On the generation of upstream large amplitude internal waves at a bottom topography in the ocean

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Introduction

Internal waves may occur in a layered ocean or fjord, where the density variation is due to a gradient in salinity or temperature. Knowledge of flows due to internal waves, their origin, propagation and implications on the surroundings is important for many reasons. Technological examples are effects on floating units for exploration or production of oil in deep water, and more generally sub-subsurface installations. Internal waves have also important implications within fields like biology and geology. Examples are mixing due to wave breaking which may stimulate the production in the ocean, and transport of larvae and sediments. Many studies of internal waves have been devoted to the propagation properties of the waves. Here we shall focus on the generation of the waves. More specifically, we shall focus on internal waves generated by tidal flow over a bottom topography, for example at a shallow fjord sill. This topic is currently receiving much attention both by experimentalists and mathematical modellers. The common aim is a better understanding and description of the origin of internal waves in the ocean or in fjords.

Korteweg-de Vries or Boussinesq models may be useful to simulate stratified flows, provided that the waves are sufficiently long to satisfy the assumptions of the models. The latter is not always the case, however. Observations of stratified tidal flows at sills reveal that rather strong nonlinear effects may take place and that many of the waves may relatively be rather short.

We shall therefore, here, employ a fully nonlinear and fully dispersive model in a two-dimensional investigation of the flow at a bottom topography. The model is a two-layer model and has a lower fluid with thickness h_1 and constant density ρ_1. The upper fluid has thickness h_2 and constant density $\rho_2 < \rho_1$. A coordinate system $O - xy$ is introduced with the x-axis along the interface at rest and the y-axis pointing upwards. We model the flow in each of the layers by potential theory. A Lagrangian method is adopted, where pseudo Lagrangian particles are introduced on the interface, each with a weighted velocity defined by $\mathbf{v}_\times = (1-\alpha)\mathbf{v}_1 + \alpha\mathbf{v}_2$, where $\mathbf{v}_1$ and $\mathbf{v}_2$ denote the fluid velocities in the respective layers, and $0 \le \alpha \le 1$. The prognostic equations are obtained from the kinematic and dynamic boundary conditions at the interface, i.e.

$$D_\times(x,y)/dt = \mathbf{v}_\times, \tag{1}$$

$$D_\times(\phi_1 - \mu\phi_2)/dt = \mathbf{v}_\times \cdot (\mathbf{v}_1 - \mu\mathbf{v}_2) - (1/2)(\mathbf{v}_1^2 - \mu\mathbf{v}_2^2) - (1-\mu)gy \tag{2}$$

where $\mu = \rho_2/\rho_1$, $D_\times/dt = \partial/\partial t + \mathbf{v}_\times \cdot \nabla$ and the gravity g acts downwards. The prognostic equations are expanded in Taylor series, keeping several terms, in order to obtain an efficient scheme. Accurate solution of the Laplace equation is obtained by application of complex theory and Cauchy's integral theorem. The model is derived in reference 1. Experimental evidence of the usefulness of the model is provided in reference 2.

Model simulations of the flow at a sill

We have chosen to model the flow at a sill in the Knight Inlet in British Colombia, as described in some recent observations by D. Farmer and co-workers (reference 3). Data representing typical conditions during tidal flows in this inlet (fjord) are

- Length scale: depth of upper layer $h_2 = 4.5m$
- Depth of lower layer (in computations): $h_1/h_2 = 32$
- Time scale and densities: $\sqrt{h_2/g}$, $\Delta\rho/\rho \simeq 0.014$
- Typical velocity in the ocean: $U \sim 0.5 - 0.8m/s$
- Nondimensional speed of geometry/flow: $U/\sqrt{gh_2} \simeq 0.07 - 0.15$
- Linear long wave speed (for comparison): $c_0/\sqrt{gh_2} \simeq 0.1183$

We have not yet been able to simulate the relevant semidiurnal tidal flow. Thus, we here apply periods of 1.2 and 2.4 hours, for convenience.

