NUMERICAL SIMULATION OF SLOSHING WAVES IN A 3D TANK

Q.W. Ma[+], G.X. Wu[+] and R. Eatock Taylor[++]

\+ Department of Mechanical Engineering, University College London, Torrington Place, London, WC1E 7JE, UK

++ Department of Engineering Science, University of Oxford, Parks Road, Oxford, OX1 3PJ, UK

Introduction

Sloshing waves are associated with various engineering problems, such as the liquid oscillations in large storage tanks caused by earthquakes, the motions of liquid fuel in aircraft and spacecraft, the liquid motions in containers and the water flow on the deck of ships. These motions are often very large and their behaviour is strongly non-linear when the excitation is large or when the excitation is near to the natural frequencies. The wave pattern may behave like a standing wave, a travelling wave or a hydraulic jump. During the process, large pressures may be created. Here we consider the sloshing waves in a 3D rectangular tank undergoing translational motions in three directions. The numerical algorithm is based on the finite element method discussed in the last workshop (Ma, Wu & Eatock Taylor, 1997).

Mathematical formulation

A Cartesian co-ordinate system, *Oxyz*, fixed with the tank is used. Its origin is located at the centre of the free surface, as shown in Figure 1. The displacement of the tank in *x*, *y* and *z* directions are defined as:

(1) $X_b = [x_b(t),\ \ y_b(t),\ \ z_b(t)]$

The total velocity potential ϕ can be split into:

(2) $\phi = \varphi + xu + yv + zw$

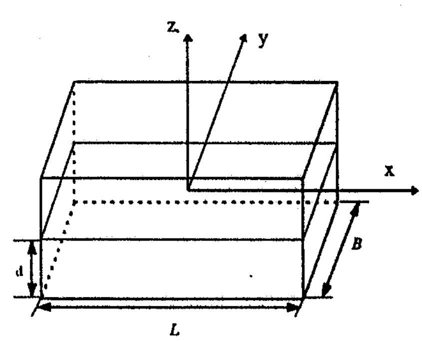

Figure 1 Tank and co-ordinate system

where u, v and w are the components of $U = \frac{dX_b}{dt}$ in the *x*, *y* and *z* directions, respectively. φ in (2) satisfies the following equations:

(3) $\nabla^2\varphi = 0$ in the fluid

(4) $\frac{\partial\varphi}{\partial n} = 0$ on the side walls

(5) $\frac{\partial\zeta}{\partial t} = -\frac{\partial\varphi}{\partial x}\frac{\partial\zeta}{\partial x} - \frac{\partial\varphi}{\partial y}\frac{\partial\zeta}{\partial y} + \frac{\partial\varphi}{\partial z}$ on the free surface

(6) $\frac{\delta\varphi}{\delta t} = \frac{\partial\varphi}{\partial z}\frac{\partial\zeta}{\partial t} - \frac{1}{2}\nabla\varphi\bullet\nabla\varphi - g\zeta - x\frac{du}{dt} - y\frac{dv}{dt} - \zeta\frac{dw}{dt}$ on the free surface

where ζ is free surface elevation measured in *Oxyz* and $\frac{\delta\varphi[x,y,\zeta(x,y,t),t]}{\delta t} = \frac{\partial\varphi}{\partial t} + \frac{\partial\varphi}{\partial z}\frac{\partial\zeta}{\partial t}$. These equations are then combined with the initial conditions which can be given as:

(7) $\zeta(x,y,0) = 0 \quad \varphi(x,y,0,0) = -xu(0) - yv(0)$

Results

In the analysis below, some parameters are nondimensionalized as follows:

$(x,y,z,L,B,a) \to (x,y,z,L,B,a)d$, $t \to \tau\sqrt{d/g}$, $\omega \to \omega\sqrt{g/d}$

We first consider a 2D case in which $L=2$, $B=0.2$ and the motion of the tank is governed by $u(\tau)=a\omega\cos(\omega\tau)$ and $v=w=0$. Figure 2 shows the history of the wave elevation at $x=-1.0$ with $a=0.00186$ and at four different frequencies either higher or lower than the natural frequency $\omega_0=\sqrt{(\pi/L)\tanh(\pi/L)}$. It shows that the numerical results are in an excellent agreement with the linearised analytical solution (Faltinsen, 1978).

