Long time evolution of gravity wave systems

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Introduction

Four main avenues for the non-linear prediction of conservative wave evolution exist: kinetic equations, the narrow banded cubic PDE of Schrödinger type (called here NLS), NLS plus (Dysthe), fully non-linear computational (FNL). The first three comprise the weakly non-linear family, based on truncation of expansions beyond third order in wave steepness, ka. Dysthe corrects a crucial shortcoming of the NLS, which predicts only symmetric wave envelopes. They are both derivable from kinetic equations. The latter permits evolution in both horizontal dimensions on the free surface and it is inherently free of narrow banded assumptions. It does, however require a pre-selection of the pertinent modes, a critical process. The preferred form of the kinetic equations which originated with Hasselman (1962) and Zakharov (1968), is that of Krasitskii (1994) preserving the Hamiltonian form of the free surface problem, discovered by Zakharov. These important Krasitskii equations have hardly been applied. Neither have these four major methods been previously compared with each other systematically, or with experiments. This was our purpose.

Recently at the OEL-UCSB we have carried out systematic experiments in a large wave tank[1] (50 m l, 4.2 m w, 2.1 m d) on the evolution of a carrier wave seeded with side bands, Tulin & Waseda (1997), as well as analytical–computational studies of evolution using various avenues, including Krasitskii (Oshri, 1996)

Meanwhile one of us (ML) has implemented a high resolution fully non-linear calculation method based on boundary integral equations. The method adopts the particular Eulerian-Lagrangian approach of Dold and Peregrine (1986). A significant speed up of the computations is obtained by coupling the spectral convergence properties of the Euler Mclaurin quadrature formula with a fast summation multipole expansion technique allowing for an $\mathcal{O}(N \log N)$ operation count and an $\mathcal{O}(N)$ storage requirement. This development allows for the high resolution prediction of wave trains with $\mathcal{O}(10^2)$ waves.

Here, in collaboration, we present some of our first results showing comparisons of the evolution of a system beginning as a carrier wave plus small closely spaced side-bands, $(\omega_c \pm \delta\omega)$, which begins as a Benjamin-Feir instability. Four different methods are used: FNL, Krasitskii, NLS; Dysthe. In particular, Krasitskii's four-waves reduced equations for discrete wave systems (Krasitskii, 1994) are solved and, for the first time using this model, the evolution of an arbitrary number of wave components is allowed for. More specifically, in the computations shown below, the number of waves is always large enough to achieve the invariance of the results under further refinement (in most of the computations at least 24 equally spaced wave components are used).

With FNL as a benchmark, the rank order of performance was: Krasitskii, Dysthe, NLS. The success of Krasitskii is due to the large number of waves allowed for: by reducing the number of waves Krasitskii fails and, eventually, NLS-like results are recovered.

Results

The experiments, covering a range of ka_c and of $\delta\omega$, followed the evolution of a carrier plus seeded sidebands over about one cycle of modulation, usually ending in breaking (cfr. fig. 1). The lower sideband, $-\delta\omega$, always grew relative to the upper one, $+\delta\omega$, in contrast to well known results based on NLS but as predicted by Krasitskii (with a number of waves large enough) and Dysthe.

In particular, this behaviour is numerically studied for $ka_c = 0.1$ (upper plot in fig. 2) where FNL, Krasitskii, Dysthe and NLS spectra are contrasted. The excellent performance of Krasitskii in predicting even the waveforms can be appreciated in the lower diagram.

The energy was markedly discretized in the experiments, with a spreading toward higher frequency modes, both free and bound. Only after breaking was widening of the major spectral peaks noticeable, probably due to breaking. This consistent discretization in the first cycle validates the kinetic equation approach, which considers only modes capable of interaction in the conservative theory of interaction. Breaking first occurs at a small value of ka_c, near 0.1, as observed in Su & Green (1985) and in the ocean.

[1]The OEL wavemaker is of plunging hydraulic type, computer controlled, designed and built–in–house and featuring an innovative plunger design