The simulations (figures 1 and 2) reveal that a depression of the interface develops at the lee side of the sill. This depression may become rather significant as time goes on. Above and slightly ahead of the sill the interface becomes elevated. Between the elevation and depression a wave train is generated propagating upstream. The number of waves increase with time. When the oscillatory (tidal) flow slackens, these waves may travel by their own into deep water, and evolve according to dispersion and nonlinearity. The figures reveal that the generated waves have amplitude up to about twice the depth of the thinner (upstream) layer. The simulated wave trains bear strong resemblance with observed waves at the Knigth Inlet. Both the wave length, amplitude and number of waves are in fact rather close. Similar waves are observed at other places, like e.g. at the Gibraltar.

The large scale observations reveal also that a rather deep depression is formed behind the sill. It is rather evident that strong mixing takes place in this depression (reference 3), and it is unclear how this can be modelled. Another phenomenon which is frequently observed at the Knight Inlet is the generation of mode 1 and mode 2 depression solitary waves. These waves have quite large amplitude (reference 4).

Returning to the theoretical model, we found in reference 1 generation of pronounced upstream solitary waves. They were generated due to a moving geometry in the thinner of the two layers. In a fjord the geometry is usually present in the thicker of the (two) layers. The question is still open: how are solitary waves generated at such a geometry? At the workshop more detailed and extensive results will be presented.

This work was conducted under the Strategic University Programme 'General Analysis of Realistic Ocean Waves' at the University of Oslo, financed by The Research Council of Norway.

References

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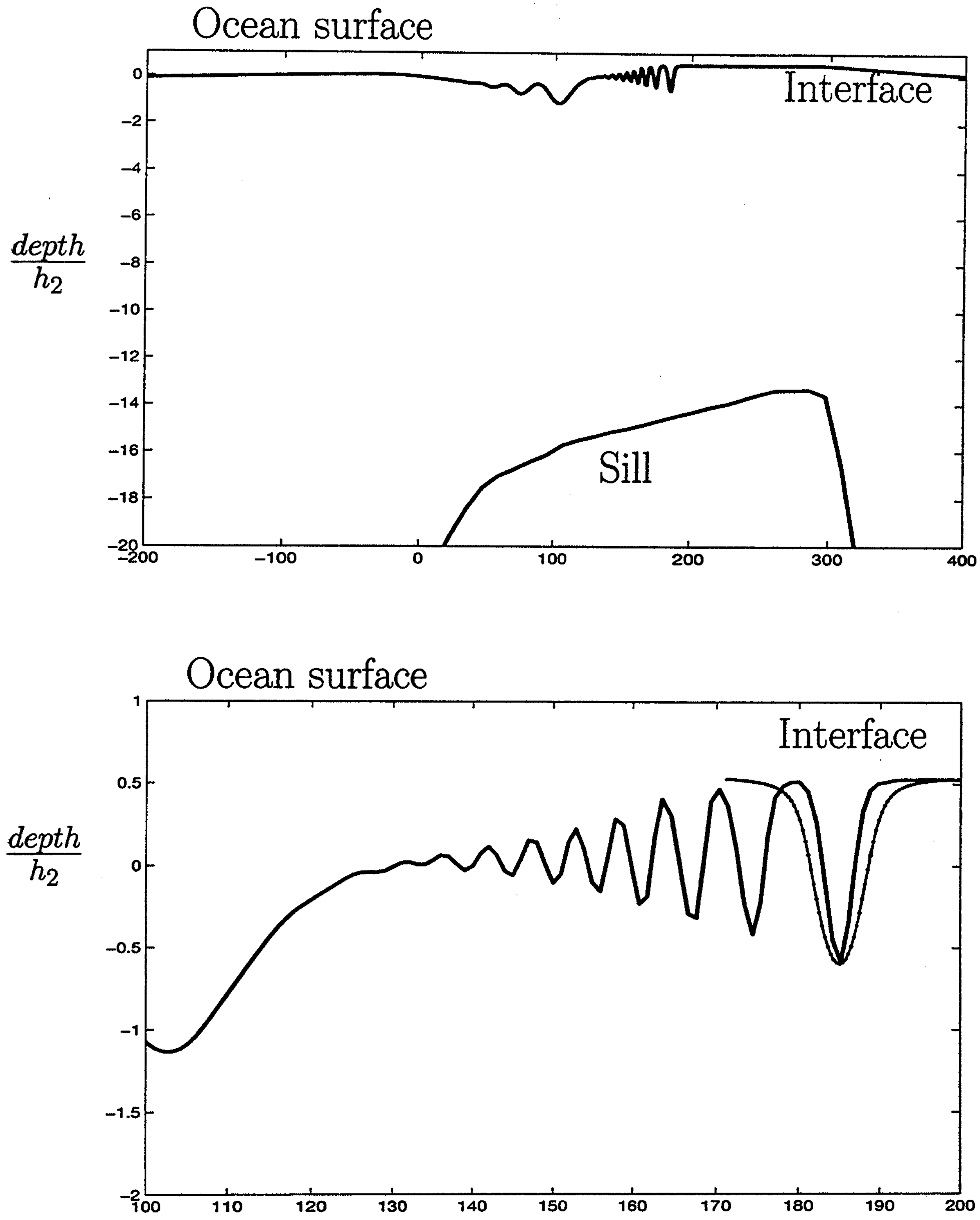

