GEOMETRIC SYNTHESIS OF 2D SUBMERGED BODIES

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1. INTRODUCTION

Consider the cross-section of a slender submerged body symmetric with respect to the vertical axis and let *d* be the distance between the origin O of the coordinated system and the free surface. The point O is within the cross-section and along the symmetry axis. If S is the cross-section area and $B_1 = (C_M+1)S/2\pi$ let $a = (B_1)^{1/2}$ be the characteristic length, where C_M is the heave added mass coefficient in *infinite* fluid, namely, in a fluid region without free surface. The heave potential in this case can be expressed by means of the Fourier (Laurent) series

$$\phi(x,y) = B_1 \cdot G(z) = B_1 \sum_{n=1}^{\infty} \frac{a^{n-1}}{(n-1)!} \frac{D_n}{D_1} \frac{\partial^{n-1}}{\partial z^{n-1}}\left(\frac{\sin\theta}{r}\right);$$
$$G(z) = -\sum_{n=1}^{\infty} (-i)^n \cdot D_n \cdot \left(\frac{a}{z}\right)^n, \qquad (1a)$$

where y is the vertical coordinate, z = x + i.y is the complex variable and G(z) is the complex potential.

The coefficients $\{D_n \,;\, n = 1,2,3,.....\}$ define completely the geometry of the cross-section and one can introduce then the *function of form*

$$F(Ka) = \sum_{n=0}^{\infty} (-1)^n \frac{(Ka)^n}{n!} \frac{D_{n+1}}{D_1}. \qquad (1b)$$

It can be shown (see Aranha & Pinto (1994)) that the sectional heave exciting force due to a harmonic wave with amplitude A, frequency ω and wavenumber $K = \omega^2/g$ is asymptotically given by the expression

$$f_3(t) = \rho S(C_M + 1).F(Ka).\frac{dw_0}{dt};$$
$$w_0(t) = \left(\frac{\partial \phi_I}{\partial z}\right)_{x=z=0} = -i\omega A e^{-Kd} e^{-i\omega t}, \qquad (2a)$$

with an error of the form [1 + O(δ)] where

$$\delta = (C_M + 1)\cdot K^2 S \cdot F(Ka)\cdot e^{-2K(d-a_0)} \approx (Ka)^2 \cdot e^{-2Kd}. \qquad (2b)$$

In the above expression a_0 is the radius of the circle that circumscribes the cross-section and since $\delta \le 0.135(a/d)^2$ the error is of order 3% when the equivalent cross-section radius a is half the distance d between the point O and the free surface.

Notice that $F(Ka) \to 1$ when $Ka \to 0$ and so (2a) recovers the inertia term of Morison formula in the low frequency limit; in this sense this expression represents an extension to the whole range of frequencies of this well known formula. Also, $F(Ka) \equiv 1$ for a circle, indicating that Morison formula can be used in the whole range of frequencies for this geometry.

2. GEOMETRIC SYNTHESIS

The importance of such approximated solution is that it enables one to address, within certain limitations, the inverse problem, namely, the one where the behavior of the exciting force is defined and the geometry of the cross-section is then obtained. By defining a convenient *function of form* F(Ka) one can determine the coefficients $\{D_n\,;\, n = 1,2,3,.....\}$ and so the geometry of the cross-section that it is associated with the chosen F(Ka). The purpose of this work is to present an example of this geometric synthesis and an experimental validation of the final result, by direct measurement of the exciting force in the wave tank. The example chosen was fitted to provide a simple geometry, that could be easily built and such that the final result could have been obtained by an ad hoc extension of Morison formula to the whole frequency range.

Consider then the *function of form*

$$F(Ka) = \cos^4(\alpha Ka) = \frac{3}{8} + \frac{4}{8}\cos(2\alpha Ka) + \frac{1}{8}\cos(4\alpha Ka). \qquad (3a)$$

The geometry related to this one-parameter functions of form are such that the heave exciting force have a very flat zero at the frequency $Ka = \pi/2\alpha$. Expanding (3a) in power series one can determine the coefficients $\{D_n\,;\, n = 1,2,3,.....\}$ from (1b) and using them into the expression for G(z) the following complex potential is determined (see Simos (1997)):

$$G(z) = -\frac{3a^2}{8}\frac{i}{z} - \frac{a^2}{4}\frac{i}{z+2\alpha a} - \frac{a^2}{4}\frac{i}{z-2\alpha a} - \frac{a^2}{16}\frac{i}{z+4\alpha a} - \frac{a^2}{16}\frac{i}{z-4\alpha a}. \qquad (3b)$$

For $\alpha = 0$ the complex potential G(z) represents a circle with radius a centered at the origin O; as α increases this circle is continuously distorted and for α large enough one obtains *five* circles with centers placed at { $z = (0,0)$; $z = (\pm 2\alpha a,0)$; $z = (\pm 4\alpha a,0)$} and with radius {$(3/8)^{12}a$; $a/2$; $a/4$} respectively.

