SECOND-ORDER WAVE DIFFRACTION BY THIN VERTICAL BARRIER IN DEEP WATER

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1. Introduction

The importance of the inclusion of *nonlinear* effects in the calculation of wave forces on offshore structures is well known. The motivation for this arises primarily due to the need to calculate more accurate wave force predictions than those provided by the *linear* scattering theory which is based on the assumption of *small* wave amplitude. Fully nonlinear prediction on the wave force on a body is a very difficult task. However, many researchers extended the linear theory to *second-order*, although the *second-order theory* is also limited to *small* amplitude waves. But some *nonlinear* effects on a body have been successfully predicted by a second-order analysis, which could not be predicted by the linearised theory. The second-order theory for wave diffraction by a submerged cylinder in deep water or in water of uniform finite depth has been investigated earlier in the literature by many researchers (cf Ogilvie(1963), Vada(1997), Wu and Taylor(1990), McIver and McIver(1990), Wu(1991,1993) and others).

In the present paper we have studied second-order theory for water wave scattering by a thin vertical barrier, which may be surface piercing or submerged in deep water. The second-order reflection and transmission coefficients have been obtained analytically in terms of quantities obtainable explicitly on the basis of linerearised theory. Numerical results for the second-order reflection coefficients against the wave number for two configurations of the barrier are depicted graphically. It is observed that in the low frequency range, the second-order efficet is significant.

2. Formulation of the problem :

Let a thin vertical barrier represented by $x = 0, y \in L = L_j (j = 1, 2)$ be present in deep water, where the y-axis is taken vertically downwards through the barrier and the origin lies on the mean free surface, and $L_1 = (0, a)$ for a surface piercing barrier and $L_2 = (b, \infty)$ for a submerged barrier. A uniform wave train is incident on the barrier travelling from the direction of the positive infinity value of x. The motion in the water is assumed to be irrotational, so that it can be described by a potential function $\Phi(x, y, t)$. Let $y = \eta(x, t)$ denote the free surface depression below the mean free surface $y = 0$. Assuming the *steepness* parameter ϵ to be small, Φ and η may be expanded as

$$\Phi = \epsilon\Phi_1 + \epsilon^2\Phi_2 + \dots \tag{1}$$

and

$$\eta = \epsilon\eta_1 + \epsilon^2\eta_2 + \dots \tag{2}$$

where the *first-order* potential function Φ_1 gives rise to the *linearised theory* of water waves while Φ_2 gives rise to *second-order* theory.

Let the incident first-order wave have amplitude a and frequency σ, so that $\Phi_1^{inc}(x, y, K, t)$ is represented by

$$\Phi_1^{inc}(x, y, K, t) = Re\left[-\frac{iga}{\sigma}\phi^{inc}(x, y, K)e^{-i\sigma t}\right] \tag{3}$$

with $\phi^{inc}(x, y, K) = e^{-Ky - iKx}, K = \sigma^2/g, g$ being the gravity.

Then the first-order potential function $\Phi_1(x, y, K, t)$ can be expressed as

$$\Phi_1(x, y, K, t) = Re\left[-\frac{iga}{\sigma}\phi_1(x, y, K)e^{-i\sigma t}\right] \tag{4}$$

where $\phi_1(x, y, K)$ satisfies

$$\nabla^2\phi_1 = 0 \text{ in the fluid region ,} \tag{5}$$

$$K\phi_1 + \frac{\partial \phi_1}{\partial y} = 0 \text{ on } y = 0, \tag{6}$$

$$\phi_{1x} = 0 \text{ on } x = 0, y \in L, \tag{7}$$

$$r^{1/2}\nabla\phi_1 \text{ is bounded as } r \to 0, \tag{8}$$

where r is the distance from a submerged edge of the barrier,

$$\nabla\phi_1 \to 0 \text{ as } y \to \infty, \tag{9}$$

$$\phi_1(x,y,K) \sim \begin{cases} e^{-Ky}(e^{-iKx} + R_1(K)e^{iKx}) & \text{as } x \to \infty, \\ T_1(K)e^{-Ky-iKx} & \text{as } x \to -\infty, \end{cases} \tag{10}$$

In the condition (10), $R_1(K)$ and $T_1(K)$ denote *first-order* reflection and transmission coefficients. We may suppose that $\phi_1(x,y,K)$ is a known explicitly including $R_1(K)$ and $T_1(K)$ for $L = L_j (j = 1,2)$ (cf Dean(1945), Ursell (1947)).

