A Boussinesq-panel method for predicting the motion of a moored ship in restricted water

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This abstract describes a practical approach to predicting the non-linear wave-induced motions of a moored vessel in restricted waters. A combination of Boussinesq theory for wave propagation over mildly-sloping bathymetries and linear potential theory for wave-body interaction is employed to produce engineering answers to real problems with modest computational effort. The moored body problem in the near-shore region is in some ways more complicated than the open water case and in other ways simpler. While a deep water irregular incident wave is readily decomposed into a Fourier sum of long-crested waves, no such simple description is available for the waves in a semi-enclosed variable depth basin. On the other hand, when the structure is moored in finite-depth or shallow water, the relatively short mooring lines make large amplitude response less likely, while the protective surrounding boundaries of the harbour or bay tend to ensure small wave steepness and the release of bound harmonics, reducing the importance of higher-order wave forcing. In deep water, second- (or higher-) order wave forcing is typically necessary to induce a resonant response of the moored structure, while in restricted waters the release of low frequency energy will often resonate with natural sloshing-type modes in the basin, which can lead to resonant response of the moored structure at first-order in the wave forcing.

Based on these observations, we focus here on obtaining an accurate description of the wave flow at the moored ship, including any natural sloshing-type modes which may be present, and on the sensitive dynamics introduced by a non-linear system of mooring lines and fenders. Non-linear wave-body interactions are however, ignored. Two well established methods are combined to make the calculations. First the wave field in the bay or harbour is computed using modified Boussinesq theory [2], an efficient method for predicting non-linear wave propagation over variable bottom topography. The flow is then linearised locally, by assuming that in the near vicinity of the ship the waves obey the linear free-surface boundary condition in water of constant depth. Linear potential theory [3] is used to describe the hydrodynamic interaction between the incident wave and the floating body, where the Haskind relations are invoked to determine the wave diffraction force from the locally linearised Boussinesq solution. Finally the equation of motion is solved in the time-domain, including the instantaneous point restoring force due to each mooring line and fender, to compute the motion response of the body. Viscous and fender friction damping forces may become important when the motions include resonant response with little or no wave damping, and approximations to these forces are included by means of empirically derived coefficients.

A brief mathematical expression of the ideas expressed above follows, more details can be found in [1]. The hydrodynamic interaction between the fluid and the floating body is assumed to be well described by linear potential theory. That is to say, regardless of the non-linear, and/or refraction-diffraction effects (either from the variable bottom topography or the boundaries of the bay) which have influenced the wave flow on it's way to the ship; in the local vicinity of the ship the flow is assumed to satisfy the linear free-surface boundary condition in uniform water depth. It is also assumed that certain non-linear external forces can be applied to the body, without violating the original hydrodynamic assumptions (*i.e.* body motion and wave steepness remain small.) The condition of small motions will typically be ensured by the mooring system

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in near-shore regions. Under these assumptions the equation of motion for the body can be written in the following convolution form

$$\sum_{k=1}^{6}(M_{jk}+a_{jk})\ddot{x}_k(t)+\int_0^t d\tau\, K_{jk}(t-\tau)\dot{x}_k(\tau)+C_{jk}\,x_k(t)=F_{jD}(t)+F_{j\mathrm{nl}}(t); \qquad j=1,6. \quad (1)$$

In this expression, $x_k(t)$ represents the position, and angular rotation of the body in 6 rigid-body degrees of freedom in Cartesian coordinates where x_1 is alligned with the ship axis pointing forward, and x_3 is oriented vertically upwards. An over-dot indicates differentiation with respect to time. The body's linear inertia matrix is M_{jk}, and the hydrostatic restoring-force coefficients are given by C_{jk}. The force due to radiated waves generated by the body's motion is expressed by an impulsive contribution a_{jk}; plus a convolution of the radiation memory function K_{jk} with the body velocity $\dot{x}_k$. The wave exciting force $F_{jD}(t)$ is obtained by employing the Haskind relations, which express the force in terms of incident wave quantities and solutions to radiation problems

$$F_{jD}(t)=\int\!\!\int_{S_b} d\vec{\xi}\, p_I(\vec{\xi},t)\,n_j(\vec{\xi})+\rho\int_{-\infty}^{\infty}d\tau\int\!\!\int_{S_b}d\vec{\xi}\,\phi_j(\vec{\xi},t-\tau)\dot{\phi}_{In}(\vec{\xi},\tau). \quad (2)$$

In Equation (2), ϕ_j is the solution to the j^{th}-mode impulsive velocity radiation problem, which is defined by the boundary condition $\vec{n}\cdot\vec{\nabla}\phi_j=n_j\,\delta(t)$, ($\delta(t)$ the Dirac function) while ϕ_I is an incident wave potential satisfying the linear free-surface boundary condition and inducing a first order dynamic pressure p_I. (The term "incident wave" is used to refer to the wave undisturbed by the presence of the body.) A subscript n indicates the operation $\vec{n}\cdot\vec{\nabla}$, with $\vec{n}$ the normal vector to the equilibrium wetted body surface S_b, while n_j is the generalised unit normal in six degrees of freedom. The first term in Equation (2) is often referred to as the Froude-Krilov force, while the second term describes the scattering of the incident wave by the body when fixed to its initial position. Any non-linear external forces, such as those due to the mooring system or viscous/frictional damping, are included in the equation of motion via the term $F_{j\mathrm{nl}}(t)$.