The second case considered is a 3D problem in a square tank of $L=B=4$, which moves in a vertical direction with an initial horizontal disturbance defined by:

$$(8)\quad w(\tau)=\omega_z a_z\cos(\omega_z\tau),\quad u(\tau)=v(\tau)=\begin{cases}0.0283 & \tau=0\\ 0 & \tau>0\end{cases}$$

We have made calculation for four different amplitudes and frequencies. The corresponding wave history recorded at one corner is presented in Figure 3 where ω_0 given above is also a natural frequency of this square tank. The wave elevation due to purely vertical motion is theoretically zero. It, however, can become quite large when a small initial horizontal perturbation exists, as can be seen from Figures 3b to 3d. Furthermore, these large responses are not in the forced frequency but in one near to ω_0. A similar phenomenon was also reported by Su and Wang (1986) when they considered the motion at about twice the natural frequency.

In the third case, the tank of $L=B=8$ undergoes only horizontal motions defined as $u(\tau)=v(\tau)=\omega a\cos(\omega\tau)$ with $a=0.0372$ and $\omega=0.9999\omega_0$. A travelling wave can be observed in Figure 4 which shows the sequence of a wave crest moving from the corner $(-L/2,-B/2)$ to the corner $(L/2,B/2)$. Figure 5 gives the wave history at the two corners. It can be seen that the wave can become very sharp. Figure 6 illustrates the pressure history at two points, which behaves like pulses hitting the walls of the tank repeatedly.

In the fourth case, the tank of $L=8$ and $B=4$ is moving with velocities $u(\tau)=\omega_x a_x\cos(\omega_x\tau)$, $v(\tau)=\omega_y a_y\cos(\omega_y\tau)$ and $w=0$ where $a_x=0.0372$, $a_y=0.0186$, $\omega_x=0.9999\sqrt{(\pi/L)\tanh(\pi/L)}$ and $\omega_y=0.9999\sqrt{(\pi/B)\tanh(\pi/B)}$. Some typical snapshots of the wave profiles are illustrated in Figure 7. The travelling wave is also evident in this case.

The last case we considered corresponds to very shallow water. The tank of $L=B=25$ is moving only in horizontal directions with $a_x=a_y=1.2$ and $\omega_x=\omega_y=0.998\sqrt{(\pi/L)\tanh(\pi/L)}$. A hydraulic jump has been observed in this case, as shown in Figure 8. It should be noted that there are some higher frequency waves superimposed on the wave system in our case. Huang and Hsiung (1996) also observed the hydraulic jump based on a shallow water formulation but no higher frequency waves seem to exist in their analysis. More results will be presented in the workshop.

Acknowledgements

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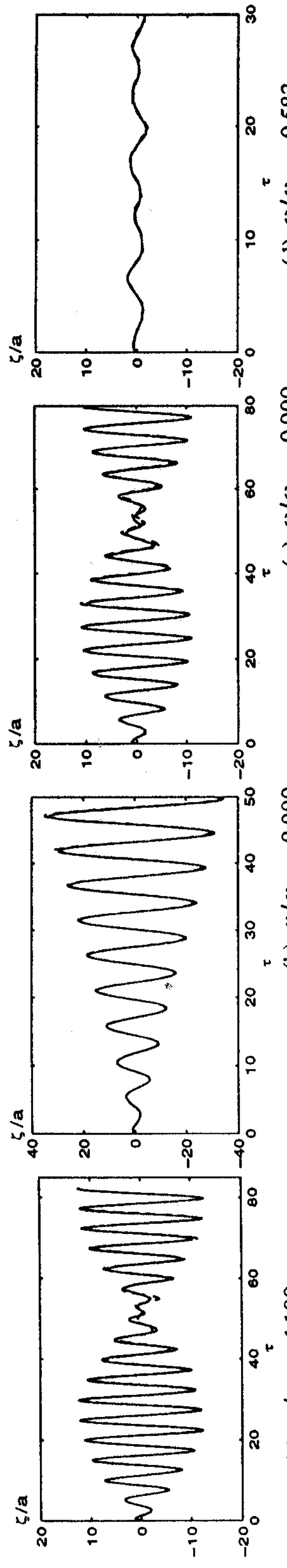
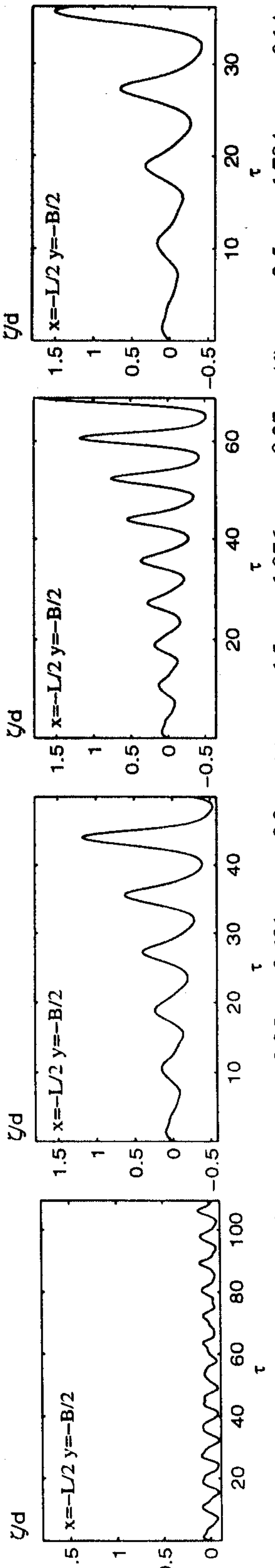