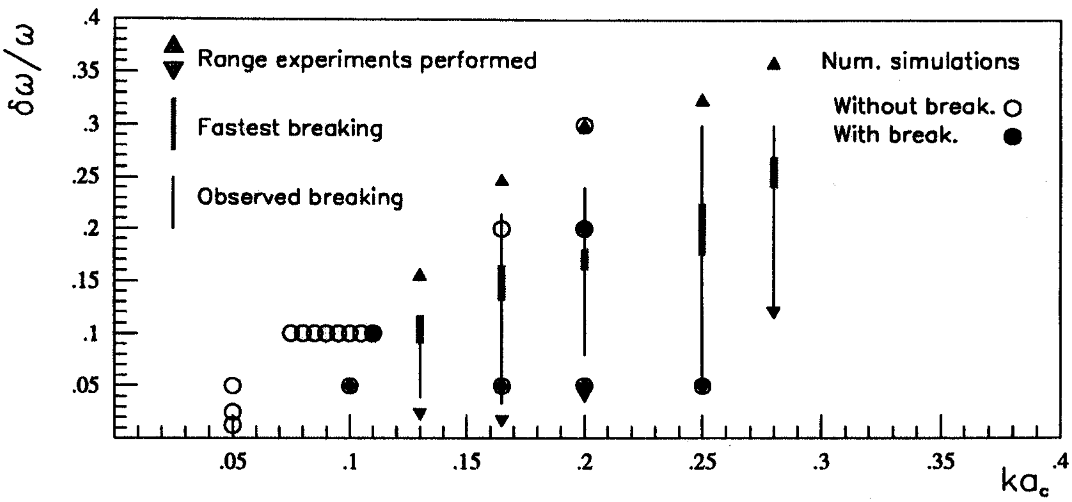

Figure 1: Summary of the experiments [4] and of the numerical computations performed. The experimental area where breaking events are observed is denoted by thin lines. Thicker lines indicate intense breaking close to the wave maker. Circles refer to numerical computations.

In the present computations, as breaking conditions are approached, differences appear in the predictions of weakly and fully non-linear methods. In particular, with small changes in ka_c the relative performance of Krasitskii, best of the weakly non-linear models, rapidly deteriorates in the second cycle of modulation (cfr. figures 3-4). We speculate that this is because, due to the weakly non-linear assumption, Krasitskii is unable to take into account the strong non-linearity at peak modulation that alters the energy exchange among wave modes, albeit in small quantity.

The high resolution of FNL is evidenced by its ability to follow waves through deformation to breaking; in figure 5, $ka_c = 0.11$, we note that the appearance of two simultaneous breakers may be very unusual. The general ability of FNL to predict the onset of breaking is shown by comparison with experiments (cfr. fig. 1). Clearly the FNL can prove very useful in the further study of evolution and breaking. The further development of Krasitskii may also prove worthwhile, as its use can be extended to two surface dimensions. In the long run, unfortunately, the use of any of these methods, even in one dimension, will fail after breaking. The experiments here clearly showed that breaking radically changes the evolution of the wave system. There have been attempts to deal with this using weakly non-linear theory, but no predictions have yet been tested by experimental comparison.

References

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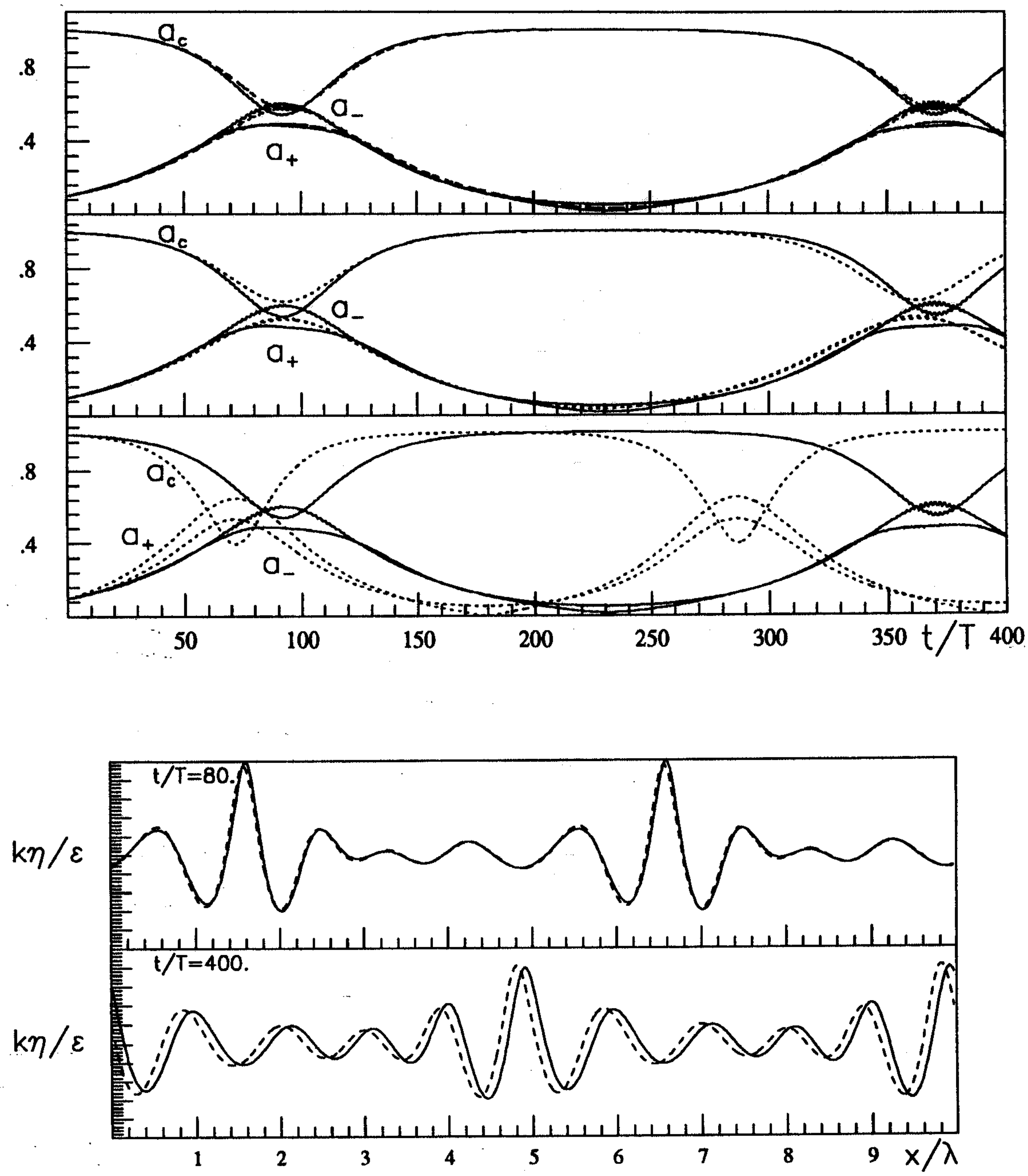