Note that the strictly linear version of Equation (1), (*i.e.* with $F_{j\mathrm{nl}}=0$) can be expressed in the frequency-domain by taking the incident wave to be a Fourier sum of time harmonic waves, $\zeta(t)=\mathrm{F}^{-1}\{\tilde{\zeta}(\omega)\}$, where F is the Fourier tranform operator and ω the radian frequency. The response is then $x_k(t)=\mathrm{F}^{-1}\{\tilde{x}_k(\omega)\}$ and Equation (1) becomes

$$\sum_{k=1}^{6}\left\{-\omega^2[M_{jk}+A_{jk}(\omega)]+i\omega\,B_{jk}(\omega)+C_{jk}\right\}\tilde{x}_k(\omega)=\tilde{F}_{jD}(\omega); \qquad j=1,2,...,6. \quad (3)$$

It is convenient (and more efficient) to compute the neccessary radiation and diffraction quantities in the frequency-domain, and then inverse Fourier transform before solving Equation (1).

To obtain the incident wave quantities required by the Haskind relations (2) we employ Boussinesq theory. Boussinesq theory provides an efficient means of solving the Laplace equation, along with the non-linear free-surface condition and Neumann conditions on an arbitrary (mildly-sloping) bottom and an enclosing boundary. The vertical variation of the flow is expanded in a power series, and modified formulations retain good non-linear and dispersion characteristics for relatively steep waves up to kh=3 (k the magnitude of the wave number and h the local water depth.) As a result of the Boussinesq calculations, the incident wave free-surface elevation $\zeta(\vec{x},t)$, and depth-averaged horizontal velocity $[u'(\vec{x},t),v'(\vec{x},t)]$ are known at a set of points $\vec{x}$ on the intersection of the still water plane with the body, where

$$[u',v']\equiv\frac{1}{(H+\zeta)}\int_{-H}^{\zeta}[u(\vec{x},z,t),v(\vec{x},z,t)]\,dz, \quad (4)$$

and $H(\vec{x})$ is the still water depth at the point $\vec{x}$. The flow in the vicinity of the structure is assumed to be a superposition of linear free waves in water of constant depth h, the average still water depth under the vessel. Thus the surface pressure and vertical component of velocity are expressed via the linear free-surface boundary conditions as $p_0(\vec{x},t) = p(\vec{x},0,t) = -\rho\dot{\phi}_I(\vec{x},0,t) = \rho g\zeta(\vec{x},t)$, and $w_0(\vec{x},t) = w(\vec{x},0,t) = \dot{\zeta}(\vec{x},t)$. These quantities are Fourier transformed to get $\tilde{p}_0$, $\tilde{u}'$, $\tilde{v}'$, and $\tilde{w}_0$. Consistent with linear theory, all frequency domain quantities are assumed to have a hyperbolic depth variation so that

$$c(\vec{x},z,t) = F^{-1}\left\{\tilde{c}_0(\vec{x},\omega)\frac{\cosh[k(z+h)]}{\cosh(kh)}\right\}; \quad w(\vec{x},z,t) = F^{-1}\left\{\tilde{w}_0(\vec{x},\omega)\frac{\sinh[k(z+h)]}{\sinh(kh)}\right\}, \qquad (5)$$

where $c = [u,v,p]$ and the linear dispersion relation $\omega^2 = gk\tanh(kh)$ relates the the wave number to the frequency. Equation (4) reduces to a relatively simple relationship between the depth-averaged and still water level velocities in this case, $[\tilde{u}_0,\tilde{v}_0] = [\tilde{u}',\tilde{v}']\frac{kh}{\tanh(kh)}$. This linearization procedure provides an incident wave pressure, and normal component of velocity (in either the time or the frequency domain), which can be combined with the radiation potential through the Haskind relations to obtain the complete diffraction exciting force on the body.

An experiment with a ship tied up in an L-shaped harbour was recently performed at the Danish Hydraulic Institute and it provides a good test case for the numerical model. The geometry of the experiment is shown in Figure 1. The ship was a 1/80 scale model of a 72,000 metric ton total displacement LPG carrier, water depth was 24 meters at full scale and the mooring lines and fenders were linear springs. This figure also compares the measured and computed incident wave elevation spectra at the ship's center. It is important to note that although the wave-maker signal was a strictly linear signal based on the Pierson-Moskovitz spectrum with T_p = 11s and H_s = 4.9m, (thus containing no energy below approximately f = .05 Hz) non-linear interaction among the waves, and reflection from the tank boundaries, produces significant low-frequency sloshing-mode energy in the basin, and this effect is well captured by the Boussinesq calculations. The Boussinesq model was run with nearly 100,000 grid points and 18,000 time steps (Δx = 6 m, Δt = .22 s) corresponding to just over an hour of full-scale simulation time. The beach was modelled by applying a porosity layer to that portion of the free-surface. CPU time for the Boussinesq calculation was approximately 8 hours on one 120 MHz node of an IBM RS/6000 sp-2 machine. The hydrodynamic coefficients of the ship were computed using WAMIT with 800, 1600, and 2400 panels to show convergence at 259 frequencies. The 2400 panel run required approximately 4 CPU hours. Linearisation of the Boussinesq incident wave, Fourier transform of the coefficients, and integration of the equation of motion to perform the simulation required approximately 2 CPU minutes. Figure 1 also shows the response amplitude operators of the ship in surge, sway, roll and yaw (heave and pitch are unaffected by the mooring system and show good agreement with the experiments.) The coefficient of dynamic friction at the fenders was measured as part of the experiment, and this was included in the calculations as a point force in opposition to the ship's velocity in surge and angular velocity in roll. The model does a good job of capturing the non-linear response of the vessel, although (not surprisingly) it appears to be neccessary to include viscous roll damping in the calculations.