(a) $\omega/\omega_0 = 1.100$ (b) $\omega/\omega_0 = 0.999$ (c) $\omega/\omega_0 = 0.900$ (d) $\omega/\omega_0 = 0.583$

Figure 2 Wave history at $x = -1.0$ for different frequencies (solid line: analytical; dashed line: numerical)



(a) $\omega_z = 0.5\omega_0 = 0.359$ $a_z = 1.0$ (b) $\omega_z = 0.95\omega = 0.681$ $a_z = 0.8$ (c) $\omega_z = 1.5\omega_0 = 1.076$ $a_z = 0.27$ (d) $\omega_z = 2.5\omega_0 = 1.794$ $a_z = 0.14$

Figure 3 Wave history at a corner due to the motion of eq. 8($L = B = 4$)

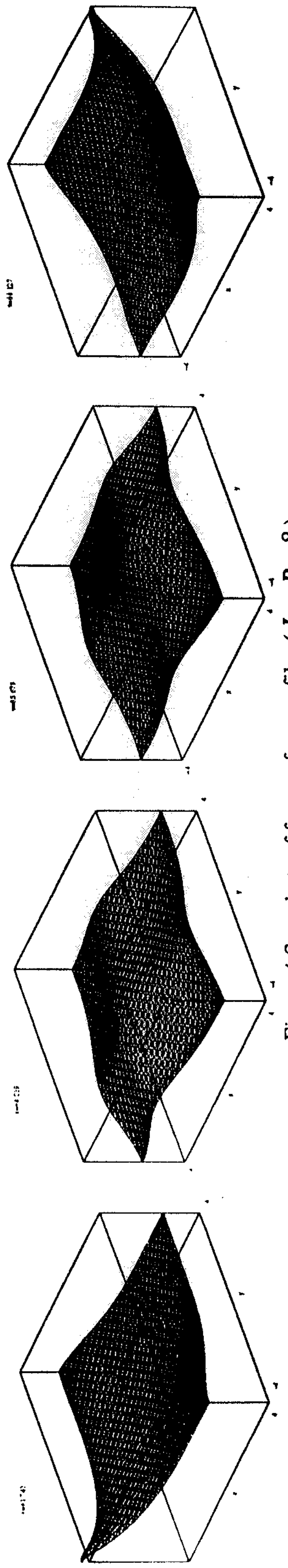


Figure 4 Snapshots of free surface profiles ($L = B = 8$)

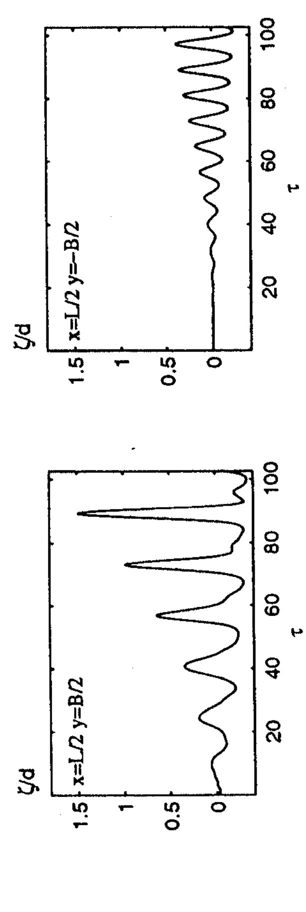


Figure 5 Wave history at two corners ($L = B = 8$)