Figure 2: $ka = 0.10$–case. Top: time evolution of the Fourier components (amplitudes are normalized with the initial value of a_c); from top down the FNL solution (solid lines) is contrasted with weakly non–linear predictions (dashed lines): Krasitskii–model (upper), Dysthe equation (middle) and non–linear Schrödinger equation. Bottom: wave patterns at the first (upper) and second (lower) maximum of the modulation cycle; FNL: solid lines, Krasitskii–model: dashed–lines.

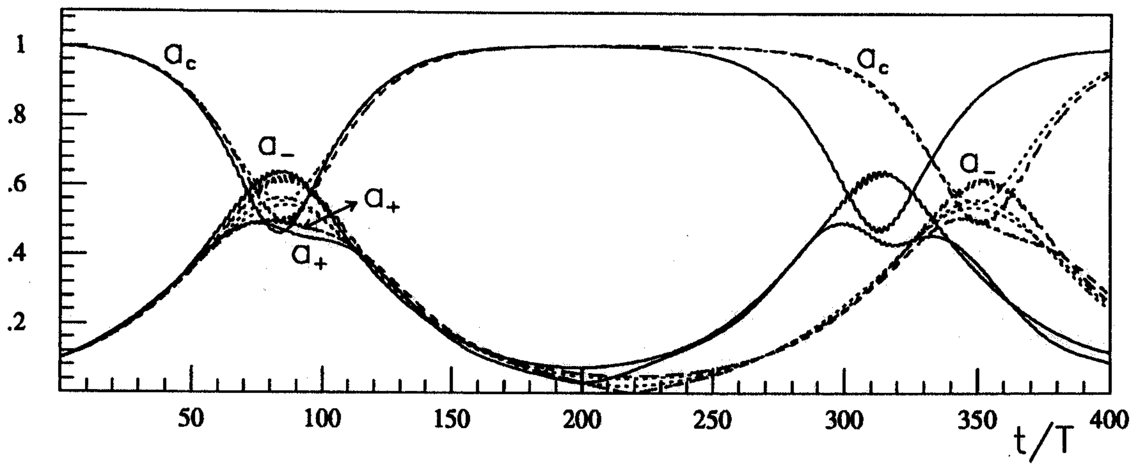

Figure 3: ka = 0.105–case. Time evolution of the Fourier components of the wave system (amplitudes are normalized with the initial value of a_c). Solid lines: FNL, dashed lines: Krasitskii–model