References

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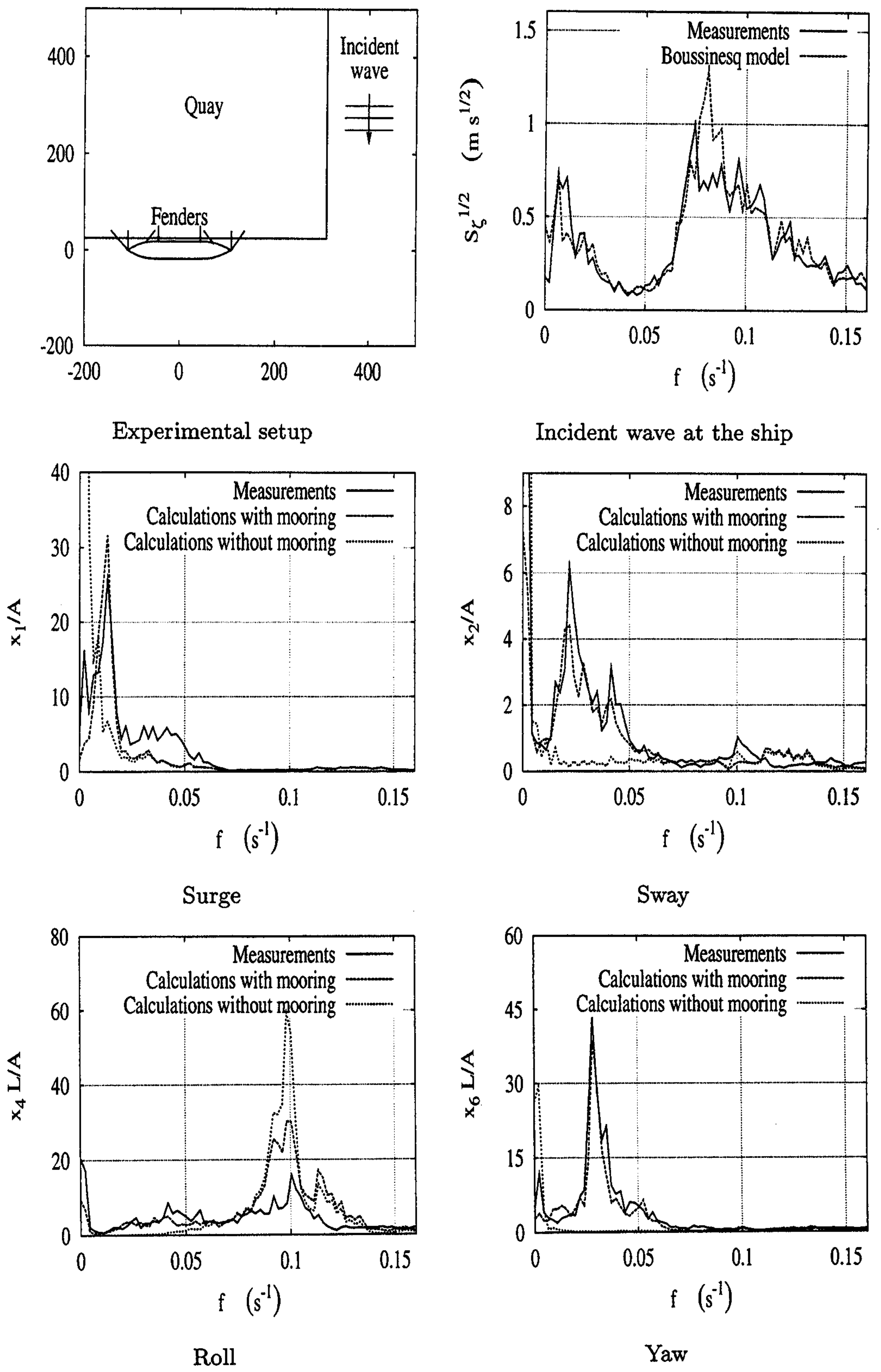

Figure 1: Comparison of the calculations with experimental measurements for a ship moored in an L-shaped harbour. Results are at full-scale. Calculations without the mooring system are also shown for reference. L is ship length and A wave amplitude.