Figure 1: Development of interface. Model simulations of the flow at Knight Inlet. Oscillatory flow. $U = U_0 \sin \omega t$, $U_0/\sqrt{gh_2} = 0.08$, $\omega t = 0.84\pi$, 1.2 hours period. Bottom figure: blow up of upper figure. Dots in bottom figure: solitary wave solution (for comparison).

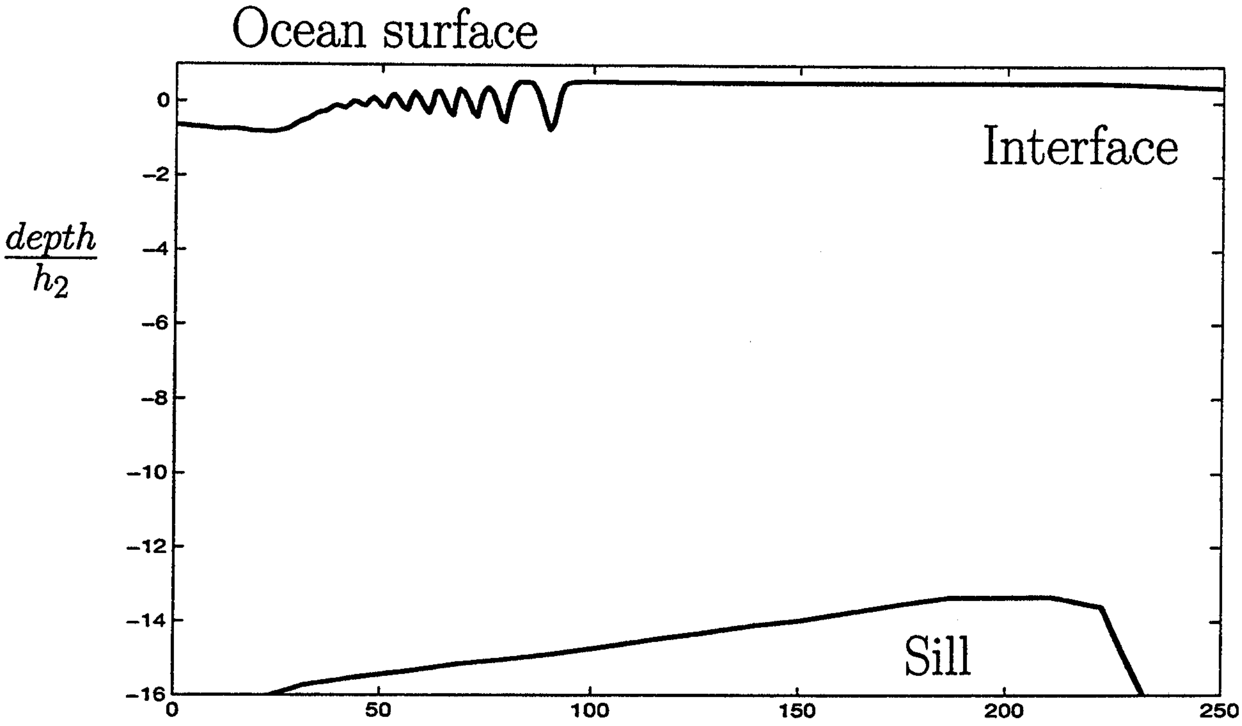

Figure 2: Same as figure 1, but $\omega t = 0.53\pi$, 2.4 hours period.