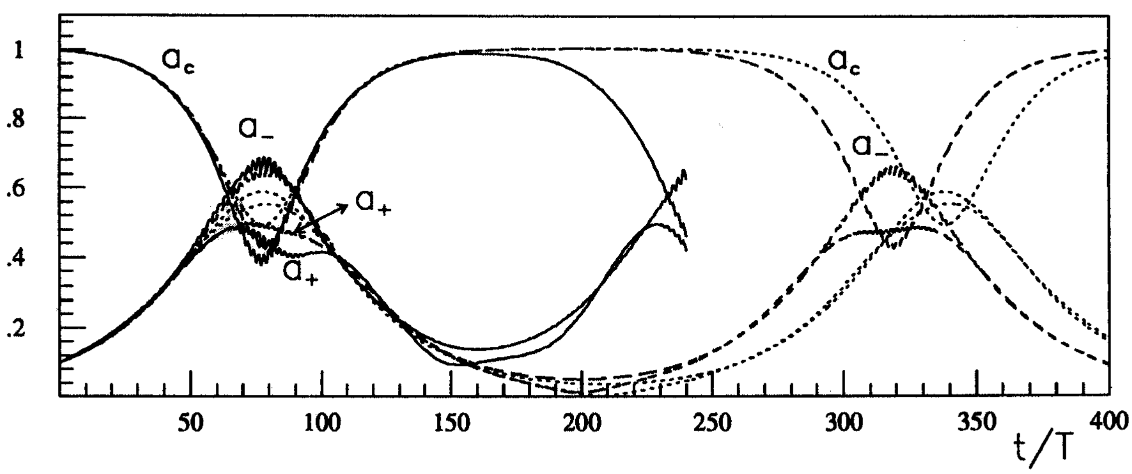

Figure 4: ka = 0.11–case. Time evolution of the Fourier components of the wave system (amplitudes are normalized with the initial value of a_c). Solid lines: FNL, dashed lines: Krasitskii–model.

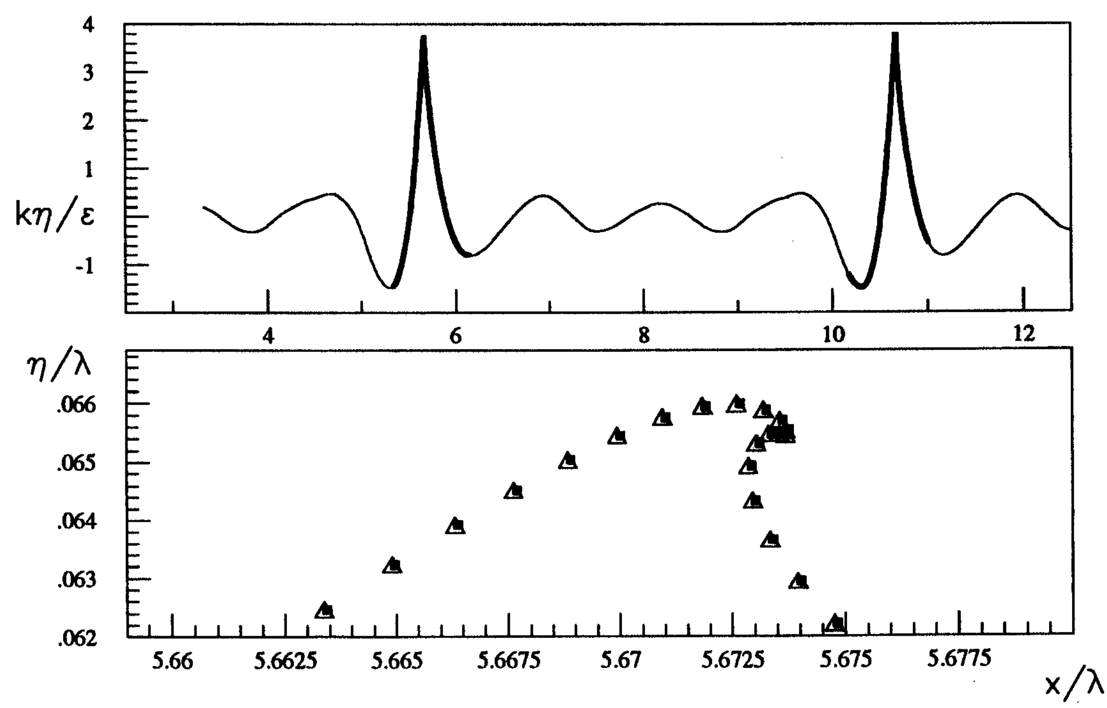

Figure 5: Final stage in the evolution of a perturbed wave train of initial steepness ka = 0.11. In the lower diagram the two plunging breakers are superimposed by translation and plotted in natural scale.