The *second-order* potential function $\Phi_2(x,y,t)$ can be expressed as (see McIver and McIver (1990))

$$\Phi_2(x,y,t) = \Phi_s(x,y) + ct + Re\left[a^2\sigma\phi_2(x,y)e^{-2i\sigma t}\right] \tag{11}$$

where Φ_s, ϕ_2 are respectively the *steady* part and the *double-frequency* part of $\Phi_2(x,y,t)$, c is a constant that only affects the position of the mean free surface. We are interested in ϕ_2 only. It satisfies (5), (8), (9) together with

$$4K\phi_2 + \frac{\partial \phi_2}{\partial y} = f(x) \text{ on } y = 0 \tag{12}$$

where

$$f(x) = \frac{i}{K}\left[(\frac{\partial \phi_1}{\partial x})^2 + \frac{3}{2}K^2\phi_1^2 + \frac{1}{2}\phi_1\frac{\partial^2\phi_1}{\partial x^2}\right](x,0), \tag{13}$$

and

$$\phi_2(x,y) \sim \begin{cases} iR_1(K) + R_2e^{-4Ky+4iKx} & \text{as } x \to \infty, \\ T_2e^{-4Ky-4iKx} & \text{as } x \to -\infty. \end{cases} \tag{14}$$

As $x \to \infty$, the first term in the asymptotic relation (14) arises due to the interaction of the first-order incident and reflected waves, while the second term represents a free outgoing wave with frequency 2σ. There is no contribution to the incident wave in deep water at second order. Again, as $x \to -\infty$, the only term in the asymptotic relation (14) arises due to a free outgoing wave with frequency 2σ. R_2 and T_2 are called the secon-order reflection and transmission coefficients. These can be determined in terms of ϕ_1.

3. Second-order reflection and transmission coefficients

Let $\psi(x,y) \equiv \phi_1(x,y,4K)$ be the first-order potential due to an incident wave field with frequency 2σ propagating from the direction of positive infinity so that $\psi(x,y)$ is known for $L = L_1$ and L_2. We use the Green's integral theorem to the functions $\phi_2(x,y)$ and $\psi(x,y)$ in the region bounded by

$$y = 0, -X \le x \le 0^-, 0^+ \le x \le X; x = 0^{\pm}.0 < y < a; y = Y, \ -X \le x \le X; x = \pm X, 0 \le y \le Y,$$

for the surface-piercing barrier problem and in the region bounded by

$$y = 0, -X \le x \le X; x = \pm X, 0 \le y \le Y; y = Y, \ -X \le x \le 0^-, 0^+ \le x \le X; x = \pm 0, b < y < Y,$$

for the submerged barrier problem.

Now making $X, Y \to \infty$ we obtain the following expression for the second-order reflection coefficient R_2, given by

$$R_2 = \lim_{X\to\infty} \left[-i \int_{-\infty}^{X} \phi_1(x,0,4K) f(x) dx + iR_1(K) \left\{ R_1(4K) e^{4iKX} - e^{-4iKX} \right\} \right]. \qquad (15)$$

Again, an expression for the *second-order* transmission coefficient T_2 is obtained by applying Green's integral theorem to $\chi(x,y) \equiv \phi_1(-x,y,4K)$ and $\phi_2(x,y)$ in the same region mentioned above and making $X, Y \to \infty$ we obtain

$$T_2 = \lim_{X\to\infty} \left[-i \int_{-\infty}^{X} \phi_1(-x,0,4K) f(x) dx + iR_1(K) T_1(4K) e^{4iKX} \right]. \qquad (16)$$