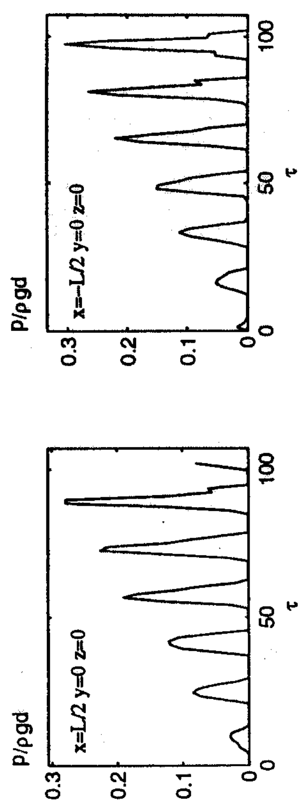


Figure 6 Pressure history ($L = B = 8$)

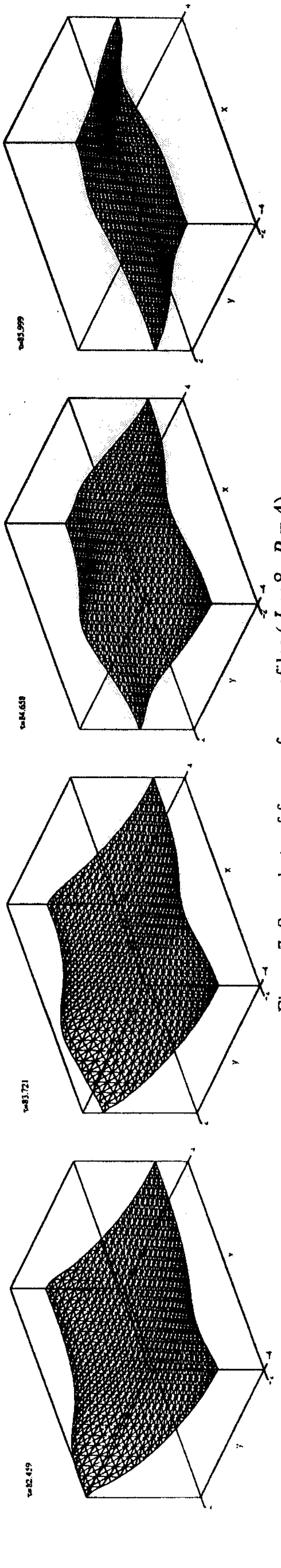


Figure 7 Snapshots of free surface profiles ($L = 8$, $B = 4$)

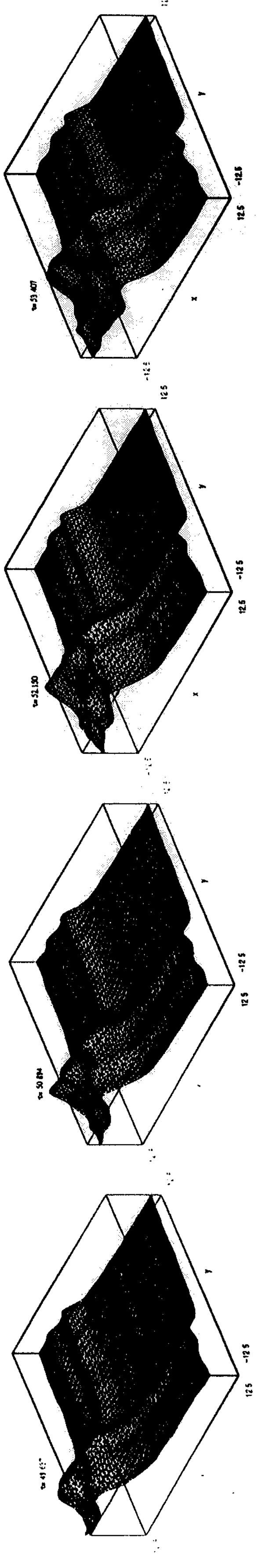


Figure 8 Snapshots of free surface profiles ($L = 25$, $B = 25$)