WATER IMPACT ON CYLINDRICAL SHELLS

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The plane unsteady problem of deflection of a cylindrical circular shell in collision with an ideal incompressible liquid is considered. The Wagner approach and the method of normal modes are used to take into consideration fluid-structure interaction. The analysis is focused on strain-time histories of the inner surface of the cylinder. The computed results are compared with the model experiments by Shibue *et al.* (1994) for steel shells and by Arai & Miyauchi (1998) for an aluminium shell. The predicted elastic response agrees fairly well with the measured response for the steel cylinder with high rigidity and not so good for the cylinders with low rigidity.

Hydrodynamic loads on entering shells peak just after the impact and decay with time. The strains approach their maximum values much later, when the hydrodynamic pressures are already low. The time, when the strains reach their maximum values, can be estimated as a half of the period of the lowest mode of the cylinder vibration. For cylinders with high rigidity this period is small, which indicates that the strains peak at small penetration depth, where the Wagner theory can be used to evaluate the hydrodynamic loads. At the initial stage of the impact the deformations of the liquid volume are infinitesimal, which allows us, as a first approximation, to put the boundary conditions on the undisturbed initial level of the liquid and to linearize them and the equations of motion near the initial rest state.

1. Formulation of the problem

Initially the liquid is at rest and occupies a lower half-plane ($y' < 0$), and the elastic cylinder touches its free surface ($y' = 0$) at the single point taken as the origin of the Cartesian coordinate system $x'Oy'$ (dimensional variables are denoted by a prime). At the initial instant of time ($t' = 0$) the cylinder of radius R starts to penetrate the liquid vertically with its initial velocity being V. The shape of the entering cylinder and the cylinder velocity are changed owing to the interaction of the elastic cylinder with the liquid. The presence of the contact points between the free surface and the elastic body is the main feature of the problem. The positions of these points are unknown in advance and must be determined together with the liquid flow and the shell deflections.

We shall determine the elastic cylinder deflection, the bending stress distribution and the contact point positions under the following assumptions: (i) the liquid is ideal and incompressible; (ii) the liquid flow is plane, potential and symmetrical with respect to the y'-axis; (iii) the shell thickness is constant and small in comparison with the other two dimensions; (iv) external mass forces and surface tension are absent; (v) the period T of the lowest elastic mode vibration of the circular shell is small compared with the ratio R/V; (vi) the dimension of the wetted area of the entering cylinder is the monotonic function of time.

The problem is considered in non-dimensional variables. The scales are: $L = \sqrt{RVT}$ is the length scale, T is the time scale, V is the velocity scale of the liquid flow, $\rho VL/T$ is the hydrodynamic pressure scale, where ρ is the liquid density (Ionina 1998). The coupled problem has the form

$$\ddot{w} + \frac{ET^2}{\rho_0 R^2(1-\nu^2)}(w - v_\theta) + \frac{ET^2h^2}{12\rho_0 R^4(1-\nu^2)}(v_{\theta\theta\theta} + w_{\theta\theta\theta\theta}) = \frac{\rho L}{\rho_0 h}p_0(\theta, t) \quad (-\pi < \theta < \pi), \tag{1}$$

$$\ddot{v} + \frac{ET^2}{\rho_0 R^2(1-\nu^2)}(w_\theta - v_{\theta\theta}) - \frac{ET^2h^2}{12\rho_0 R^4(1-\nu^2)}(v_{\theta\theta} + w_{\theta\theta\theta}) = 0 \quad (-\pi < \theta < \pi), \tag{2}$$

$$v(\theta, 0) = w(\theta, 0) = 0 \qquad (-\pi < \theta < \pi), \tag{3}$$

$$v_t(\theta, 0) = -\sin\theta, \;\; w_t(\theta, 0) = -\cos\theta \qquad (-\pi < \theta < \pi), \tag{4}$$

$$p = -\varphi_t \qquad (y \le 0), \tag{5}$$

$$\varphi_{xx} + \varphi_{yy} = 0 \qquad (y < 0), \tag{6}$$

$$\varphi = 0 \qquad (y = 0, \mid x \mid > c(t)), \tag{7}$$

$$\varphi_y = w_t \qquad (y = 0, |x| < c(t)), \tag{8}$$

$$\varphi \to 0 \qquad (x^2 + y^2 \to \infty), \tag{9}$$

$$\frac{1}{4}c^2 + \frac{2}{\pi}\int_0^{\pi/2} w[c(t)\sin\theta, t]d\theta = 0. \tag{10}$$