These expressions for R_2 and T_2 have been derived by McIver and McIver (1990) for any *submerged* body.The integrands of the integrals in the relations (15) and (16) do not decay as $x \to \infty$. McIver and McIver (1990) suggested that the integrals must be combined with other non-zero terms in the expression before the limit is taken. A computable form of R_2 is thus given by

$$\begin{aligned} R_2 \;=\; & -iR_1(K)T_1(4K) - i \textstyle\int_0^\infty [\phi_1(x,0,4K) f(x) + \phi_1(-x,0,4K) f(-x) \\ & -4iKR_1(K) \left\{ e^{-4iKx} + R_1(4K) e^{4iKx} \right\}] \, dx. \end{aligned} \qquad (17)$$

and that for T_2 is given by

$$\begin{aligned} T_2 \;=\; & iR_1(K)T_1(4K) - i \textstyle\int_0^\infty [\phi_1(-x,0,4K) f(x) + \phi_1(x,0,4K) f(-x) \\ & -4iKR_1(K)T_1(4K) e^{4iKx}] \, dx. \end{aligned} \qquad (18)$$

4. Numerical results :

The expressions for R_2 anf T_2 are given by (17) and (18). $|R_2|$ and $|T_2|$ are evaluated numerically for a number of values of non-dimensional parameters Ka (for partially immersed barrier) and Kb (for submerged barrier). $|R_2|$ and $|T_2|$, for partially immersed barrier are depicted graphically against the wave number Ka, in figs 1 and 2 respectively. It is observed that for very small and very large wave numbers $|R_2|$ and $|T_2|$ become assymptotically small. This shows that the second-order effect is neglegible for very small and very large wave number while in the intermediate range it is significant in the case of the surface-piercing barrier.

Again, in figs. 3 and 4., for submerged barrier, $|R_2|$ and $|T_2|$ are depicted against the wave number Kb. Here also similar behaviour for large wave number is observed. However, while for partially immersed barrier configuration both $|R_2|$ and $|T_2|$ tend to zero as $Ka \to 0$, for submerged barrier configuration, both $|R_2|$ and $|T_2|$ oscillate for small Kb. This may be attributed due to the interaction between the free surface and the sharp edge of the submerged barrier.

Numerical computations of $|R_2|$ and $|T_2|$ for the submerged plate occupying the position $x = 0, a < y < b$ can be carried out by using the same formulae (17) and (18), since the first-order potential function $\phi_1(x, y, K)$ for this case is known (cf Evans(1970)).

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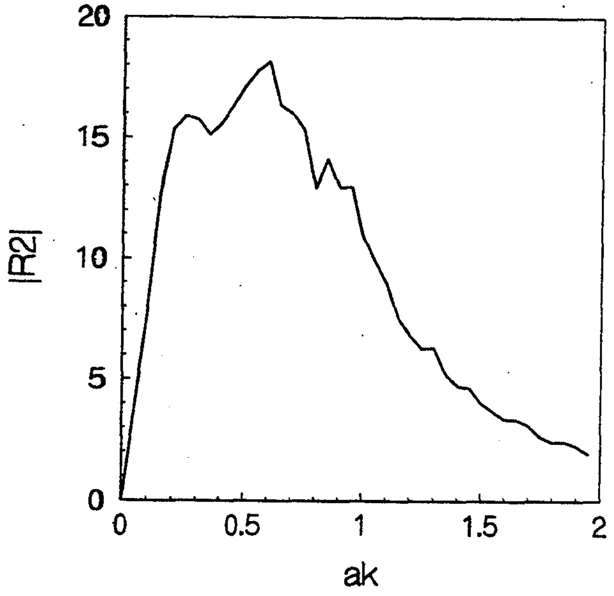