The figure below shows how the geometry changes with the increase of *tuning parameter* α. Observing that the standard Morison formula can be used for a circle, irrespective of the value of the wavenumber, one obtains from this formula applied to the five circles exactly the expression (2a;3a). This result not only enhance the confidence in the proposed approximation but also displays a simple geometry that can be easily built in order to check experimentally expression (2a).

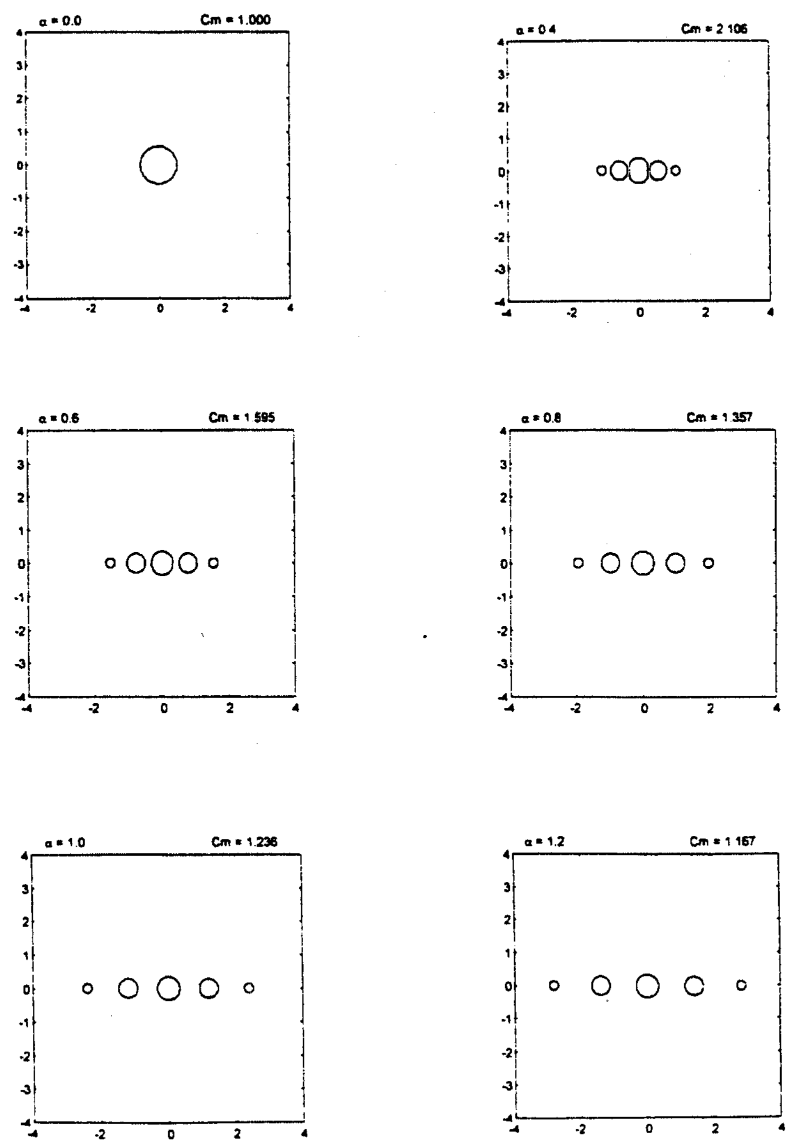

FIG.(1): Geometric synthesis of the function of form (3a) for different values of the parameter α.

3. EXPERIMENTAL SET UP

Five cylinders with length $l = 0.950$ m and with diameters {0.150m; 0.122m; 0.061m} were fixed in a rigid frame with the centers equidistant from each other by an adjustable distance b. The length l is a bit smaller than the wave tank width and if $S = 0.047$ m^2 is the total cross section area, then $a = (S/\pi)^{1/2} = 0.122$ m and the parameter α is defined by the equality $b = 2\alpha a$. So by changing b one can change the value of the *tuning parameter* α. Two load cells were placed along the transversal arms of the frame, the line joining the load cells being coincident with the longitudinal axis of the wave tank. A low amplitude harmonic wave was then imposed by the wave maker and the resultant heave force was obtained by the sum of the forces in each load cell.

Preliminary experimental results seems to confirm the proposed approximation, a consistent set of experimental results being planned to be presented at the workshop.

Different *functions of form* can be synthesized leading to geometries that can be usefull as cross sections of the pontoons of a TLP or a semi-submersible platform. The same approximation can also be developed in 3D and, in particular, for a body of revolution, where a stream function can be introduced, the same Hamiltonian approach can be used to generated the body geometry.

4. REFERENCE

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