where r, θ are the polar coordinates, $\theta = 0$ corresponds to the lowest point of the body, w and v are the radial and angular components of absolute displacements of the shell elements, respectively, ρ_0 is the density of the shell material, E is the elasticity modulus, ν is the Poisson's ratio, $p(x, y, t)$ is the hydrodynamic pressure, $p_0(\theta, t)$ is the external (hydrodynamic) load, which acts on the shell. Within the contact region, $|x| < c(t)$ and $|\theta| < \theta_c(t)$, we obtain $p_0(\theta, t) = p(x(\theta, t), y(\theta, t), t)$, where $x(\theta, t)$ and $y(\theta, t)$ are the horizontal and vertical coordinates of the entering elastic cylinder with $x(\pm\theta_c(t), t) = \pm c(t)$. At the initial stage of the impact, where $\theta_c \ll 1$, the approximate formulae $x \approx \theta/\gamma$, $\theta_c(t) \approx \gamma c(t)$ are valid, where $\gamma = L/R$. Dot stands for the time derivative. The initial conditions (3) and (4) imply that the shell is undeformed before the impact and moves vertically. It should be noted that we do not make the non-elongation assumption at the neutral surface in the equations of the shell dynamics (1) and (2). In order to derive equation (10), which is to determine the dimension of the contact region (see Korobkin 1996), the undeformed shape of the shell close to the impact point is approximated by parabolic contour.

The solution of the boundary-value problem (1)-(10) is sought, according to the method of normal modes, in the form

$$\varphi(\theta/\gamma, 0, t) = \sum_{n=0}^{\infty} \varphi_n(t)\cos n\theta, \quad p(\theta/\gamma, 0, t) = \sum_{n=0}^{\infty} p_n(t)\cos n\theta,$$

$$w(\theta, t) = \sum_{n=0}^{\infty} a_n(t)\cos n\theta, \quad v(\theta, t) = \sum_{n=1}^{\infty} b_n(t)\sin n\theta,$$

where $-\pi < \theta < \pi$, the principal coordinates $a_1(t)$ and $b_1(t)$ describe the rigid motion of the entering cylinder, and the principal coordinates $a_0(t)$ and $a_n(t)$, $b_n(t)$, $n \geq 1$, the elastic deformation of the shell. It should be noted that $p_n(t) = -\dot{\varphi}_n(t)$, $n \geq 0$, which follows from (5) and $\varphi(\theta/\gamma, 0, t) \equiv 0$, where $\gamma c(t) < |\theta| < \pi$ (see the boundary condition (7)).

It is convenient to introduce the new unknown functions $g_n(t) = \dot{a}_n(t) + (\rho L/\rho_0 h)\varphi_n(t)$ and $r_n(t) = \dot{b}_n(t)$ and take the dimension of the contact region c as the new independent variable with time t being the function of c. The differential equation for the function $t(c)$ follows from (10) after its differentiation with respect to c. Equations (1), (2), (5)-(10) provide the nonlinear system of ordinary differential equations with respect to the functions $a_i(c)$, $g_i(c)$, $i \geq 0$, and $b_n(c)$, $r_n(c)$, $n \geq 1$. This system is solved under the initial conditions

$$\begin{gathered} a_0 = 0, \quad g_0 = 0, \quad t = 0, \\ a_1 = 0, \quad g_1 = -1, \quad b_1 = 0, \quad r_1 = -1, \\ a_n = 0, \quad g_n = 0, \quad b_n = 0, \quad r_n = 0 \quad (n \geq 1). \end{gathered} \tag{11}$$

The initial-value problem is solved numerically by the fourth-order Runge-Kutta method with uniform step Δc. Finite number of the normal modes, N, is taken into account with $a_n \equiv 0$, $b_n \equiv 0$, $g_n \equiv 0$, $r_n \equiv 0$ for $n \geq N+1$. The limitations for the step Δc have been discussed in detail by Korobkin (1998). It is revealed that calculations with $N = 15$ provide the almost exact solution, which varies a little with the number of modes N increase.

