite-depth mean drift forces and moment on surge, sway and yaw direction are computed using the formula in [5]. In particular, when there is no external work on a body, the formula which is positive definite provides more accurate results.

3 Computational Results

Fig.1 shows the added mass and damping coefficient of the heave motion. The ship model is a mathematical hull of parabolic shape. The beam(B) and length(L) ratio of hull is 0.15, and the draft(T) and length ratio is 0.1. As expected, unified theory is in very good agreement with WAMIT, especially in low frequency range. Fig.2 shows the wave excitation force for heave. The agreement with WAMIT is also favorable. Fig.3 shows the motion RAO of heave and pitch. It is interesting that the result of strip theory is not bad at low frequencies although strip theory is not accurate for the hydrodynamic coefficients. It is because the dominant force for motion at low frequency comes not from the mass and damping but the restoring force Fig.4 shows the longitudinal mean drift force at head sea. In order to get an accurate value of this parameter, the accurate computation of the Kochin function, i.e. the velocity potential, is essential as well as the motion RAO. Therefore Fig.4 indicates the accuracy of all solutions in the linear problem. As expected, unified theory provides the closer result to WAMIT than strip theory.

4 Acknowledgement

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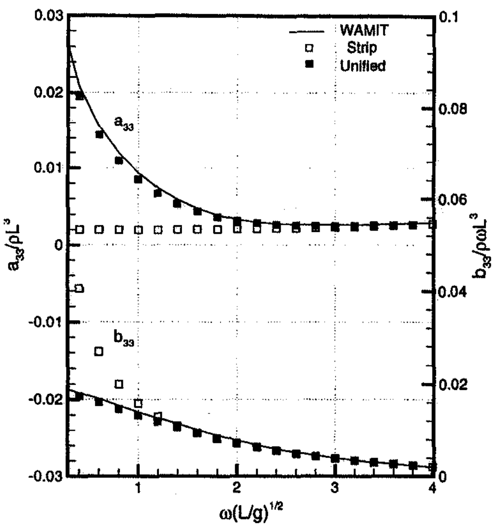

Figure 1: Added-Mass and Damping Coefficient : Heave, h/T=1.25

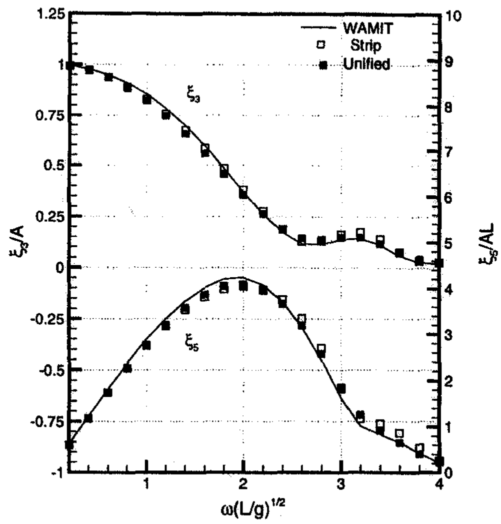

Figure 3: Motion RAO : Heave and Pitch, Head Sea, h/T=1.25

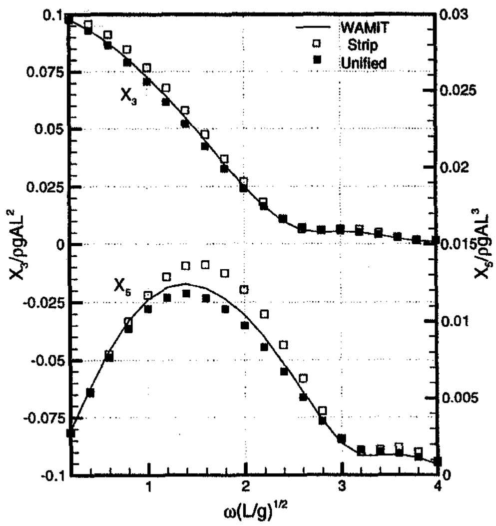

Figure 2: Wave Excitation : Heave and Pitch, Head Sea, h/T=1.25

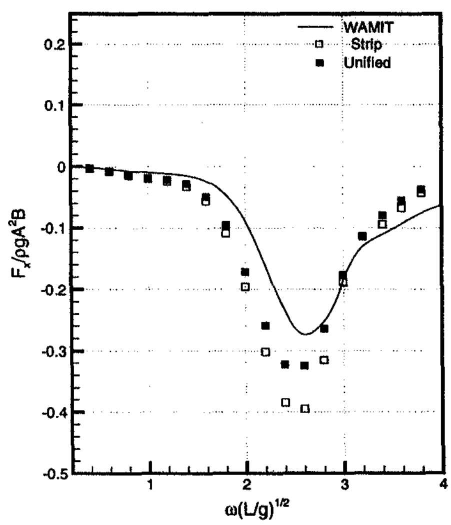

Figure 4: Longitudinal Mean Drift Force : Head Sea, h/T=1.25