Fig. 1 Second-order reflection coefficiet for surface barrier

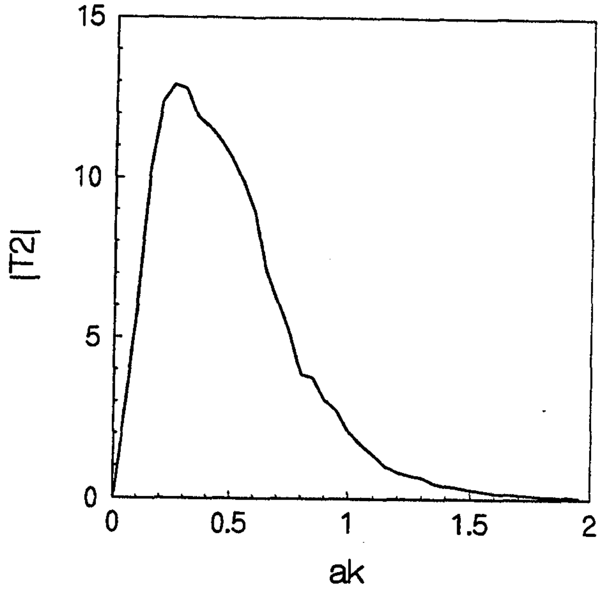

Fig. 2 Second-order transmission coefficient for surface barrier

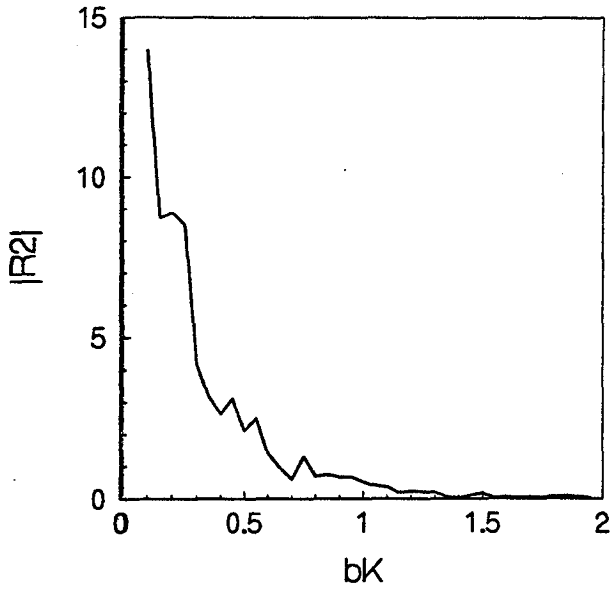

Fig. 3 Second-order reflection coefficient for submerged barrier

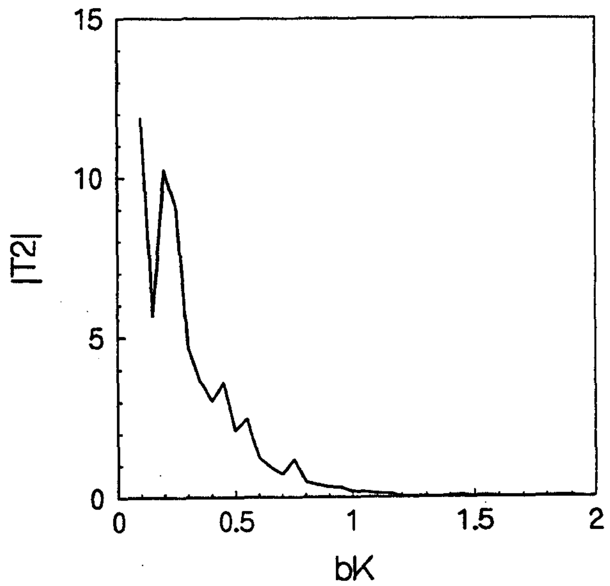

Fig. 4 Second-order transmission coefficient for submerged barrier