2. Numerical results

Experiments with cylindrical models were carried out by Shibue *et al.* (1994) and by Arai & Miyauchi (1998). The models used by Shibue *et al.* (1994) are made of steel with length of 600 mm and the outer diameter of 312 mm, which falls down on two-dimensional water way from the height of 1 m. The first cylinder was the thick model with 5.1 mm thickness and the total weight of 23.8 kg. The second one was the thin model with 1.0 mm thickness. Only the thick model is considered below. The model used by Arai & Miyauchi (1998) is made of aluminium with length of 600 mm and the outer diameter of 306 mm. The total weight of the model including strain gauges, cables, etc. is 5.2 kg. The thickness of the shell is 3 mm. The strains at the

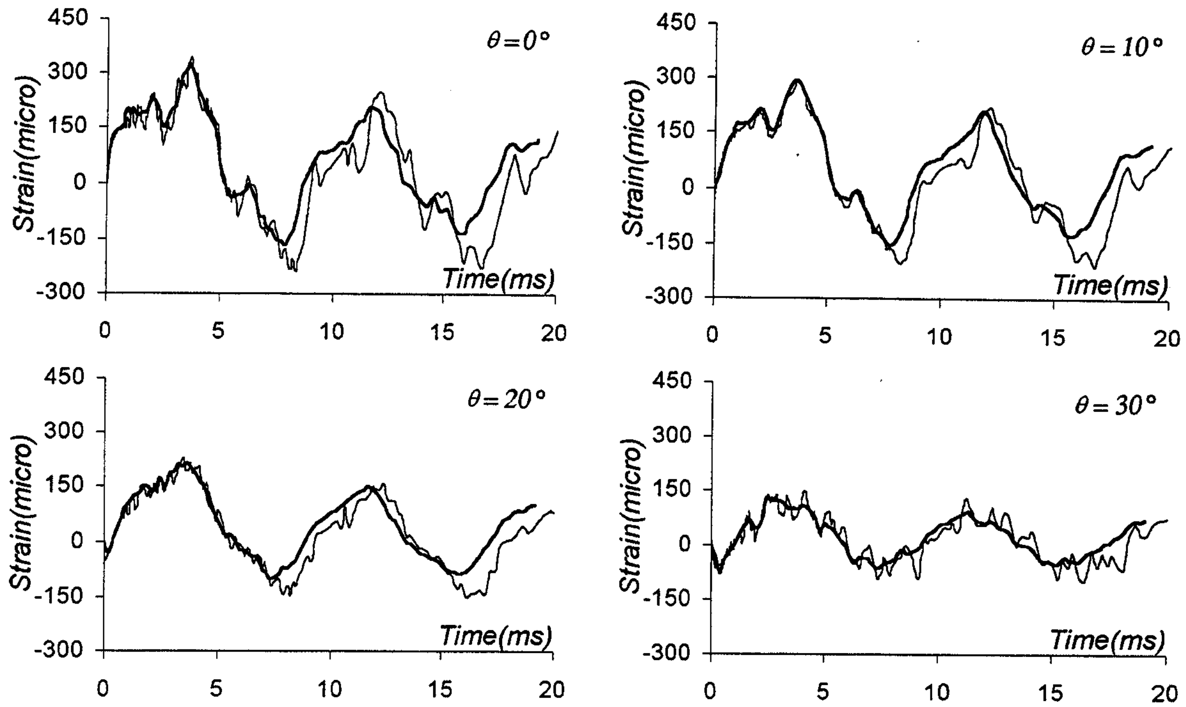

Fig. 1. Calculated (thin line) and measured (thick line) strains for the steel cylinder with 5.1 mm thickness at the angles of 0, 10, 20, 30 degrees (initial impact velocity is 3.5 m/s).

bottom of the models, $\theta = 0°$, appears about 4 ms after the contact for the steel cylinder and about 7 ms for the aluminium one.

Numerical simulations of the steel cylinder impact were carried out in the following conditions: $R = 0.156$m, $h = 5.1$ mm, $m = 23.8$ кg, $E = 206 \cdot 10^9$ Pa, $\nu = 0.3$, $\rho_0 = 8067$ kg/m^3, $V = 3.5$ m/s. The density of the shell material ρ_0 was evaluated under the assumption that the total mass of the model is uniformly distributed over the cylinder. In order to evaluate the initial impact velocity V, the calculations with five "dry" modes were performed for different impact velocities to determine the maximum strains, σ_{max}, at the model bottom. It was found that the function $\sigma_{max}(V)$ can be approximated as $\sigma_{max}(V) = 14.386V^2 + 45.139V$, where σ_{max} is in microstrains. Experiments provide $\sigma_{max} = 322$ microstrains, which approximately corresponds to the impact velocity of 3.5 m/s. Figure 1 shows calculated and experimental strains at different angles measured from the cylinder bottom. Calculated results are obtained with 15 "dry" modes. It is seen that the strain histories are predicted fairly well with the present approach at the initial stage, $0 < t' < 9$ ms, which is enough to evaluate the maximum values of strains. Calculations with $V = 4.4$ m/s, which is the initial impact velocity for free fall of a body from the height of 1 m without taken into account the air presence, overpredict the strain maximum almost twice. This means that the impact velocity is an important parameter of the process and its accurate evaluation is significant for adequate numerical simulations of the impact. It should be noted that the present approach makes it possible to evaluate the change of the velocity of the body rigid motion with time. We found that the velocity drops from 3.5 m/s at the contact instant, $t' = 0$, down to 2.5 m/s at $t' \approx 5$ ms and to 2.0 m/s at $t' \approx 15$ ms. Elastic deformations of the shell are quite small: the normal deflection $w'(\theta, t')$ is less than 1 mm and the angular displacement of the shell elements is less than 0.4 mm.

It was revealed that the assumption of the neutral surface non-elongation, which essentially simplifies the equations of the shell dynamics and is in common use in structural analysis of shell, may provides misleading results for the amplitudes of strains and their time histories.

Numerical simulations of the aluminium cylinder impact were performed in the following conditions: $R = 0.153$ m, $h = 3.0$ mm, $m = 5.2$ kg, $E = 74 \cdot 10^9$ Pa, $\nu = 0.34$, $\rho_0 = 3040$ kg/m^3, $V = 3.43$ m/s. The density of the shell material ρ_0 and the initial impact velocity V were evaluated in the same way as for the steel cylinder. It should be noted that the impact velocity is almost the same as for the steel cylinder, which implies that

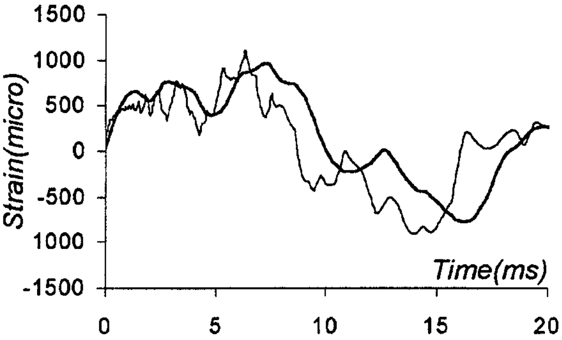

Fig. 2. Calculated (thin line) and measured (thick line) strains for the aluminium cylinder with 3.0 mm thickness at the bottom of the model (initial impact velocity is 3.43 m/s). Calculations are performed with 15 "dry" modes.

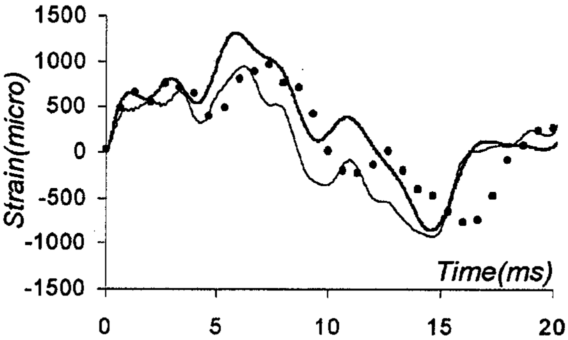

Fig. 3. Calculated and measured strains for the aluminium cylinder with 3.0 mm thickness at the bottom of the body ($V = 3.43$ m/s): —, full nonlinear hydrodynamic model and three-mode approximation for the shell dynamics (Arai & Miyauchi 1998);—, present calculations with five "dry" modes; • • •, measured strains (Arai & Miyauchi1998).

its value depends mainly on the body geometry and its weight. The density $\rho_0 = 3040$ kg/m^3 is greater than the density of aluminium, 2700 kg/m^3, which is due to the additional weight of the model, which comes from cables and connectors (see Arai & Miyauchi 1998). This additional weight is about 11% of the total mass for the aluminium cylinder and just 3% for the steel cylinder. The present approach deals with isotropic shells and effects associated with additional weights are not taken into account. It is well known that additional masses attached to a shell can essentially reduce the lowest frequency of the shell free vibration. This effect can be observed in Figure 2, where the calculated results are compared with the experimental ones by Arai & Miyauchi (1998). The calculations were performed with 15 "dry" modes. It is seen that the calculated maximum strain at the bottom of the cylinder appears about 2 ms before the measured strain does. We expect that the agreement between the calculated and measured strain-time histories would be better with more complicated shell model, where the presence of additional weights is account for.

It should be noted that the numerical calculations by Arai & Miyauchi (1998), where fully nonlinear hydrodynamic model and the three-mode approximation for the shell dynamics were used, demonstrate similar defects (see Figure 3).

We conclude that the present approach based on the Wagner theory and the method of normal modes can be recommended to obtain both the estimations of maximum strains and strain histories for not very flexible and isotropic shells.

This research was supported by Russian Foundation for Basic Research Grant N 96-01-01767 and by SD RAS integrate project N 43.

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